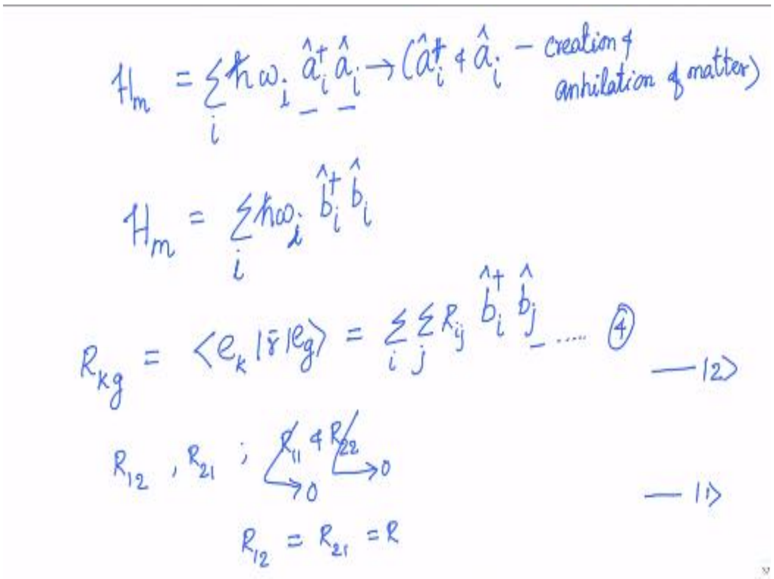
**Optical Spectroscopy and Microscopy**
**Prof. Balaji Jayaprakash**
**Centre for Neuroscience**
**Indian Institute of Science – Bangalore**

**Lecture – 18**
**Fundamentals of Optical Measurements and Instrumentation**

Hello and welcome to the lecture series on optical spectroscopy and microscopy. So far what we have seen is that how do we actually treat the light matter interaction in terms of purely quantemic, I mean both the light as well as the matter in terms of accounting for quantized nature of both of them. So there are initial few steps that we had to dwell on, one is to talk about introducing the notion of Fock states or the number entity states and then operators for creation and annihilation operators called creation and annihilation operators and their usefulness we dwelt upon a little bit.

Then we said we could expand a given operator, how do we do that in some eigen space. So if it turns out that I had written down if you had actually if you can expand in its own space, then we could write the operator, let us say the Hamiltonian of the matter and in terms of its eigenkets or an equivalent of that here in terms of the creation and annihilation operator.

**(Refer Slide Time: 02:13)**




So what we did was we said we could write this in terms of H cross summation on i omega i. So what I had done was that in the last class, I had just to keep the uniformity so that you can actually relate to what we have done it before. I had used a dagger a and a cap of i for representing the where in this place here right. This represents the creation and annihilation operators of the matter okay. So cap i are the annihilation operator of the matter. Since we will be using this in conjunction with the creation and annihilation operators of the radiation itself, it is good to make the distinction.

So I am going to change my matter operators, so I am going to call them as b okay. So it is just arbitrary name. So if I do that, then we have the creation operator corresponding to the matter we call it as bi and annihilation operator corresponding to matter we call it as bi is that we can take this expansion of the operator and plug it in to the expression that I have stated without actually proving how it came by which is this expression, expression number 1 of the previous class describing the interaction Hamiltonian in terms of ek and ek dagger of the radiation mode and ek r eg.

So if I do that, the first thing why we came here is to be able to write this element right Rkg, Rkg is our this matrix element. So remember, so Rkg we would like to write it, the actual expression for that is ek  r eg. So, now this operator the R operator we would like to write in terms of the creation and annihilation operator in a similar procedure, it is not ditto the same but I mean pretty much the same except for slight differences here and there.

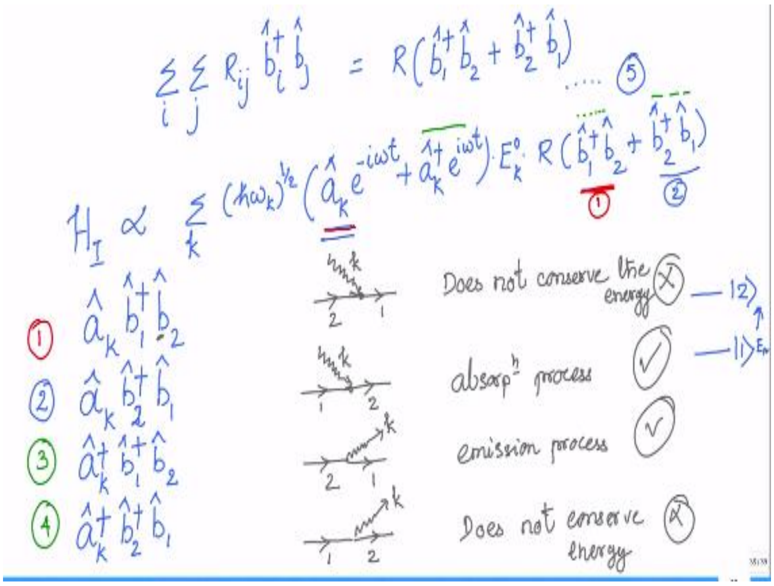
You can actually go ahead and do it and then if when you do that what you will end up getting is the following, is you are expanding it in terms of i j and we have Rij and bi cap dagger and bj this okay. So all that I have done is I have retained the Rij without actually putting in because see there we actually put in the h cross omega i in this expression right after this, we actually wrote down this because what I explicitly did not state but what I implicitly assumed that if i were to be the eigenspace of the Hm okay.

So then we could write it as h omega i. If I do not know anything about that, then the general way to do this would be to keep this like Rij, but nevertheless you would have had chance of writing this in terms of the creation and annihilation operators, in this place the creation and annihilation operators would correspond to the k and g’s okay, so the j and i and j’s we could write it in terms of that. So if you take, now given this you can already see where I am going since I have written like this, I am going to put this back into my original Hamiltonian interaction, but before doing that, it is convenient to simplify some terms.

Number 1, let us investigate here 2-level atom okay. So it has only 2 levels and let us call that as number 1 and number 2 okay. It could be in that order of energy. So we are going to investigate the transition happening between these two okay way the down to up or up two down either way, alright. So then, a corresponding Rij or Rkg would be we have R12, now R21, apart from that R11 and R22 okay. So later on, we will see that for a 2-level atom, these will go to 0 okay.

As a very simple symmetric argument we can actually present and then tell that they are to be 0, they are to go to 0, they will vanish which means that leaves R12, R21 okay. So that simplifies quite a bit and what we could actually now do is that we could actually write down the equation number 4.

**(Refer Slide Time: 10:16)**




So Rij summation of range Rij b becomes summation over i, summation j, Rij bi dagger bj cap becomes equal to R because in here all those terms, right of the possible 4 four terms now what we have, so after possible 4 terms, 1, 2, 3, and 4, these 2 will go to 0, so the only two of them that will collapse into R, so I am going to call this as R. So then, this whole thing becomes only just one which is R times b1 cap dagger and b2 cap + b2 dagger b1 cap okay, excellent. So now, let us use this, so this is my equation number 5.

After making assumptions about the nature of the system as 2-level system okay, so now when you do that, we can actually substitute this number 5 into the interaction Hamiltonian okay and when we do this, then I just drop down quite a few of the proportionality constants and then I summed it over state k right of the electromagnetic radiation modes each with energy h omega key to the half and times ak cap e to the power -i omega t + ak dagger e to the power i omega t R, so I am going to replace, so remember we look at the equation number 1, let us go to the equation number 1.

The whole idea is that we want to replace this right, ek r eg right, I missed this, so we have to write this, and then this whole thing we are going to replace it by our new description, our new expansion. So before writing that R I should not miss E0 k referring to the amplitude of the electric field times R okay. We have to write this which is b1 dagger b2 cap + b2 dagger b1 cap. So this expression is going to have 2 into 2, 4 terms. Now, let us look at this 4 different terms.

So the first term in terms of the operators here is going to have ak corresponding to the annihilation operators of ak, ak b1 dagger b2 that is our first term, right. So that is this corresponding to this, we call it as 1 okay. So I am going to call this as term number 1, that is my term number 1 and the same thing okay. So this is term number 1, I think term number 2 will be given by same this with respect to this, so that is going to be ak cap b1 cap dagger b2 dagger b1 cap, alright, and then the third similarly we could think of taking this.

So, I am going to use, just to distinguish, so what I am going to do is I am going to this with 1 and 2 alright, so that will be the third expression, so and then fourth one okay. The third one would be ak dagger ak cap b1 dagger and b2 cap while fourth will be same ak cap except now here you have b2 dagger and b1 cap. Now what does these 4 different terms represent physically, alright? So now look at this, this tells you that you are having the matter at state b1 okay, I mean you are creating the matter in state b1 and destroying matter in state 2 and destroying photon of state k okay.

So if we were to remember our 2-level system, so let us it is like this right okay. So that is our energy axis. So in this case, what is happening is we are starting with, okay I am going to take away these arrows because we are going to draw arrows now to represent different processes that these operators correspond to, okay. So our system starts from state 2 okay and then goes to the kth photon right, the k mode photon is incident upon it, both of them get destroyed or annihilated while you are creating a new state 1.

So now if we were to follow this conventionally, it is just for us to actually equate to the physical reality here, right. It is in our notion the 1 is of lower energy and 2 is of higher energy, all that matters is that we need to be consistent in analyzing all the 4 in a go. So you cannot be taking 1 being the lower for this and then the 2 being the lower for the other okay. So if you look at in all the system 1 being the lower, so that is all we need and that is what I am going to take, but then it really does not matter whether 1 is lower or 2 is lower, the whole process will be exactly the same.

It is just that depending on which one you take, one of them will mean something while the other will be meaning something else. In this case, you can clearly see you are taking a system from state 2 and then bringing down to state 1, so you are destroying the state that is what the annihilation operator corresponding to b2 means, right. You are destroying the state, at the same time you are creating a state 1 okay, basically you are reducing the number of entities from the state 2 and increasing the number of the entities at state 1 that is all it means right, but in the process you are also reducing the number of photons of mode k.

Now, this cannot happen because this process does not conserve the energy, right, where does mean it conserve energy, so cannot happen okay because you cannot take the molecule from the excited state, bring it down to the ground state that is perfectly possible, but in the process you cannot absorb, I mean you cannot annihilate the photon, this energy, the excess energy that the matter has at the excited state 2 coming back to 1 should come out, I mean it should be preserved here.

We are providing energy and then making our system go to the lower energy state, which cannot happen. Similarly we can actually write down the description of this guy, right. So, same thing you are utilizing one of the kth mode photon because ak annihilation operator, so it is going to reduce the number of photons available by number 1 while here what we are doing is that we are taking the entities in state 2 and creating one more extra or we are creating a new state.

We have been creating a new number, increasing the number of entity in base state 2, so which means that should come on the right hand side. So, we have actually created a new state from state 1 because you have annihilated state 1 here, b1 right, okay. Now, this is nothing but your absorption process, good. So, this is the instant or this is the point at which the interaction is happening, so what we are drawing this wiggly arrows is basically way of I kind of introduced Feynman diagrams in this way okay.

There are very natural ways to build up, but this is convenient here and I am just introducing it here. So now you can actually see why it really does not matter whether I took 1 as a very low energy state or 2 as our lower energy state, had I taken 2 as a low energy state what would have happened is that you would have seen that the first expression would correspond to absorption process while the second would have corresponded to at non-conserved transition, right, I mean a transition or an interaction where the energy is not conserved.

So, I would have said oh it is not possible okay. So, this clearly is possible it conserves energy, so it is definitely possible and now let us look at this. So what we have done is that we have created a k mode photon. So from the point of interaction, we are actually creating kth mode photon and in order to do that what we have done is we have removed one of the entities from state 1 which means it should come on the left hand side, so it is contributing, it is reducing its number by 1 the state 1 and you have also created, it is exactly the other way around.

So this is you have created the dagger is on the +1 state, so they should have been 2 and this is 1, correct because the dagger are the creation operators and the dagger is on the state 1, which means the state 1 increases by one number, which means from interaction point to later it has gone up by 1 that is how it should be represented and then while on the other b2 is an annihilation operator, there is no dagger here. So basically, it should have reduced its number from wherever it is by 1.

So now this what you are seeing is that we started out from state 2 and then during this process, we have created a state 1, in the process we have also created a kth mode photon okay. So, what I am going to do is, so this is emission okay that is perfectly fine right, because when the excess energy on moving the system from high energy to lower energy state is coming out as an increased number of photons okay that is completely conserved. So this process is again possible.

On the other hand, let us look at what is happening here; what we are doing is that we are creating a kth photon. So, let us write down, so we are creating one, we are increasing the number of the kth mode photon by 1, while we are also increasing the number of state 2 because b2 dagger right. So it is a creation operator, so which means there should be 2 and we are destroying okay, we are destroying our annihilation operator of one state okay. So now, again this does not conserve energy, right, so it is not possible.

So what we have done is that we have actually now collapsed these 4 possible transitions into only 2 possible transitions that are allowed okay. So that is number 2 and number 3 based on the conservation of energy arguments, okay, just so that I do not lose you guys. What we have done is we started out by writing the Rkg term in the interaction Hamiltonian that I have just stated this is how the interaction Hamiltonian would be, Rkg term in terms of creation and annihilate matter, Fock states corresponding to the matter and the creation and annihilation operators of them.

So we called it as bi dagger and bj and using this if you were to write down, there are quite a few number of terms, if you have to preserve the multi eigenkets state, then there were quite a few possible energy eigenstates. So then incredibly large number of terms that we need to write down or write down for, so we went to a simplest case scenario where let us concentrate on 2-state system, typically when you are looking at the transition, we are typically talking about transition occurring between 2 states.

So it is okay to think in terms of a 2-level atom, the atom has only 2 levels and we are going to look at how the transition happens between these 2 levels and when we did that, we got 4 terms even this case and then I said these R11 and R22 owing to symmetry arguments will be going to 0. We would see this more explicitly later on in the course, but right now just I am going to state it that if you are interested, you can actually go ahead and calculate 2, but think about that and why it would be going to 0.

The point here is the this will be going to 0 and you could in principle then think about only 2 of these matrix elements R12 and R21, I am going to set both of them equal to 1, the same R. When I did that so now you can write down this series of terms in a compact form as R of these two operators, then we put that expression for Rij into our interaction Hamiltonian and then what we said is that okay, now let us look at the various possible terms arising from this interaction, so each correspond to a different process.

So we analyze this and then say that of the 4 possible processes each corresponding to one of these terms, only 2 of them are physically possible because the energy need to be conserved, so we know that that needs to be true, so which means only 2 of them can happen. So what are the two, these two, one corresponding to absorption process and the other corresponding to the emission process. So now, let us take those and then calculate the transition rates corresponding to these processes okay. How do we do that?

What we are going to do is we are going to neglect the terms 1 and 4, so we know that these two are the terms that are available and since they are additive, they are coming as addition and then what we are going to do is we know that the rate of the transition goes as the square of the matrix element involving the interaction Hamiltonian, so we are going to put back that and then write down these explicit operations okay and then see what happens during an absorption process and in the emission process. We will do that in the next class, alright. Thank you.