

**Optical Spectroscopy and Microscopy : Fundamentals of Optical Measurements
and Instrumentation**
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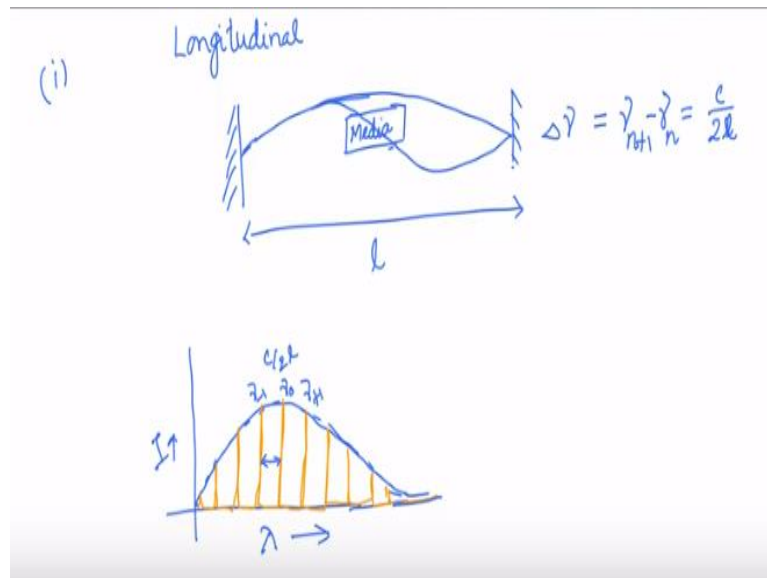
Lecture - 35

Hello and welcome to the lecture series on the Optical Spectroscopy and Microscopy. So far we have been talking about different ways of generating pulse laser output specifically in the nanosecond domain. And while they the nanosecond domain pulses can provide with very high energy we would be focusing in this lecture on the following lectures on a different kind or a different class of lasers.

That can generate femtosecond picoseconds to femtosecond lasers, laser pulses, thanks to a methodology called mode locking. And we will talk about this mode lock, the principles behind the mode lock. Again a little bit in a semi quantitative way. The idea of the motivation here is to be able to understand this device and the working principle so that eventually we are going to take you all to a laboratory setting.

And then show you laser and we will be showing you a mode lock laser and its insights and how it is operating. And these are the lasers that are typically used and it comes to state of the art microscopic techniques such as the multiphoton imaging and so forth. They are also used heavily to follow the ultrafast dynamics reaction dynamics or for that matter any dynamics that requires short pulses, okay. Now looking into the mode locked lasers themselves.

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Let us look at the laser cavity once more. We have talked about laser cavities and the modes that can exist in a laser cavity. We talked about a transverse electromagnetic mode determining the profile beam profile properties of the laser. And we also talked about what are called as the longitudinal modes of this of a given cavity. We said that not every single frequency or a wavelength that is emitted by this media can be supported by this cavity.

Because we need the condition that the end mirrors are nodes or where the reflection happening. So the wavelengths that can be supported by the cavities need to be obeying a condition given by this constraint where the length of the cavity determines the modes that it can be sustained. We said the mode separation or the delta nu of different ends is given by basically this $\mu n + 1$ minus μn .

So we can actually it is no need to write n. So this basically $\mu n + 1$ minus μn , $n + 1$ th mode and the n th mode has to be equal to that of C by $2l$. Now we also saw that given the fluorescence profile of the media, so we typically plot in the λ axis but essentially some energy axis. So if you look at that, then the profile that is that the wavelengths that are allowed take the form something like this, okay.

Now the spacing between these two or any two for that matter is C by $2l$. Now the fact that there are these different frequencies, right frequencies or the modes of operations of the laser. The laser can actually have a frequency that corresponds to

this central wavelength μ_0 or μ_{-1} or μ_{+1} . The difference between the frequencies is in the order of C by $2l$. I mean it is exactly C by $2l$.

So the fact that any of these frequencies can operate of course, there is an intensity distribution. So accordingly, you can think of these laser emitting frequencies of different lights of different frequencies with different intensities all operating and then the intensity is fluctuating on their own timescales. Now this notion of multiple frequencies that can simultaneously exist inside a cavity.

And all of them can get amplified though some, some frequencies maybe getting more amplified than the other gives rise to the possibility that if we were to sync all of these lights operating on slightly different frequencies in a I mean their phases, sync the their phases, then you can think of constructing an superposition of these waves or a pulse train whose envelope the outer envelope is determined by the phase locking of this superposition of the phase locked frequencies.

And the possibility of generating a narrower pulse, okay. Now it is not a given that you would end up having a narrower pulse. But then the motivation comes again from the original uncertainty principle analysis that we have done during the start of the course, where we have said if we want to shrink the pulse shrink the light to a shorter pulse, one thing that we need to do is to expand the intensity or the spectrum in the frequency domain, right.

We need to have more, we need to increase the bandwidth of the laser and clearly the bandwidth of the laser is very limited. However, what you can think of here is an alternative wherein you have a laser cavity within which simultaneously these different frequencies are operating, okay. So you can think of in some ways multiple super fine monochromatic lasers all operating in synchrony, okay.

This synchronous operation, right this operation of locking all of this lights in phase is called as mode locking. Now how do we go about analyzing this mode locked lasers and where does this how do we actually motivate to bring in this bandwidth and the time pulse duration into the picture, we will see in detail.

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Intensity = $|E_0|^2 \cos^2(\omega t - kx)$

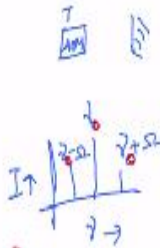
$T = T_0 [1 - \delta(1 - \cos \Omega t)]$
freq. of operation of AOM

$\Rightarrow T_0 [1 - 2\delta \sin^2(\frac{\Omega}{2} t)]$... ①

$A_y(t) = T \cdot E_0 \cos \omega_0 t =$

$= T_0 A_{y0} [(1 - \delta) \cos \omega_0 t + \frac{1}{2} \delta (\cos(\omega_0 + \Omega)t + \cos(\omega_0 - \Omega)t)]$

$\Omega = c/\lambda$



First thing is to realize that the intensity of given monochromatic view, you can write it as intensity can be written as amplitude square cos square omega t minus k x, alright. So now what since there are many frequencies that are simultaneously operating. So now what we are going to do is that we are going to write down way we are going to write down a way of phase syncing all of this oscillations.

And the one way to actually do that is to think of an acousto-optic modulator present inside the cavity and that acousto-optic modulator okay modulating the transmission of the cavity. The idea here is we are going to look at the if the transmission is modulated at a certain frequency, now this allows you to actually have all these frequencies all these different frequency start at a given point in time, okay.

Once they start depending on this different wavelengths, they will acquire different phases, but the point is when they come back all since, they are the supported modes of the cavity, they will all be at the same phase because this is where they started. This is like the $t = 0$ of that the wave time. So if that being the case the acousto-optic modulator frequency would definitely interact with the superposition of this wave.

So that being the motivation we can start to think of writing the transmission of this cavity modulated by the AOM as follows. It is a function of time and when you do that the transmission oh, by the way, this analysis and many of this pulsed laser analysis please refer to the book in the reference Laser Spectroscopy from the by

Demtroder. And we can write down the transmission as a time moderator transmission.

T_0 being the amplitude of this modulation and again $1 - \cos \omega t$ as follows. The idea here is the frequency of operation of my AOM is given by that is my frequency of operation of AOM, okay. So now what we are doing is that the light the disk the transmission that is seen by the light is given by $1 - \cos \omega t$, which is essentially the, it is basically the loss that is incurred during this, this is the loss term.

And $1 - \cos \omega t$ is going to be my the transmission term. I am multiplying by the general transmission coefficient T_0 itself which is at $T = 0$. Then whatever the transmission by the AOM because of the presence of the AOM that is what you are writing it here. When you do this, then this whole thing can be actually simplified into or rewritten as $T_0 (1 - \cos \omega t)$.

So where we are going with this is you will see when any such super imposition of modulating transmission is given to the lasing cavity. The laser not only supports the oscillation of the fundamental cavity, but also the oscillation of it also allows the modes that are next to each other, right. One mode I mean the mode that is just before and the mode that is just after.

If I show that then what we can extrapolate from there is that we can look at the frequency comb here and other λ comb. So just the way when I am actually tuned to the center wavelength μ_0 it not only supports the μ_0 operation, but it also supports $\mu_0 - 1$ and $\mu_0 + 1$. Similarly, we can actually walk through this entire comb by when you are talking about supporting the $\mu_0 + 1$, you also support the $\mu_0 - 2$.

I mean $\mu_0 - 1$ if we are talking about it then $\mu_0 - 2$ and $\mu_0 + 2$, so on and so forth. Thereby you, what you end up getting is a nice little pulse train of all where all the frequencies are operating in sync, okay. So I am just going to show that and we are going to show that the AOM transmission when you are operating like that.

The resultant amplitude will have frequency components not just only from the mode that we are actually considering but the same mode the K , let us say the μ central frequency will be seen by the cavity as also as $\mu - 1$ as well as $\mu + 1$ because of the frequency modulation from AOM, okay. So the transmission can be written down like this.

Now what we are going to do is that we are going to say okay hey look, now this is the time profile of the transmission. So the net amplitude if I can think of, of mode μ , okay as a function of time is basically this transmission right times the amplitude that we have actually written earlier, right? So that is basically T_0 times okay now I think E of μ_0 , 1 minus oh sorry, I can write it down. I am sorry.

We can write down this expression as essentially T times the amplitude function $E \mu \cos$ this is the central frequency, okay. And now clearly this can be simplified and by including the transmission that we have expressed in this equation if you use this transmission here and simplify, you can show that the amplitude does not only have its own original μ_0 .

But in fact what you will see is that you will end up having $\cos \omega t + \frac{1}{2} \delta \cos(\omega \mu + \omega t) + \cos(\omega \mu - \omega t)$. So what you see is that the amplitude not only has $\omega \mu$ of t , but it also has some frequency and the different frequencies right $\omega \mu + \omega$ the capital ω the AOM operating frequency $\omega \mu - \omega$ the capital ω .

Again the AOM operating frequency or in other words if I had to mark the central frequency μ here okay and then we are plotting the intensity or the amplitude of the light wave. So you have the central frequency μ , this is the frequency. So let us call that as μ_c . You do not only have μ_c , but you also have $\mu_c + \omega$ the capital ω and $\mu_c - \omega$ the capital ω . It is very characteristics of an interference pattern, the beat frequencies we know.

So in the beat frequency you not only have the carrier central carrier frequency but also you have an envelope which is the difference as well as the sum right. So basically we are generating that. As a result it allows us to be able to operate this I

mean, the moment you realize this, it gives us a very nice opportunity for us to operate sorry this is a correction here. So this μ_c minus capital ω .

So this is a very nice opportunity wherein you can match this ω to the C by $2l$ right the next mode in this frequency in this domain, right. So if we were to match this ω to C by $2l$ so then this whole thing becomes, even though we are we are trying to modulate the frequency of μ_0 , that modulation supports not only this μ_0 , but also μ_1 and μ_{+1} . That is the interpretation right?

So you have your $\mu_0 \cos$ of ωt , there is no issue there right? You have μ_0 oscillating nicely. And you also in addition to that you have this and this term. And if I were to set this ω to C by $2l$, then this whole frequency would be my central frequency minus 1, we call it as, let us say, μ_{-1} , and this is sorry μ_{+1} , and this is μ_{-1} . It is because it is plus ω and minus ω .

The central frequency here is basically μ_0 , alright this is my μ_0 . So in effect, by matching this to C by $2l$, you have what we have achieved is we have brought in more modes, more frequencies to operate in sync. So we can actually go ahead and walk through each and every peak starting from μ_0 to μ_1 μ_2 μ_3 in one side and μ_{-1} and so forth.

When you do that, what you will see is that the both since it is supporting both the sides like the minus one side and the plus one side, each one of them gets enhanced I mean in amplitude as supported by that of the as much as it can be supported by the gain bandwidth. As a result, all of them by operating this AOM at a frequency given by C by $2l$, you are bringing all of them in sync.

Which is intuitively if you think about it all that you are saying is that okay, I am going to put in a switch and then operate at a frequency, which corresponds to the frequency or the time period that the central frequency takes it to come back for it in one round trip. If when you do that, because of the modulation it imposes, it kind of sinks not only that central frequency, but a slower traveling slightly longer wavelength light or the mode also gets synced.

All of them gets synced at that point in space. And as a result, the entire cavity starts to operate in a phase locked manner. When you do the phase locked operation, the immediate advantage that you see is so the immediate operation that you see is that constructive superposition of all these wavelengths resulting in a constricted time profile for the output laser light.

And we will see in the next class how do we actually obtain this in a mathematical form and what is the relationship between the frequency content of this superposition and the pulse duration of the light pulse that we have generated? I will see you in the next class. Thank you.