Biomechanics
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Lecture - 80
Singularity, Gimbal Lock, Advantages and Disadvantages of Parameterization Methods

Welcome to this video on biomechanics. We have been looking at some practical applications and how to measure segmental kinematics. We looked at some of the methods, some of the approaches. We will continue our discussion on this topic.

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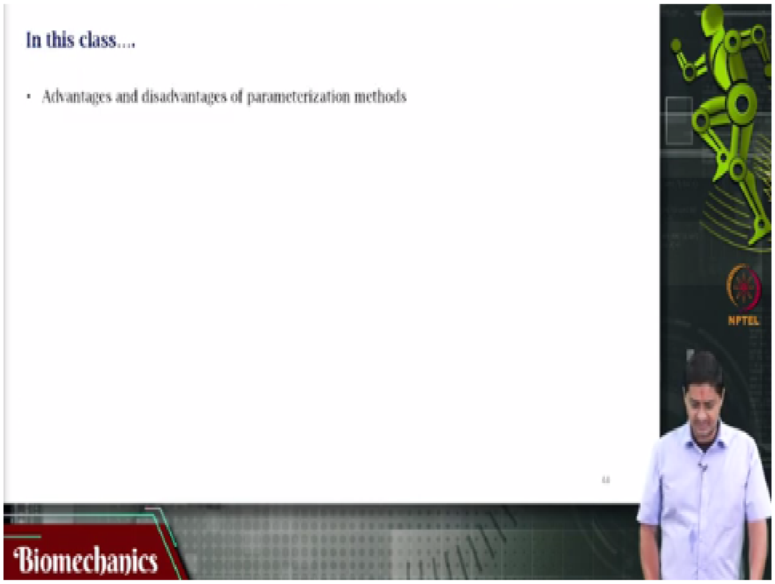

So, in this video we will look at some of the advantages and disadvantages of specific parameterization methods.

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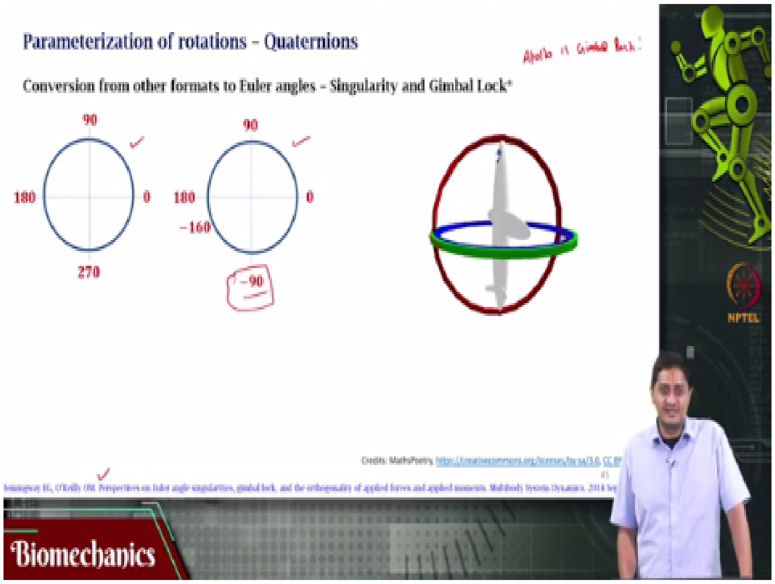

So, we looked at rotation matrix, we looked at Euler angles and we looked at quaternions. So, these methods have their own specific advantages and disadvantages. The most important advantage of the Euler angle method is that it is intuitive because we understand angles, humans understand change in orientation or angles. And so, either angle being an angle it is, somewhat easy to relate to it is possible to develop an intuitive understanding and this is critical.

Because mostly we are trying to make meaning out of data and if data has no meaning, then it is always somewhat scary or we are always apprehensive because we do not understand what this means. So, but that does not necessarily mean however that those that we understand and have an intuitive understanding are necessarily the best ones. So, let me repeat that for clarity. We prefer to have data in forms that we understand well or that we relate to well and things that we have an intuitive understanding for.

However, just because we have an intuitive understanding say for Euler angle does not mean that that is the best method, there may be other methods. But then if we have a method for which we do not have an intuitive understanding and that is the better method that is the best method. Then it will require the analyst or the experimenter to somehow convert this unintelligible data from a different format to something that we can understand.

So, it is a complication of many factors but it all comes down to this. You want accuracy, the question is do you want accuracy the answer is yes. Do you want an interview understanding? The answer is yes. But it turns out that it may be that both of this may not be available in a given method. So, there is a need for us to convert from one modality to another modality. As engineers this is something that we do all the time most of the time.

Many other different domains what we do is a different form of conversion. Mostly we end up doing conversions from one reference frame to another reference frame from one coordinate frame to another coordinate system. This is something that we do because it turns out certain types of analysis are best performed in a certain coordinate frame. But understanding of that happens in a different coordinate frame.

This is like regular work for you know proper engineers this is like regular business. This is our bread, butter, water this is our regular thing. So, this is not new to us but we still need to understand what are the specific you know advantages and disadvantages. One specific disadvantage with Euler angles is that the problem of singularity. What is this problem? We described this in one of the previous videos.

We called this problem as Gimbal lock; technical differences exist between Gimbal lock and singularity. Earlier we call it as Gimbal lock technically apparently it is a singularity problem which I called somewhat imprecisely as Gimbal lock. But in this field people sometimes interchangeably use singularity and Gimbal lock even experts even authors in the field sometimes interchangeably use singularity and Gimbal lock.

Although these two things are not exactly the same. So, what is this Gimbal lock? What happens is when two of the axes are aligned when the so in this case you have this Gimbal you have this aircraft that is going and there is a middle Gimbal there is a central Gimbal and there is one on the outside that is green in colour that is one on the inside that is blue in colour. So, if and when this brown Gimbal hits 90 degrees what happens is that the blue Gimbal aligns with a green Gimbal.

Now if the data from the blue Gimbal rates are some value that could be either because the blue Gimbal itself is moving or because the green Gimbal is moving that is an uncertainty as to from where this is coming, it is not clear. So, this is the famous problem of Gimbal lock or singularity. This is a relatively advanced concept somewhat difficult to understand and synthesis and so do read about this do spend some time reading about this.

Sometimes when we use coordinates as 0, 90, 180 and 270 in one case and in the other case I am using 0, 90, 180 and - 90 on the other side. Now what happens is that whenever there is a shift from one side to the other suddenly there will be a jump in the coordinates. This jump is called as coordinate singularity. This is somewhat common in measurement systems that use this kind of representation.

For example, this is relatively common in the electromagnetic tracking system that we use in our lab and in the IMUs that we use in our lab relatively common, there are ways of addressing this. So, this story of Gimbal lock became famous or notorious with the story of the upper low spacecraft that went to the moon NASA's moon mission. So, in that spacecraft of course when it is a spacecraft you need to have a relatively accurate idea of the orientation of the spacecraft location.

Orientation of the spacecraft is an absolute must because if you do not have it then you do not know where you are going. Even relatively small changes in the approach of the spacecraft in early stages if left uncorrected could land the spacecraft somewhere else or you might even miss the moon landing. Of course, that depends on where the initial deviation happens and whether it is left incorrect and so on and so forth.

But the point is that it is extremely important for these kind of launch vehicles to have an extreme level of accuracy of the orientation and the location position orientation idea. So, they had a inertial measurement unit that was composed of you know two Gimbals giving it a three degree of freedom. So, it is possible for the system to have three Gimbals and have one extra degree of freedom, four degrees of freedom.

Remember this was the 1960s today's initial measurement units are less than a centimetre squared in size. I do not have an exact idea of the size of the Apollo 11 inertial measurement unit but I am assuming that it was not small. So, the engineers the NASA scientists who developed this inertial measurement unit had constraints on size and weight, how much size and weight they can expend to create this inertial measurement unit.

So, they just decided to go with one less degree of freedom just two Gimbals and three degree of freedom as opposed to three symbols and four degrees of freedom. So, they went with one less degree of freedom. They were very well aware that if you have a three degree of freedom system there is a possibility that two of the Gimbals will align or there will be a Gimbal lock situation that will be created that may be created.

And if it is created or if it happens then it will be required for this initial measurement unit to be recalibrated or readjusted by measuring the orientation with respect to the stars manual intervention required by the astronauts that is a little bit of work. So, the engineers were very well aware but they decided to conserve or save space and wait. So, they went for a relatively simpler system that had the risk of taking this Gimbal lock position and what happened?

The reverse fears became true what happened was that there was a Gimbal lock. So, what happened was that at a particular orientation of the spacecraft. The Gimbals became coplanar and there was of course a control mechanism a preventive mechanism that was kept to avoid this kind of a situation. Whenever you are approaching see this Gimbal lock would happen when the brown symbol reaches 90 degrees.

So, when you are reaching 90 degrees close to 90 degree say 80 say 75 onwards you have a control mechanism that prevents Gimbal lock from happening. But somehow that did not work. For some reason the control mechanism that was present to avoid the Gimbal lock also did not work. This made the entire inertial measurement or orientation system completely ineffective. So, for some time the spacecraft did not have an idea of where or how it was oriented this is a dangerous situation.

So, for this to be corrected it required that the spacecraft be itched away from this and away from this locked situation and then reorient the symbols such that are recalibrate these Gimbals by using the stars as the reference position. This requires some kind of manual intervention. So, the story is that finally they managed this situation and while they got in touch with the ground crew at NASA, they said something similar to or something along the lines.

Hope you will send the third symbol as a Christmas gift or something like that it is a famous story of the upper low 11 Gimbal lock situation. Pointing to the particular case when you know by the way this involved human lives and if this had become a big problem these people could have just been stranded for all you know and that is an extremely dangerous situation. So, it is important or crucial for you to decide how you are going to orient your measurement system.

It is an inertial measurement unit is absolutely important for you to decide how you will orient it. It should be oriented such that the axis that is likely to go into lock position or the axis in the middle almost never goes to 90 degrees or approaches that 90 degrees. Even as it is approaching the problem becomes severe it is so 88, 87. So, it does not have to be exactly 90 from about 75, 80 onwards the problem begins to build up.

So, there is a particular orientation where you will not require that middle symbol to ever go above say 65, 70 degrees. If there is that orientation that is where you align that Gimbal so that is important. So, the middle axis should be aligned to that axis that is that does not require more than 75 around about 70, 75 degrees of orientation change. The range must be within that level and the other two axis should not change more than 180 degrees.

Because if it changes more than 180 degrees the 181st degree will be measured as - 179 degree due to this situation due to this you know - 90 + 90 type of measurement system. There may be a jump in the angle and you actually do not know where this jump is coming from what is the source of this. It is an actual measurement and because there might be actual movements that may be very fast these jumps will appear as nice and it will be incredibly hard for you to signal process this and remove this.

So, it is best to orient your measurement system such that the middle Gimbal is aligned along an axis that is not expected to move above say 75, 80 degrees maximum alternative I say for example and the other two axes are not expected to move more than 180 degrees well below 180 degrees. This is how you align your measurement system this is very important. So, because everybody has an IMU everybody will measure this.

But unless you have an idea of how to place your IMU on how to do the recording all the data that you get is just nice. So, we need to understand what is the measurement system that we use and what is it measuring and what is the meaning of those measurements. Otherwise, there is no specific utility or use for these measurements. So, for more stories on this Gimbal lock situation read for a very good technical treatise on this topic.

The perspective on Euler angles and singularities Gimbal lock and orthogonality of applied forces and applied moments by Hemingway and Rayleigh, a nice paper where they discuss the some of the details of this Gimbal lock situation. Of course, do check a Wikipedia page on you know upper low 11 Gimbal lock situation. And of course, Wikipedia is Wikipedia so you cannot be 100 sure of the source of this. So, but there is a nice anecdotes and titbits.

So, technical anecdotes are technical anecdotes but remember technical anecdotes stop were there they are just anecdotes. So, do read up about this topic. There are many other places and some nice videos are also available on this. But this is something that you need to work for relatively long time to develop an intuitive understanding by the way this is not intuitive to start with. So, something to keep in mind. So, let us proceed with our discussion.

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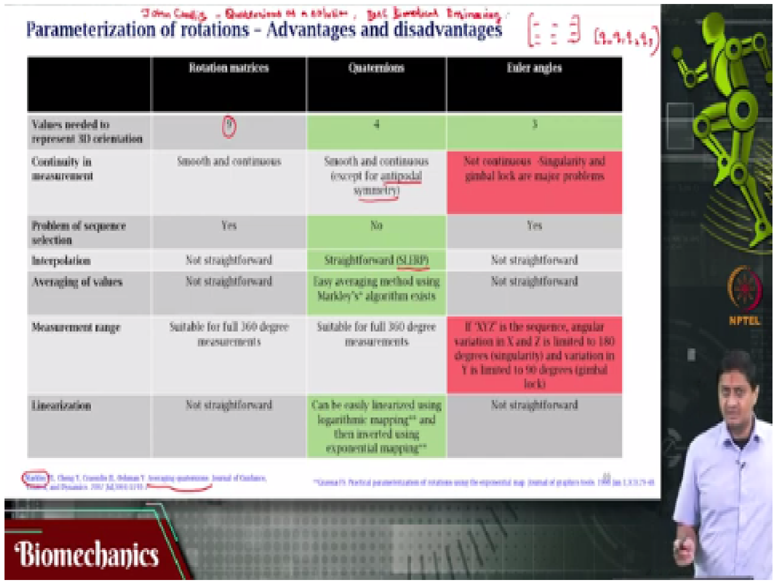

So, we look at quaternions, rotation matrices and Euler angles and their advantages and disadvantages in terms of the values that are needed to represent the orientation the various things. So, remember there is no single perfect method, the specific method that you want depends on your needs your application and your expertise. So, many different things exist. How many values are needed to represent 3D orientation.

In rotation matrices you need well 3D orientation means it is a 3 by 3 matrix 1, 2, 3, 4, 5, 6, 7, 8, 9 values. In quaternions you need four values we saw that what are these? One real part and then the imaginary parts q 0, q 1, q 2, q 3, q 4. However, you say that in Euler angles you just need three angles about the three-principle axis. Actually, you need more you also need to have an idea of the sequence in which you are measuring.

If you want to measure continuously you actually need more but the measurements itself you just need three the angles about the three axes. Rotation matrices will update smoothly and continuously from measurement systems. So, in quaternions the measurement is relatively smooth and continuous except when there is the antipodal symmetry problem. Do check what this is and relatively advanced concept.

I am leaving it as an exercise for you to check but the point is that it is not a major or critical issue. It happens but it is not a huge problem. In Euler angles other angles are not continuous that is the Euler angle singularity or Gimbal lock problems these are critical problems. So, we described this in one of the previous classes. Remember if one of the angles goes to 90 degrees then the whole system has only a single degree of freedom.

That is the difference between the other two angles and that is it or something like that. Remember we made a rotation matrix and we showed that there is essentially a single degree of freedom or there is a lot of uncertainty in terms of measurements. Is a sequence selection a problem in rotation matrix? Yes, in Euler angles absolutely yes you need to know the sequence in which you are measuring.

By the way the choice of the sequence depends on the particular measurement that you want. Again, remember you cannot measure in any which way you want; you need to align your angles or you need to choose your sequence such that you are going to get the maximum possible reliability in your measurements. Minimal chances of this kind of Gimbal lock sequencing or singularity type of errors or these are serious errors that cannot be corrected by software.

This is an experimental measurement or artefact that you cannot overcome by using some algorithm, it is impossible to do that. So, you need to measure carefully when you do that. So, the choice of sequence depends on the specific application which is why you need to know what is it that you are measuring and what are the specific ranges that you expect to happen. If you do not know what you are measuring or the specific task or the specific case for which then you must first know the range.

First identify the range and then decide the orientation. So, it is not a trivial decision that any new can make without understanding the problem. New B are expert anybody can make this mistake but you need to have an understanding of what is the measurement that you are making. In quaternions you do not have this problem of sequence selection. This problem is not there in quaternion but what is the disadvantage?

Because you do not have an intuitive understanding that is a disadvantage. Then can you interpolate if I measure at a relatively slow speed or if I want more data points can I interpolate?

Interpolating in rotation matrices and in order angles is a non-trivial problem it is not straightforward. In quaternions also it used to be a relatively difficult problem but then people developed this interpolation algorithm called slurp algorithm.

After which it became a straightforward situation straightforward problem to be solved. Now everybody is using, slurp algorithm to interpolate. So, interpolation although is not straightforward in all the three has been made straightforward in quaternions by the development of this slurp algorithm. Similar algorithms for a similar simpler straight forward algorithm for interpolation of Euler angles and rotation matrices do not exist or not yet available.

They are at least they are not simple or straightforward. How do we average measured values? This averaging of measured values is not a straightforward problem in all the three methods. But in quaternions there is an relatively simple or easy averaging method that was developed by Markley. This is called Markley algorithm so that algorithm is used to average the quaternions. What is this algorithm going to do?

It essentially creates an eigen value decomposition of the set of all quaternions that you want to average and the eigen vector corresponding to the eigen value that has the highest value that or the highest eigen value is the average. So, the details of this algorithm are available in this famous paper by Markley. The paper is called averaging quotations, two-word name for a paper averaging quaternions that is it.

Just Google it, averaging quaternion by Markley you will get this paper that the details of this algorithm are available. So, essentially what this requires is a creation of an eigenvalue decomposition of the set of all quaternions that you want to average and then finding the eigen vector corresponding to the largest eigen value. Measurement range is suitable for full 360 degrees rotation matrices and quaternions are suitable.

Euler angles because of this sequencing or this singularity symbol log problem that is a problem. If x, y, z is the sequence for example x and z the measurements are limited to 180 degrees and y is limited to 90 degrees. Practically y is limited only to about 75, 80 degrees not more because of the singularity Gimbal lock problems. Linearization of this data sets if it is represented in rotation matrices or in Euler angles these are not straightforward.

Again, in quaternions also these were not straightforward but people have developed algorithms that have created a logarithmic mapping and an exponential mapping method to linearize. So, from reading this table what is the message that you are getting? The message is that it seems like all the three methods are relatively hard, rotation matrix quaternions, Euler angles. But it seems like quaternions has a set of tools that helps you work with them.

You can linearize them using logarithmic mapping and exponential mapping, you can average them using Markley algorithm and you can interpolate them using the slurp algorithm. Similar simple straightforward algorithms are not well developed for rotation matrices and Euler angles. Remember the application or this kind of development comes from various fields because quaternions are used in a diverse set of fields.

They are not just used in aircrafts and their orientations. They are used in aircraft and orientation but they are also used in human body orientation measurements, they are also used in animations, they are used in computer graphics, they are used in virtual reality, they are used in a whole bunch of fields. So, people from all these fields come together and they want to develop these kinds of methods. So, they can continue to use this better.

So, this has probably this is at least my guess as to what has led to the development of this nice algorithms that has made the quaternions as the method of choice for measurement. It is not that it is a method of choice but it has been made as a method of choice because of the availability of these kind of algorithms. This is the take home a takeaway message or at least this is what I understand from this table.

So, it seems like quaternions appear to be the solution or method of choice. For more details on this method refer to this paper by John Chalice something like the title of the paper is something like quaternions as a solution to problem of measurement of kinematics or something like that in the journal is BMC biomedical engineering. We will provide the link to this paper; it is an open access paper.

So, we will provide the link to this paper in our forums but you can Google this. John Charles quaternions as a solution to the problems of kinematics human measurement or something like that is a long title so I am not able to remember. But the journal is BMC biomedical engineering. By the way John Charles is my teacher so my opportunity to thank my teachers here such a nice neat paper.

Of course, it takes time for you need to read this paper many times to understand, you read once in your understanding a little bit and then you read a little bit more and then you are still not clear and so on and so forth. So, please do check if you are more interested. Remember this is something that is becoming extremely hard it is used everywhere. So, do check this out.

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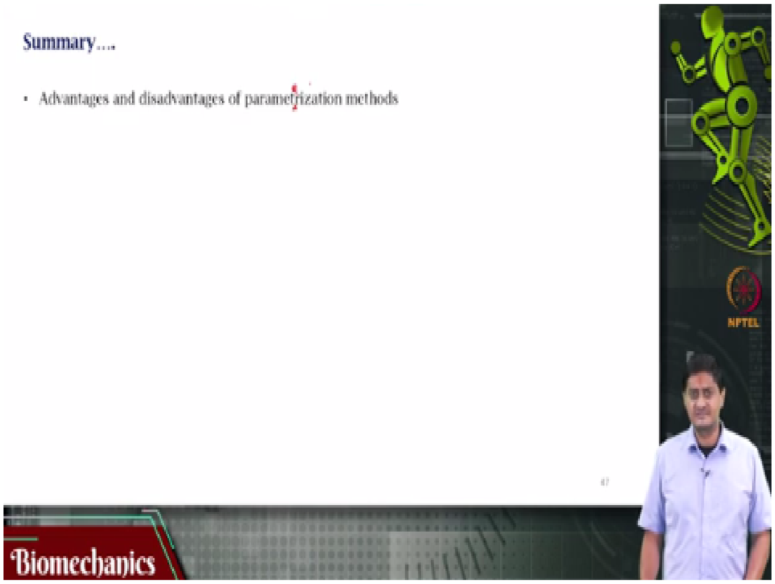

So, with this we come to the end of this video. In this video, we looked at the advantages and disadvantages of the various parameterization methods, parameterization we keep missing this e that is parameterization methods. We looked at the story of the Gimbal lock that happened in the upper lower spacecraft. And what you should do to avoid the Gimbal lock singularity situation is such that you orient your measurement system such that the middle axis is not required to measure more than say 75 degrees.

And the other two axis are not required to measure more than 180 degrees this is how you align. So, the choice of access is dependent on the specific application. So, as an experimenter you are required as the expert in the field answer experimenter you need to know how to orient your measurement system. And we also looked at the specific advantages of quaternions like the availability of interpolation algorithm slurp.

The availability of algorithms for averaging the availability of algorithm for linearization like algorithmic mapping, exponential mapping and other such methods make quaternions as the superior method or method of choice for measurement of kinematics. It also suggested some of these papers that contain all the details John Charles paper, paper by Markley, paper by Hemingway and the other papers. So, with this we come to the end of this video, thank you very much for your attention.