

Biomechanics
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Lecture - 65
Stress Strain Relations in Tendons

Vanakam, welcome to this video on biomechanics. We have been looking at biomechanics of soft tissues. In the past two videos we looked at examples of cartilage, ligaments and we started our discussion on tendons.

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In this video we will be looking at stress strained relations in tendons.

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Stress-Strain Relations

- The elastic behavior of tendons and other materials is represented as either a force-deformation or a stress-strain relation.
- In the first case, the main variable of interest is tendon stiffness (N/m) or compliance (m/N)
- In the second case, elastic behavior is characterized by Young's modulus.
- The value of Young's modulus, also called the *tangent modulus of elasticity* or *elastic modulus*, equals the slope of the stress-strain curve, $d\sigma_i / d\epsilon_i$, where σ_i is the tendon stress, that is, the ratio of tendon force to tendon cross-sectional area, and ϵ_i is the strain, the ratio of the tendon elongation to the tendon length at rest (dimensionless).





So, as we discussed previously stress strain relations of biological materials, as with engineering materials that represented either as a force deformation curve or also relationship between stress and strain. Essentially the elastic behaviour of these are determined by the relationship between the stress and strain or force and deformation, we have seen this previously. So, if you are looking at a force deformation curve the important variable of interest is stiffness or complaints.

Or if you are looking at the stress strain relationship elastic behaviour is of course characterized by the modulus, Young's modulus. But then this relationship need not be linear because if it is linear then there is a Young's model there is an Young's modulus for the material, but as we saw with the biological materials with most biological materials the stress strain curves are not linear. Then what is the modulus?

Well, you know it is essentially tangent to the stress strain curve at that point. Now that means you are looking at tangent modulus of elasticity or elastic modulus. This is the slope at a given point in the loading and loading curve, at the given point that, so it is specific to that point it is it does not work everywhere. So, unlike engineering materials where the modulus is a constant for that material that we are looking at for most materials for most engineering materials of interest.

For example, steel has a modulus it is not going to be like that. Here you have a modulus that has that is defined for that point for that stressor for that strain. So, this is for example if you are

looking at tendon stress. The ratio of tendon force to tendon cross sectional area is tendon stress and the ratio of the elongation to the rest length of the tendon is the strain and then you compute this.

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Stress-Strain Relations

- Young's modulus for tendons can be regarded as the stress required making $\Delta l/l = 1$, where l is the length of the tendon prior to the deformation.
- It can also be represented by the product of tendon stiffness and the ratio of the original length to the cross-sectional area.
- In some joint configurations, the tendon may be slack and does not produce resistive force when elongated.
- For such cases, elongation Δl is defined relative to the slack length, $\Delta l = l - l_s$, where l is the length of the elongated (deformed) tendon and l_s is its *slack length*, the length at which the tendon begins to resist external force

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So, Young's modulus can be regarded as a stress required for making you know $\Delta l/l = 1$, this is the length of the tendon prior to the deformation. Of course, you can also you know represent this as a product of stiffness and the ratio of the linked to the cross-sectional area. Important to note is that in some joint configuration's tendon might be slack and when it is slack it will not produce a resistive force when elongated.

So, you it is below the resting length it has to first reach the resting length and then it will start producing the force. In such cases the elongation or the deformation that we are looking at is depend relative to the slack length. Now you have an elongated length or the deformed tendon length and then you have the length of the tendon when it is slack that is the slack length. The length at which then the length at which the tendon begins to just resist the external force it, just begins to resist it external force.

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Stress-Strain Relations

- The slope of the stress-strain curve, in general, depends on tendon elongation.
- The curve has three regions : (a) an initial toe region where the slope increases, (b) a linear region where the slope is constant, and (c) a failure region.
- When in vivo testing is performed on humans, loads are below the injury threshold and thus the failure region is not reached

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Unlike engineering materials, for a tendon the slope of the stress strain curve in general is a function of the deformation of the tendon. So, that means very clearly that this is not an elastic material, so this is not going to have a modulus. Why? Because the slope of the stress strain curve is the modulus if the slope is a function of the deformation that means what, it is not the force that is a function of the deformation.

If the force is a function of the deformation, then I can compute. If it is a linear function of the definition then I can compute the slope and that is a constant for the material but the slope itself is a function of the deformation means as the deformation increases slope changes meaning that it is going to have non-linear elasticity. It is going to have non-linear behaviour; you its elasticity is defined at that point.

So, in general the stress strain curve of a tendon can be divided into three regions broadly, what is called as a toe region which is an initial region where the slope increases with elongation or deformation. As deformation increases the slope increases in a huge way, the slope increases remember, it is not the force that increases, of course the force will increase that does that happens even for materials with linear elasticity, even for engineering materials that is not what we are discussing.

It is the slope that increases with deformation and then you can actually find a region within which this slope is linear, a linear region in which the slope is constant. You can actually find a linear relationship between stress and strain and then if deformation continues you have what are called as failures first you have failure at the nano and micro levels and then if it continues further, you have macroscopic failure what are called as tendon ruptures, they happen.

In general, during daily life most tendons function in this region, in the toe region, some tendons function in the linear region. Very rarely during daily life tendons undergo microscopic or macroscopic failures. Of course, there is a difference when it comes to sports activities or what are not exactly daily life activities. So, when you perform in vivo testing on humans, how do you perform this by the way how do you perform in vivo testing on humans using imaging techniques, of course of you know using imaging techniques.

Loads that are below the threshold of injury and failure reason is almost never reached in daily life. And almost always the region of operation for a tendon is in the non-linear region or in the toe region. Mostly for most tendons but there are exceptions, there are some cases in which it goes to linear region but almost never goes to the microscopic or macroscopic failure regions. Of course, these failures almost always happen in sports injuries or accidents.

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Stress-Strain Relations in the Toe Region

- In the toe region, the tendon is strained to approximately 2% (range 1.5%-4%). *The only known exception is the human psoas tendon, where this region extends to 10% strain.*
- Under these low strain conditions, collagen fibers straighten and lose their crimp pattern but the fiber bundles themselves are not stretched.
- The modulus of elasticity increases with strain until it reaches a constant value at the start of the linear region. The average slope of the stress-deformation curve in the toe region is at times called the toe-region modulus.

The graph plots Stress (N/mm²) on the y-axis against Strain (%) on the x-axis. The curve starts at the origin and rises through a 'Toe region' (non-linear) into a 'Linear region'. Key points on the curve include 'Crimped fibers' at low strain, 'Straighten fibers' at the start of the linear region, 'Microscopic failure' at the end of the linear region, and 'Macroscopic failure' (Rupture) at the peak of the curve. A 'Physiological range' is indicated for the toe region. A red arrow points to the slope of the toe region, labeled 'modulus'.

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So, in the toe region, we have strain up to 2 percent approximately on average median is about 2 percent, the range is between 1.5 and 4 percent. Of course, that is an exception remember in biology exception is the rule. So, in human source tendon, the toe region itself extends up to 10 percent strain that is a very unique case. Mostly when we are discussing this region you are looking at the strain of about 2 3 percent, 1.5 to 4 percent.

These are relatively low strain conditions. Under these conditions collagen fibres get out of their crimp wavy pattern, they straighten and lose their crimp pattern but the bundles themselves are not necessarily stretched. So, the modulus of elasticity remember, what I am saying, the modulus increases with increasing strain until it reaches until the modulus reaches a constant value until it refers to modulus here.

Until the modulus reaches a constant value at the beginning of the linear region, until this point modulus continues to increase with increasing strain. So, if I have if I try to find the average slope of the stress deformation curve in the toe region in the non-linear region, this is sometimes together sometimes called as toe region modulus. Remember toe region modulus is not constant, toe region modulus changes with deformation.

But I can either find the average or median and then say that this is approximately the modulus in the toe region but that is going to be a misrepresentation of the complex reality. But sometimes depending on the selected level of analysis sometimes people use this as the toe region modulus by definition, there is no such thing as a toe region modulus you have toe region moduli in plural, you have many moduluses or many moduli in plural.

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Stress-Strain Relations in the Toe Region

- The first model of Nonlinear elasticity in the toe region assumes consecutive recruitment of individual collagen fibrils at varying degrees of crimp.
- With increasing tensile deformation, new load bearing fibrils jump into action and resist deformation.
- A nonlinearity arises because of the increasing number of fibrils contributing to the tendon resistance.
- This model explains how a large number of linear elements with different resting lengths can produce a globally nonlinear force-deformation curve.

Model of the nonlinear elasticity of the tendon in the toe region: Linear elements with spring constants k_1, k_2, k_3 come into action at certain stages of deformation x_1, x_2, x_3

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A question is how do we model the relationship between stress and strain in the toe region or in the non-linear elastic region? There are different models of modelling and there are different ways of modelling non-linear elasticity in the toe region. One model assumes that there are and in very large number of individual collagen fibrils and it turns out that they have different amounts of crimp, they are at different levels.

One fibril is having that kind of crimp, the other one is having that kind of crimp, the other one is having that kind of crimp etcetera. So, then at the level of micro fibril and at even more macro level you are going to have recruitment that happens at different resting length because uncrimping is going to happen at different lengths, is it not? So, because this one is going to have a different length at which it is going to uncrimp when compared with the third one for example.

So, as deformation increases new fibrils are recruited new load-bearing fibrils are recruited and they jump into action and they start resisting deformation. But then you might ask, hey but you just said that these are linear they are linear, I am assuming that these are linear springs with different resting lengths. But then the non-linearity comes because of a very large number of these kind of now linear fibrils with different resting lengths.

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For example, let us look at this rubber band now I am at this point I am not applying any deformation. I am not pulling on both ends you see that the red band and the green band both of

them are slack. Now as I am beginning to pull on both sides at this point for example now when you look what you are seeing is that the red band is pulled. What about this elasticity? It is elasticity is linear.

It is having a linear elasticity but its resting length is something but what about the green band the green band is still slack why is it still slack although the elasticity is the same this is made of the same material. So, elasticity is the same why is it still slack because the resting length is different. Now I am continuing to pull I am continuing to pull like that what you are seeing is that at this point at approximately this point both the red and the green band are recruited.

Now both these bands although they are having linear elasticity are getting recruited at two different resting lengths. Of course, this is just two in the case of the tendon you are going to have thousands of these fibrils are millions of collagen fibres that uncrimp at different lengths giving rise to a form of non-linearity that arises due to linear properties.

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So, what we see is the source of this non-linearity may be because of the differing resting links or the differing crimp properties of different fibrils or different collagen molecules. So, as the number of fibrils that are getting recruited changes are as the resting links are as the fibrils at different resting lengths are getting recruited, although each of them may have, they may not have by the way.

Each of them may have linear elasticity are linear properties between stress and strain. As you combine a very large number of linear elements you might give rise to a global non-linear behaviour in the whole tendon. This explains how a very large number of linear elements with differing resting lengths can produce a global non-linear behaviour. This is one model of toe region non-linear elasticity.

Why is this happening? Because of differing resting length this is x_1 this is x_2 and this is x_3 by the way these are only three lengths. What I showed in the rubber band example are only two or three bands or two or four depending on whether you consider both legs both arms of the rubber

band as one or two depending on that. There are just two bands but then in the tendon you are going to have a huge number of fibrils that uncrimp and crimp at differing lengths.

Because of this reason you are going to have a global non-linear behaviour in the tendon. So, this is one model of toe region non-linear elasticity.

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The slide is titled "Stress-Strain Relations in the Toe Region". It contains three bullet points: "Another model for toe-region nonlinear elasticity is the flexible hinge model of tendon crimp.", "This model assumes that crimped collagen fibers are lengthened until they become straight, similar to stretching a helical spring until it becomes a straight wire.", and "Stiffness increases from a low value for the spring to a much larger value for the straight wire". A diagram shows two blue zigzag lines representing crimped collagen fibers with a red wavy line between them, and a straight blue line representing the fibers after stretching. The slide also features a green robot icon, an NPTEL logo, and a presenter in a pink shirt.

Another model to explain toe region non-linear elasticity is the flexible hinge model of tendon crimp. This is like you have crimped collagen fibres that become lengthened until they become straight, this is like having a helical spring until it becomes a straight wire. Well, obviously the stiffness of the helical spring increases from a low value initially to a much larger value for the whole wire.

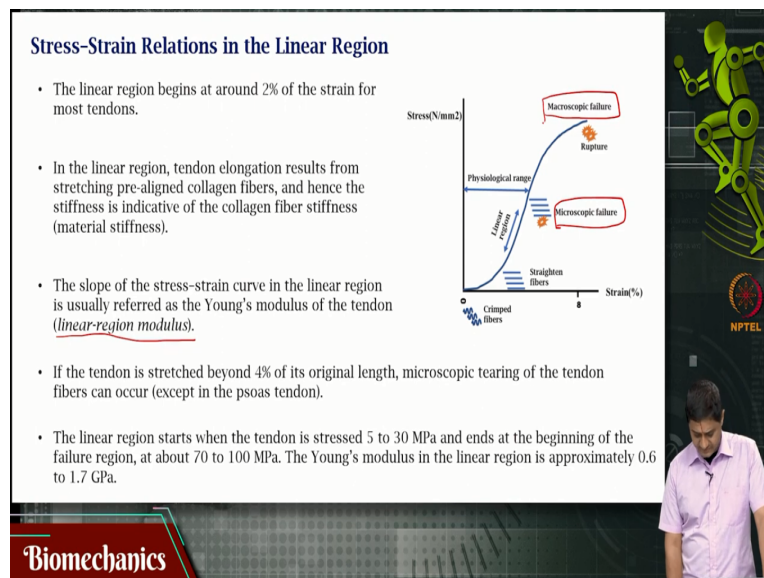
By the way this is not an exact comparison because if you have a helical spring and that you completely stretch into a wire then this is not going to go back to its state. Because you know that region or that behaviour is going to be plastic or by the time the deformation that has happened the language that has happened would have caused a elastic change or non-elastic change, beyond the region of elasticity, that is fine.

In this case they do in the case of the tendons they do in the case of the tendons they do go back to the original configuration. But the point is as they become stretched so here is a hinge as they

get stretched as they become more and more straight their material property changes because of uncrimping. Because they are being straightened and then it is showing a different property earlier it was showing the spring-like property now it is showing the wire leg property.

These are two different material properties depending on the configuration of the collagen fibrils itself. So, the amount of crimp will determine the amount of non-linearity that is there, this is another model to explain the non-linearity in the toe region of the stress strain curve in a tendon.

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For the linear region, in almost all the tendons for most of the tendons the linear region begins at around 2 percent of the strain I already mentioned, in some cases well it varies between 1.5 and 4 percent. So, the linear region can be assumed to begin at around 2 percent for most tendons. In this case the elastic modulus is one because the slope is a constant that is a linear region of stress strain curve.

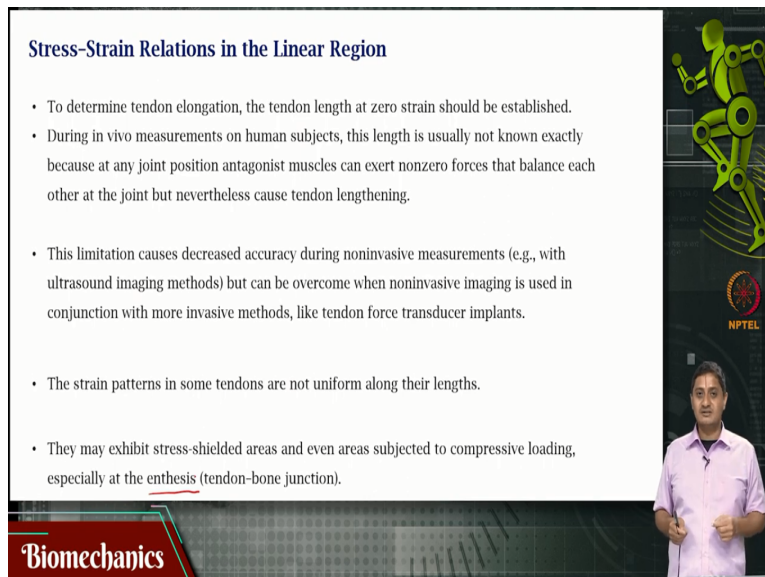
In the linear region tendon elongation results purely due to pre-aligned collagen fibres stretching, so the collagen fibres are getting stretched and because of that stiffness increases. So, this is mostly due to collagen fibre material stiffness. This is due to material stiffness of the collagen fibres. This is if I measure the slope of the strain curve in this region that is the young's modulus of the tendon itself.

But then that is saying the linearized or the Young's modulus in the linear region or the so-called linear region modulus. In most of the tendons if stretching is continued if the deformation happens or if the elongation happens beyond 4 percent, then microscopic tiring of the tendon starts. Of course, we saw some exceptions for example in the source tendon this starts after 10 percent.

But then in general this starts and for most of the tendons after 4 percent nano or micro level failures begin. If the strain continues beyond 8 or 10 percent for most tendons there is a visible failure or the macroscopic failure then happens. Mostly in the physiological region mostly the physiological range is well within the linear region, in almost all the cases it does not include the linear region in some cases it does include the linear region.

So, the daily life physiological range is much less than four percent so in a linear region it starts when a tendon is stressed between 5 and 30 mega pascals and it ends at the beginning of the microscopic failure region at about 70 to 100 mega pascals. The Young's modulus varies between tendons in this region varies or is in the range of 0.6 to 1.7 giga pascals this is the young's modulus within the linear region that is.

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Stress-Strain Relations in the Linear Region

- To determine tendon elongation, the tendon length at zero strain should be established.
- During in vivo measurements on human subjects, this length is usually not known exactly because at any joint position antagonist muscles can exert nonzero forces that balance each other at the joint but nevertheless cause tendon lengthening.
- This limitation causes decreased accuracy during noninvasive measurements (e.g., with ultrasound imaging methods) but can be overcome when noninvasive imaging is used in conjunction with more invasive methods, like tendon force transducer implants.
- The strain patterns in some tendons are not uniform along their lengths.
- They may exhibit stress-shielded areas and even areas subjected to compressive loading, especially at the enthesis (tendon-bone junction).

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Now if I am interested in finding the elongation of the tendon, I need to first find out what is the length of the tendon at zero strain when there is no strain what is the length of the tendon.

Because this is not very easy to find in human participants because at any joint position usually there will be an action of the antagonist, muscles are fully at rest only you know when the person is having some serious problem mostly muscles are active.

So, there is going to be some tone or some non-zero force that will always be there. So, and in any joint position there will be activity of the antagonist muscle that can exert or that can produce non-zero forces that will balance each other but can cause some lengthening of the tendon. So, this can cause some you know lengthening. So, you actually do not know what is the zero length, you do not know whether you are at zero length.

It is at some length there is no active force there is no actual action being performed but that does not mean that the tendon length at that point is necessarily zero, so you do not know where your zero is. This usually compromises with the accuracy of the non-invasive measurements for example ultrasound measurements methods, ultrasound elastography type of methods. It causes or it compromises with accuracy.

But when you use this non-invasive imaging in conjunction or along with more invasive methods like implanted force transducer in tendon. By the way these are not necessarily simple experiments to perform but suppose you do that when you complement or when you supplement in addition to the imaging technique you also have a force transducer that is implanted in the tendon, then this kind of limitations can be very easily overcome of course.

Of course, in some tendons the same patterns are not uniform along their lengths. So, a different point in the location of the tendon the strain pattern will be different. So, here I am not discussing stress strain curve here I am discussing where you measure the stress strain property also changes the sustained property on the tendon. Well, this we will discuss this in a little bit of more detail but just mentioning.

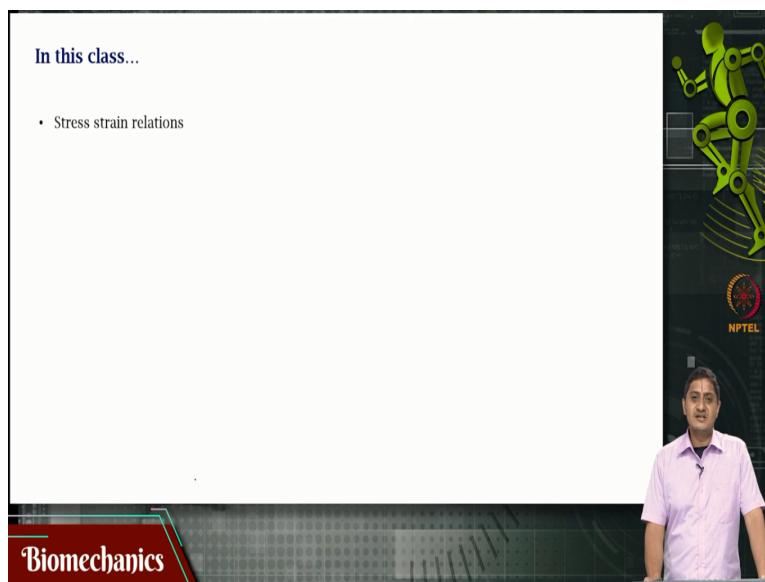
So, exactly where or which sample of the tendon you use for your testing also matters because some of them may exhibit stress shielded areas or some of them may be areas that are subjected to compressive loading. For example, at the indices is the point at which the tendon connects to

the bone or is the junction of the tendon and bone that is called as enthesis. At these points properties are likely to be very different from the rest of the tendon.

So, there are many things, there are regions of elasticity, nonlinear elasticity, linear elasticity, physiological range and outside of the physiological range that are material properties. But depending on where you are in the tendon are you closer to the muscle or are you closer to the bone the properties also change as you travel from the muscle to the bone something to keep in mind.

So, it is not uniform throughout it is not if you say this is the property of the tendon that does not mean that is a property of the entire tendon, it is that property for that sample give or take.

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So, in this video we looked at stress and strained relations in tendon, specifically we looked at two crucial models of non-linear elasticity in tendon. We discussed about linear region elasticity, we also discussed how elastic properties change as a function of where you are in the tendon are where you are measuring in the tendon. With this we come to the end of this video, thank you very much for your attention.