Optical Spectroscopy and Microscopy

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Lecture – 2
Fundamentals of Optical Measurement and Instrumentation

Hello and welcome to the course on optical spectroscopy and microscopy, and so far, we have been looking at how well we can localize the photons, particularly we were talking about the relationship between the quantum mechanical principles, particularly the uncertainty principle and how it manifests in a routine experiment that is done in the lab or in a routine instrument that is operated day in and day out by everybody.

For example, in this specific case, we are talking about a microscope and in a microscope we are trying to see how well we can separate two objects that are closely spaced. Let us say we said that we went ahead and defined what is the resolution limit in this particular case.

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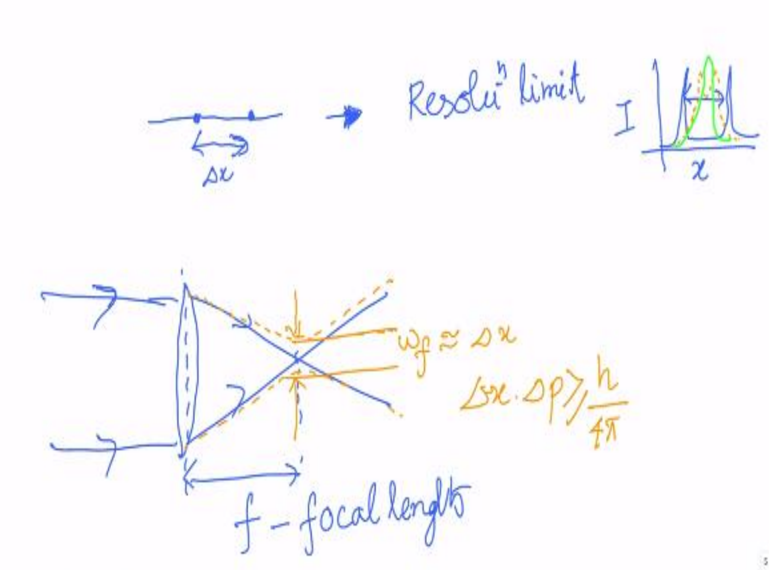

The resolution limit is defined as the closest the two objects can be and still we are able to say these 2 objects are 2 distinct objects, or in other word, let us say if you were to plot out if you have to take 2 points in space, alright, point A and point B and then if I were to image this right, if I have capture an image of this points in space, what I would do is that I would be generating equivalent points in my images and what I am going to do is an experiment wherein I am going to reduce this distance, let us call this distance as delta x.

How small a distance, I mean what is the smallest distance these 2 points can be and still in my image I am seeing it as 2 distinct points. Now that is called as a resolution limit and I said we will talk about this in more detail and in a much more elaborate manner later in the course, but then I said this is given by conventional means as resolution limit or where we can actually plot out the intensity, right. We can actually draw a small line profile across this image and then say I am going to plot the intensity as a function of space.

So if I do that then what I will see is this x and then the intensity here you will see there will be a spike, each spike corresponding to the position of the dots here. Now, we as we bring this closer and closer, so let us I think I was taking an orange line there, what is going to happen is this is going to come very close, at some point it is going to look like that before actually merging into one single line.

The resolution limit, thanks to Abbe, is defined as so if you plot out, if you change the x and then if you plot out the intensity, then resolution limit is defined as distance by which the intensity has fallen down by 1 upon e okay. So now this distance yeah we would call it as a resolution limit and so we said that this resolution limit is of the order of the wavelength right, that is what we have been seeing there right. The resolution limit is of the order of wavelength of the light.

So now today, what we are going to do is we are going to continue that formulation and then see how we can actually obtain this a relationship from the fundamental principles of quantum mechanics. The principle that we are talking about is the principle of uncertainty that is connecting, that is linking the uncertainty between the position and measurement of the position and the momentum okay of any particle. So in this particular case, what we are trying to do is we are trying to localize a photon.

I said okay now this resolution limit is tightly linked to the idea how well we can focus the light in space or how well we can localize the light photon in space. One of the ways we can localize the light photon is by focusing through a lens and we were actually constructing lens and then drawing the ray diagrams to say how to estimate the size of the beam, our size of the focus spot, alright. So if you remember, so will reconstruct the lens here, when reconstructing the lens what we see is high school physics tells us that the rays that are traveling incoming rays converge at a point which is a f from the lens.

They are converging to a point at a distance f where f is termed as the focal length of the lens. Now what I said is that while that is true in paper, but in reality what you always see is that no matter how so ever good lens that you take, it never is a point, I mean there is a finite extent, point would mean that there is no width to that spot at all, that is not true, there is a finite extent to which this light rays converge before they start to diverge after the focal point. So the right picture would be to actually draw it out something like this and what we are trying to measure is how small a spot or the width wf can be okay.

So if I get it smaller and smaller, I can localize the photon to a smaller and smaller spot. So this wf to me is equivalent to delta x, good. So if I need to know how small a delta x I can which means I want to minimize my inaccuracies in the photon’s localization. I need to maximize my uncertainty on the momentum side, right, because our delta x times delta p is greater than or equivalent to h by 4 pi.

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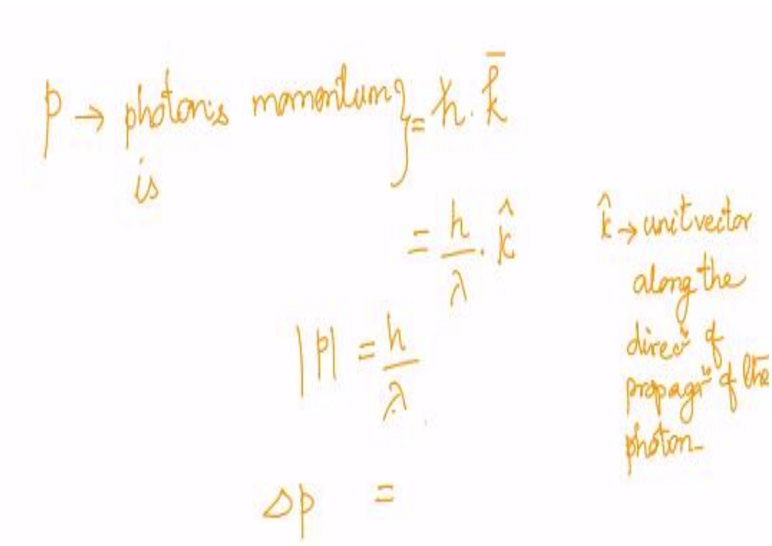

So now what we do is that we write down what the momentum of the photon itself is. So the P the momentum of the photon, photon/s momentum is given by h cross times k vector or kr or in a simple term we can write it as h by lambda times k dash okay, sorry no I am going to use a different notation here so that we do not get confused. So we can write it as kappa, we can write it as kappa cap where kappa cap is the unit vector along the direction of propagation of the photon, right.

So normally you would be more familiar with writing the photons momentum, right. We write as h by lambda, there we are actually talking about this, that is the modulus of P which is having only the magnitude, right. We are used to writing it as h by lambda, but for this discussion, I need to pay attention to the direction, so what I am going to do is I am not going to write the modulus of P, but actually I am writing the P, the vector, the momentum vector itself.

For that I need to include the direction and that direction is coming through the kappa okay, kappa cap the unit vector pointing towards the direction of the propagation of the photon itself. Now given this, then how do you write the uncertainty associated with the photons momentum? So there are 2 things to think about, one is h is the Planck's constant, so that is a constant, so there is nothing uncertain about it, but there is lambda. It is very tempting to think that the uncertainty in the momentum could come from the uncertainty in the lambda itself, but let us take a step back and then see what it means.

What it means is that the impinging light into the lens has multiple color, each lambda correspond to different colors. So you can think of that is a spread in colors of different light and then we are trying to focus, as a result you are having this spread in the localization of the photon per se. However, that runs into a big problem there. The problem being that I could in principle provide you a monochromatic light, a light with one single pure wavelength to very great accuracy okay.

Still we would see that there is a definite amount of extent to which you can actually focus and not any below than that, which means the uncertainty has to come from somewhere else, it is coming in reality, so that uncertainty if you watch carefully, one of the other parameter that could change is this key, the Kappa vector, the unit vector that is directed along the direction of propagation of the photon itself. Where is the uncertainty in that?

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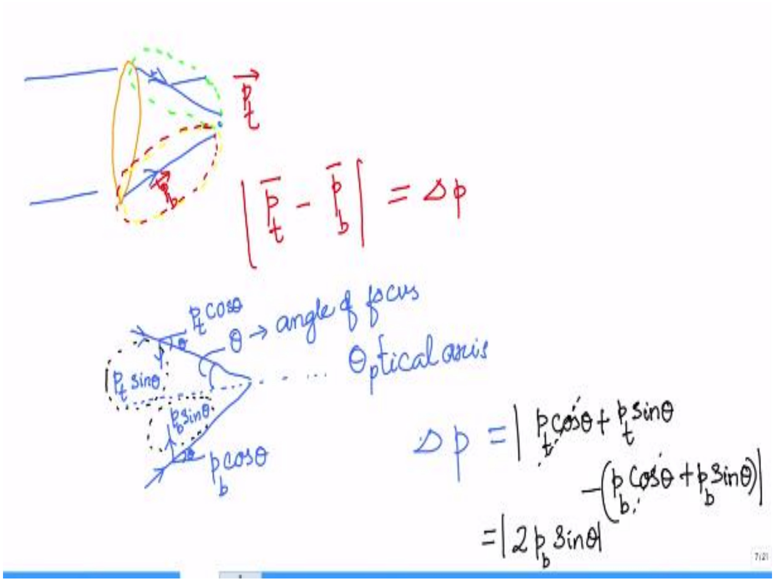

The uncertainty if you look at the ray diagram of the light that is emerging out from the lens, it would become much more clear. So, we have a lens and the light ray, I am going to go into the geometric optics where it is point it is easier to draw and propagate. So there are 2 two different directions the light rays converge onto this point, one from the top we will call that a top and the other from the bottom. So a photon that you find it here could in principal could have come from a path that is indicated by the light ray on the top.

So it could have come from a light ray that is on the top indicated by this ray or could have come from a light that originated from the bottom alright, the yellow is not very visible, so let us take the red okay. So we would not be able to tell where it came from, or in for that matter, it could have come from any line, any light ray that is in between these two okay. So what we are going to say is that there uncertainty regarding the light ray or the direction of propagation of the light because of which the photon is localized in this place.

So we need to consider the momentum, the direction of the momentum vector, particularly the K vector Kappa vector that is along this to this. So, now we are going to take two extreme cases right. The maximum uncertainty is between the top to the bottom ray, anything else is lesser than that. So our idea here is to maximize uncertainty in the momentum so that we can minimize, we can get the minimum limit associated with the delta x uncertainty with the position.

So to do that, what we are going to do is we are going to resolve this momentum, right, the momentum of the top vector we will call that as Pt, so that is the top vector and the bottom one we will call that as Pb. See ideally I mean these two are in the magnitude wise exactly same, the only difference is in the directions, right, because they are the same photons and the focus by the same light, I mean same lens. So now what we need to do is we need to figure out what is happening at this place.

We have to take the difference between Pt minus Pb and that would be our, this difference would be our delta P. Now how do we do this? We realize that it can be simplified nicely, moment we recognize that this component, right, the component on the top ray and on the bottom ray can be resolved into its components, this vector can be resolved into these components okay. So I am going to redraw this diagram for clarity.

So what we are looking at is this triangle, that is my optical axis which is nothing but a line that connects the centre of the lens with the focal point about which all the rays converge on to it, so and then we have an angle theta and it is called as the angle of focus alright. So now what I am trying to say is that I am going to resolve this momentum vectors in 2 components. So since this is two, this is theta, this is going to be theta and this component that is parallel to the optical axis would be Pt cos theta and this would be Pt sin theta.

Similarly we could resolve the light ray that is traveling from bottom towards the focus in 2 components, right. So there is a component that is in the upward direction because it is converging up so and the parallel component, this is our theta. So, now this would be my Pb cos theta and the other component would be Pb sin theta. Using this we can actually substitute in here, when you calculate the delta P, what you will realize is that the component Pb sin theta and Pt sin theta, cos terms will cancel out, how does it work.

So now, let us see. If you write down this expression delta P which is modulus of the Pt which is Pt cos theta + Pt sin theta minus Pb cos theta + Pb sin theta. Now if you look at the diagram carefully, you will realize that Pb sin theta and Pt sin theta are in opposite direction. So if you take the negative of that, so what would happen is that Pt cos theta canceling out with Pb cos theta because of the negative sign. However Pt sin theta minus Pb sin theta, the term that is left out, you will realize is they are pointing in opposite direction.

So as a result what you will get is 2Pb sin theta, they will not cancel, they would actually add up because it is sin of minus theta, so it is minus sin theta, so minus minus plus, so that will you will get the component as delta P equal to two 2Pb sin theta.

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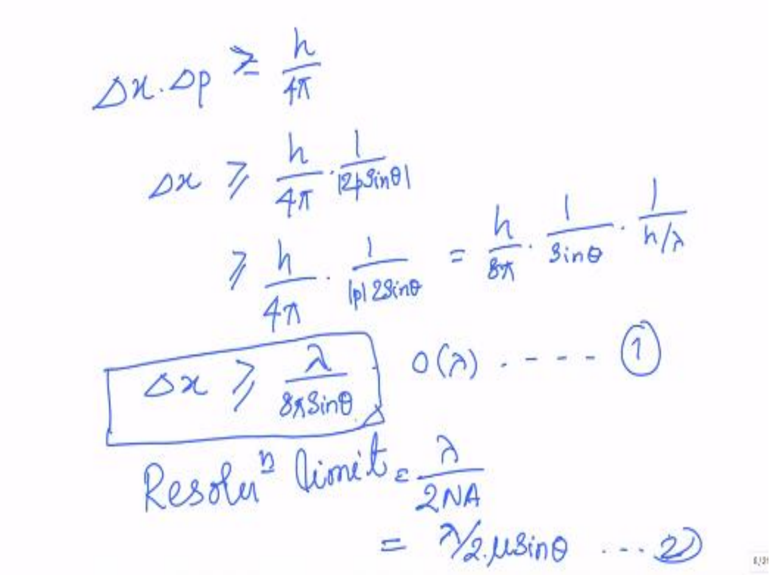

Now that is interesting because we can take this and then substitute in our expression connecting in the uncertainty principle. So what we have is delta x times delta P equals h by 4 pi, not equals but it is greater than or equal to h by 4 pi, and we have an expression for delta P, the expression is two modulus 2P sin theta right. So delta x will be greater than h by 4 pi times 1 divided by 2P, momentum of the photon Pb or Pt does not matter but both of them are equal, sin theta and this is nothing but greater than or equal to h by 4 pi 1 divided by modulus P2 sin theta, which can, this whole this term okay, can be written as h by 8 pi.

Now we know that the momentum P for a photon is h by lambda right. So if you substitute that what we will have is a sin theta term unhindered divided by the modulus P if you write it, it will be h by lambda. So all of this would imply or can be simplified as will boil down to lambda, the one the denominator goes up, divided by 8 pi sin theta. The first thing we realize is this is of the order of lambda, not just that, if you actually look at the expression, expression for the resolution limit.

I intentionally did not write down that expression before, but now we can go back and look at the textbooks or any optical literature sites talks about diffraction limit, you would come across the resolution limit to be resolution limit would be given as lambda over 2 times the numerical aperture okay. So what is numerical aperture, numerical aperture in turn would be given as lambda divided by 2 into mu sin theta. Now compare okay, this expression, you will see there is a striking similarity right.

The uncertainty principle tells you that the delta x can only be greater than this, right, and then your resolution limit well what people have gotten is this. The only difference that if you take the ratio between these two, the ratio would be actually 8 pi by 2 mu factor okay. We predict the resolution limit to be lower from quantum mechanics than what has been told, but that is only a difference in the factor, the functional form, the dependence on the lambda, the dependence on the sin theta that, the theta is the angle of focus is very very striking.

This notwithstanding if you are still wondering why and how this quantum mechanics are treatment of the matter using quantum mechanics is relevant to studying spectroscopy and microscopy, I am going to give you one more, I mean one more striking example and then show you that it is really important to pay attention to this fact that we need to take an approach of understanding or using quantum mechanics to understand how light interacts with the matter. The second one again makes use of uncertainty principle.

This starts with a slightly modified version of an experiment, I am going to call that as thought experiment, I am going to propose it as a thought experiment and then I am going to introduce a different form of an uncertainty principle and then show you how that can be useful in predicting the results there. So in summary, what I have told you is you can use uncertainty principle to arrive at an expression for the resolution limit or the localization limit of a photon which is very strikingly similar to a conventional expression that is thrown out of a kind of nowhere.

I mean you can derive it from the first principles of diffraction, but it is a lot more complicated and involved while this is extremely straightforward and comes out naturally okay, and I am now going to go into the second experiment and we will describe the experiment before actually going into the details of writing down this expression and showing how it is really predicted by quantum mechanics. The experiment is as follows.

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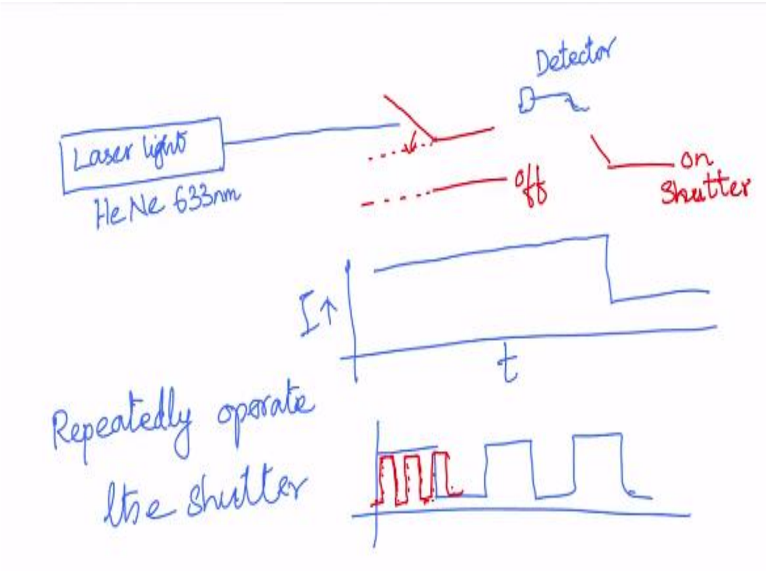

Let us take a laser light source okay. So, this could be a very ubiquitous laser such as helium-neon laser operating with a central wavelength of 633 nanometers wavelength, right. It is a red light that read HeNe that you use to do several things in the lab. So now the light beam is coming out of the laser. What I am going to do is I am going to intervene this light beam with shutter, a mechanical shutter or a light chopper whichever you feel like right. So, I am going to represent the shutter as something like this.

So when it is closed, no light, and when it is open, so okay let us represent the shutter with a different color of the pen, so red. So when the shutter is on, no light, but you can have a different configuration where the shutter is off okay, so you close down the shutter, so in which case let us represent right this is off state and this is on, off the shutter. These are the states of the shutter and what you will see is that I am going to have a small detector here, a small device that tells you whether the light is present or not okay.

If I plot the detector output, whenever there is off state, I will have the light coming through and when the shutter is on, I will have no light coming through okay, so intensity of the light as a function of time. Now, I can keep doing this off and on, off and on, right. So, then what I am going to get? So, if I repeatedly operate the shutter, what is going to happen is that the shutter is off, light is on, shutter is on, no light, again as and when you keep opening and closing, you will get the light off and on and closing on.

So now, I start to increase the speed at which I am actually closing and opening the shutter, what you would expect is that it will become shorter, the duration will become shorter if assuming that I am uniformly closing it off and on. Like that, one can conceptually ask a question how fast can I actually keep going, is there a limit to the speed at which actually I can operate? Given that it is a mechanical shutter, you can see that okay there is a limitation in terms of the instrument that you can construct and then there may be a limit.

However, conceptually if we think of that, there is no limit, you can actually operate at any speed that you would like, right, because it is like a mass less shutter and you would be able to really really go very high speed, but the question that you would like to ask is how does the detector output look like when we go actually at a very very very high speed? Would I keep getting this light all the time? It turns out that answer is a little bit a little bit more complicated.

However, what is I mean grossly true is that at some limit or some speed beyond which you will not get any light output at all despite the fact you are opening the shutter, well that is very surprising, right? You keep, you are opening and closing, only thing that you are doing is you are increasing the speed, at one speed, you start getting any light out from the device. The fast operating shutter starts to act as if it is a stop after laser light.

There are quite a few qualifiers to this statement, but that is roughly the on an average behavior that you would observe, why is that? What is happening and how is it relevant to the observations of the experiments that we are going to do and study in the course? We will do that in the next lecture and what I am going to do is I am going to make use of a different form of an uncertainty principle and tell you this is exactly what you would expect.

In fact, I would even say when you would expect this. I would even be able to predict at what speed you will be able to expect this and what governs this, alright. With that, I will hope to see you in the next lecture. Bye.