

**Optical Spectroscopy and Microscopy : Fundamentals of Optical Measurements
and Instrumentation**
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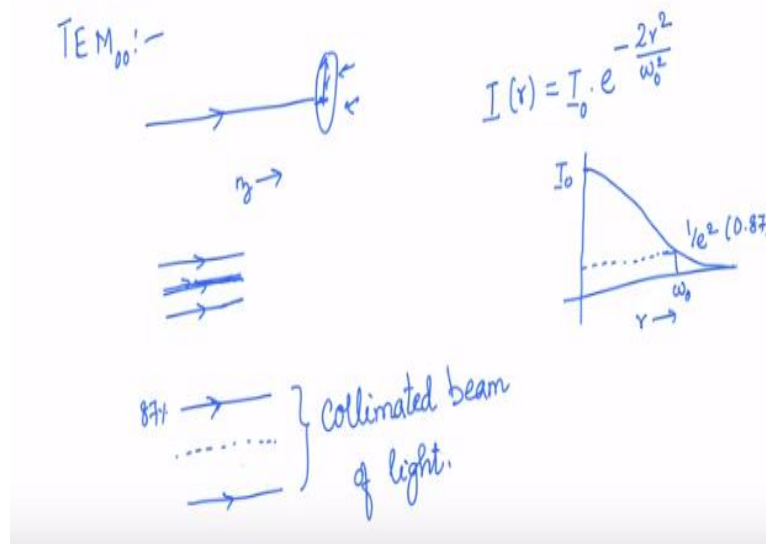
Lecture - 39

Hello and welcome to the lecture series on the Spectroscopy and Microscopy. So far what we have seen is different ways of generating the lights and different kinds of lights starting from incoherence light source to the CW lasers and the pulse lasers. So in the pulse lasers too we were talking about different ways of different ranges of pulses and then different ways of generating them.

Now what we are going to do is that we are going to look at the laser beam propagation itself in space and what how do we characterize them and this is very important because for us to understand how to manipulate these beams into different, as we look into different equipments, okay. So we need to be able to route them, reroute them and split them and whatnot.

And for us, for us to be able to do this and to be able to freely understand what is happening, we need to know some basic parameters or some basic description of this Gaussian beam propagation. And first of which is to understand that where Gaussian beam itself, right we have talked about it quite in depth in the course during the course why it has a particular shape and all that stuff.

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This is this I am talking about the transverse electromagnetic modes and the 00 mode can be quite nicely approximated as follows. So let us say the laser beam is traveling along this direction. So let us call that as our z direction. And the first thing we notice is that there is a cylindrical symmetry. So you will see that we represent it in kind of array. So the z direction is something like I mean along this axis.

So cylindrical symmetry meaning, the intensity if you measure and if nothing happens and if the beam is what is called as a collimated beam, then what we see is that the intensity that we measure from the above from the center distance r from the center, now that about which if you make a circle, they will all be having similar intensity okay. So what we are doing is that let me do it properly.

So if I pick a distance r from this central axis and then draw a circle. Now see the circle, it is a three dimensional picture. So what you have to see is that the circle will be really aligned. So you will not be able to see a circle truly when the way I am representing because it is traveling along the plane of the whiteboard. So only if you had to look from here you would see it as a circle.

And in that circle I am saying from the center I am taking an r distance. So let us not worry about this. So if you take that, then this distribution is uniform and I mean is identical. So you can express that as I the intensity at distance r as the intensity at r = 0 times e to the power minus 2r square divided by omega naught square okay. We will talk about this omega naught in a little while, okay.

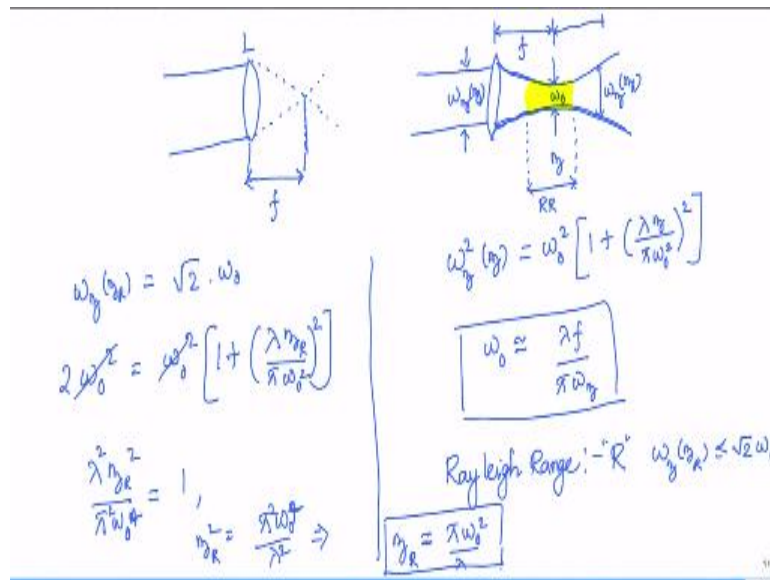
So and if you plot this it is a Gaussian. So we will start at I_0 and it drops down to I_0/e^2 okay, at $r = r_0$. So this is the r that we are doing. So that is about it is a $1/e^2$. So that is roughly about 87% 0.87. It will be 865 to be specific at r equal to r_0 , okay. So that is the characteristic of the beam when you take the cross section across it is a on a plane across its direction of the propagation.

So that is for a beam that is traveling with a uniform with all throughout this z . The r_0 , the width r_0 is uniform throughout the z . When do you think that the width will change? Width will change, so there are many ways of representing such a beam. So one is to just draw a one single line. Other thing is to say a series of lines each of them representing one ray.

And but then the density of them would be very high at the center and low and the density here represents the intensity. So now that or what we generally do is to represent it as two lines. These lines actually represent the boundaries of the beam, the boundaries here are the boundaries at which the intensity has fallen down by 87% of its central value.

So now if that boundary were to be parallel to each other, right which means the beam size is not changing across the space. Then we say such kind of light as collimated beam of light. Now or parallel beam of light. Now this collimated beam of light can be sent through various different optics.

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One of the main optics that we will be talking about is what happens to it when you actually start focusing, okay. Now from our high school optics, we know that if you have a geometry optics specifically, if you have a lens and then you send in a parallel beam of light or a or in this case collimated beam, so then they will come to focus after the lens after they travel through the positive lens.

Then the beams of the width of the beam progressively comes down and reaches to a minimum at a distance f from the lens and this f we call it as a focal length. So this is the lens and the plane that contains this we call it as a focal plane. That is perpendicular to the incident, the direction of propagation and then it contains this point we call it as the focal plane.

And the main deviation from that for a Gaussian beam is that the rays do not I mean for a geometric I mean for a reoptics they do converge to a point. However, in reality, particularly for a Gaussian beam, it does not converge into a point. Actually what it does is very unique. What it does is you take a beam of light and then focus it. It converges, it does minimize no doubt about it.

But it converges to a spot of with a definite extent. Now this spot, let us call it as omega naught. And we are going to measure our z from this distance. So and then the input or the size at the input of the lens, we can think of as omega z . So now what we can actually do is that see the z we can express or we can write down this the width of the behavior.

I mean we need to be able to write down the change in the width of the beam as we move across move along the z axis as it propagates through, right. So, of course there is a symmetry about the focal plane. So if you had to travel a distance, so this is happening at a distance f from the lens, center of the lens. So if you had to actually measure at a, measure the width at distance f from this focal plane, which is basically at 2f from the lens.

You will see the width is again your ωz , okay. Clearly that is a function of z, so we will mark it as that. So this can be written down in a very nice form as ω^2 . Specifically, it is a function of z, is given by ω^2 times $1 + \frac{\lambda z}{\pi \omega^2}$ whole square bracket ends.

So given an input beam size and the distance z from the focal point, you would be able to estimate you will and the focal point f you have to replace the z by the f to obtain what would be the spot size. And it is a we do this every now and then in the lab. So this equation can get a little untidy to calculate all that. So what we like to do is we like to actually I mean we like to get a easier or a simpler equation directly relating the ω to the input beam size.

And if you do that the ω we can actually do that, we can express this in as in terms of ω and then simplify it. So when you do that what will happen is that ω will be equal to $\frac{\lambda f}{\pi \omega z}$. In can be approximated to this. Now this is very handy and useful, okay.

Now if you watch carefully this $\frac{\lambda f}{\pi \omega z}$ is nothing but it is very much related to your the original uncertainty principal treatment where we got that $\sin \theta$, the numerical aperture equivalents, right. And then the spot size you will see that it is pretty equivalent to that. Just let us wait for a little bit and then we will see that it is this is nothing but exactly that.

But apart from ω the spot size at the focus there is also another quantity that is of very practical value, which is if you actually look at this the rate at which the size is changing that the beam size is changing is not uniform, okay. It is rapidly

going down initially followed by decrease in the rate at which it is going down almost around the focus the beam size hardly changes.

So now this region about which the omega naught okay, the omega naught does not change, okay. So I am talking about this region where the omega naught does not change in its extent. We would like to know is called as is an important region because that would in turn tell you the spatial restriction in the z dimension, okay.

Just away omega naught tells you the spatial restriction in the radial direction in the Z dimension the spatial restriction is given by this yellow shaded region that we have looked at. And it is more formally defined as Rayleigh range. It is defined as the distance R of z at which the it is not R of F actually, so the distance R, let us call it as distance R, the distance R at which the omega z, okay.

Let us call that as z R, okay is equal to R lesser than root 2 omega naught, okay. What we are saying here is that we will call this distance, it happens in both the directions. So this distance at which okay, so at which the beam size has gone up only by about 40% which is the root 2 times right 1.414. So only by about 40%.

That distance we will take it as the distance where the beam size has not changed and that we will denote that as a Rayleigh range. And you can we can actually easily estimate that from this expression what we are actually trying to do is to say that my omega z square or omega z, I am going to set it as the omega z R, right? That is what we are I am trying to estimate and I am going to do that I want to know the z R.

So omega z, z R is basically root 2 omega naught. So if I substitute that into this expression, what we have is two times omega naught square equals omega naught square 1 plus lambda. That z is basically my z R divided by pi omega naught square whole square end the brackets and that goes off. So one could now solve for z R. So this would be lambda square z r square by pi omega naught square equal to 1 by omega naught square. So from then twice of that should be your Rayleigh range.

And oh, this is omega naught to the power 4, right. It is square, so it should be omega naught to the power 4. So that would imply, so that would imply z R equals pi omega

naught square by lambda and then the range itself would be twice of that okay. So and then we can substitute the omega naught in terms of the lambda f by pi omega z to get the whole expression for the z R in terms of the input beam size and the focal length of the lens that we use and the wavelength of the light that we are actually using.

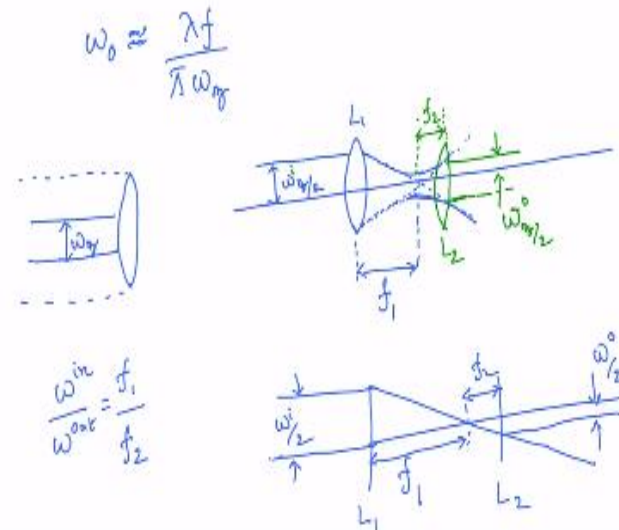
So these two are very important quantity. One, omega naught; two z R the Rayleigh range simply because these two determine the our ability or quantify the our ability to restrict the light in space, okay. And for a Gaussian beam which turns out to be the case for or good approximation for the output of the beam from the any of these lasers is true.

So these properties given when you put the, I mean the light through the lens, then we need to know what happens to that light. And we describe that by these two parameters. And if you are given these two, then you know all that you need to know about that thing under the assumption we are talking only about the TEM 00 mode approximated to Gaussian, alright.

What we will do now is so what we will do now is that so that these two describe the basic properties of the beam. Now I am going to use I am going to demonstrate a little device, a telescopic device to change the beam parameters of that we have described right now. So typically, when you want to have a very nice defined focus, as you can see from this expression for omega naught.

So as you can see from this omega naught expression, we need to have few things. One, I am going to rewrite this expression back lambda f by pi omega z.

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For a given lens and the light right given lens and the light the f and λ is fixed. So only thing that you can change is the ωf , which is the input beam size. So if you were to actually want to generate the sharp, the smallest beam possible, then you want to have as wide a beam as possible that is coming into the lens.

So let us take a lens and as we keep increasing this size what this equation predicts is that you will get smaller and smaller ω naught. So clearly the largest size that you could have is as big as that of the lens. And anything beyond that you are actually throwing away the light or not utilizing it. And at that boundary, the focus is limited by diffraction.

So what you call it as the I mean, if you have a very good focal length and stuff like that, we call it as a diffraction limited spot. But of the given lens which means given f and λ we can predict what would be the spot size. And so which means you need to be often able to freely change the size of the beam. The way you do that is through this device that I was talking to you about, is the telescope.

And it is nothing but a two lens system and okay I represent the central axis, central optical axis goes through while beam with incoming width of ωz , right would be represented. One of the rays the top rays we can represent it like this. So the property of this is to say that hey look in the geometric optics, it would have passed through something like this.

But in real optics I mean the Gaussian optics is going to go through like that. And we can think of this taking a shape. I am not drawing the entire curve, but now it can focus and then start expanding out. Now depending on where you keep your second lens right? So this is our lens 1 and the minimum spot size is happening at the focal length f_1 .

Now depending on where you place your second lens either close or further, you will end up getting I mean cutting off this beam at different sizes. So let us say if you had to keep up a second lens somewhere around here. So is it green here? Now and if the second lens, the second lens is focal length so to speak is matched to this distance.

If we keep the second lens and then the second lens' focal length is also matched, then what is going to happen is that this lens is going to collimate this beam out like this, such that your output would be ωz_0 , output width would be something like that. This is very similar to our the telescope in the geometric optics, right. Except, we are here talking about a Gaussian beam focusing beam is a Lorentzian profile.

But, the dotted lines so just to make sure that we follow we can follow it easily I am actually representing these straight lines, right? The straight dotted lines represent the geometric optics or what is happening at the focus, right? So then if you look at it, it is nothing but two triangles, right? So that is your triangle number 1 formed by the lens L_1 with $\omega z_0/2$ input that continuing and then you are interjecting here.

So this beam width, this beam width here we are talking about this $\omega z_0/2$, alright? So I am just going to draw this triangles properly so that it becomes easier to understand sorry. This is lens L_2 , then this is L_1 and this distance is focal length f_1 and this distance is focal length f_2 and this is your width by 2, half width of the output beam and this is half width of the input beam.

So from this geometry you can actually see that the ω_{in} by ω_{out} is basically the ratio of this side to this is f_1 by f_2 . So by choosing these the focal length of these lenses, the various focal length of these lenses, we could either constrict the beam or expand the beam. And these kind of operations are very useful for generating light that is of that is of a, that is spatially restricted in nature.

Which we use for various imaging and probing systems in a localized manner, spatially localized manner. What we will do is in the next class next lecture, we would focus on using this and setting up I mean, how do we I mean practically go about doing this in a lab, in a lab setting, okay. And for that, we need to understand a few tips and tricks. And then once we understand and then see it and demonstrate ourselves that this light is actually expanding, then we would move on to the next few things.

So in order to go into that practical mode, we would need a little bit of practical I mean introduction to the practical aspects of the lasers themselves, how do you measure the beam and how do we actually make the beam go through a particular path and so on. That we would see briefly in the next lecture, following which we will go to the demonstration, okay. I will see you the next class.