Biomechanics
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Lecture - 55
Elastic Properties and Stress Strain Relations

Vanakkam. Welcome to this video on biomechanics. We have been looking at mechanics of biological materials. Specifically, in the last few videos, we were looking at bone as a biological material or as a biomaterial. In the last video, we looked at the Wolffe law of bone remodeling, and Hookean and non-Hookean behavior and deviation from Hookean behavior.

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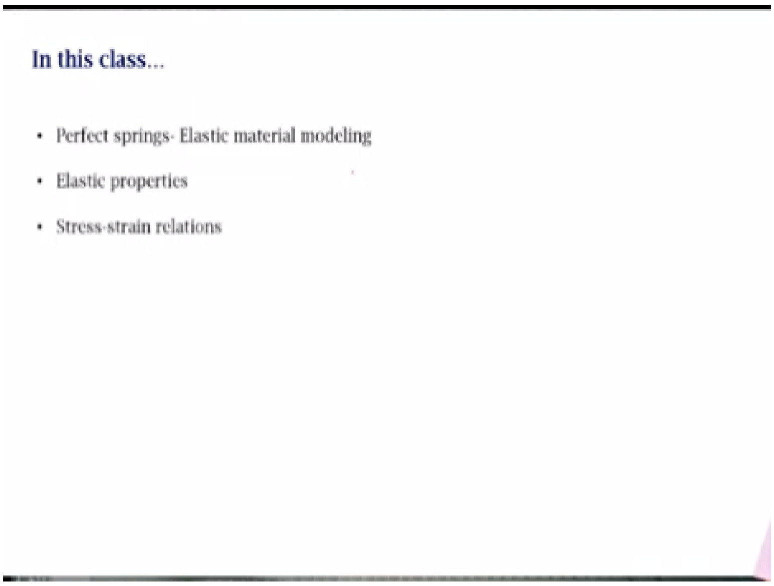

In this video, we will be looking at perfect springs or ideal springs and how would you model elastic materials, elastic properties of biological materials and stress-strain relations.

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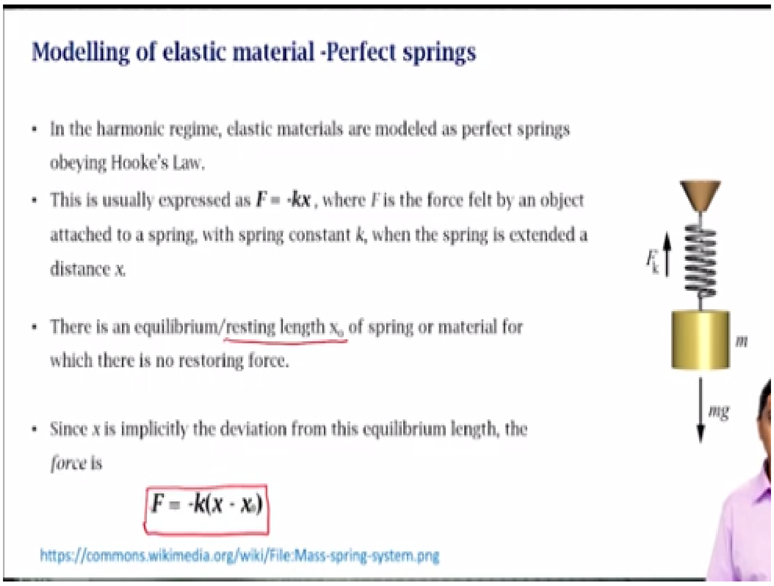

So if you have an elastic material, you model them as a perfect spring that obeys Hooke's law. So you know what the perfect spring equation looks like. The force that is developed in the spring is -kx, where the force is, F is the force that is felt by the object attached to the spring. The spring constant itself is k, when the spring is extended to a distance of x.

Also remember, there is a particular point at which the spring does not develop a restoring force or it does not develop a force or it does not respond. This length at which the spring does not develop a restoring force is called as resting length or equilibrium length, is called x naught. Now if the length of the spring is at this point, then there will be no force that it will be developing.

That also means that if there is a deviation in length from this point, the force that will be developed will be linearly dependent on the deviation from this resting length. So the resting length is x naught, the more you deviate from this resting length, the more will be the force that will be developed by this spring, but not just that. I mean more means how? It will be linearly dependent.

For linear springs, it will also be linearly dependent. That is the relating quantity will be a constant or the relating coefficient will not have x on it. So something to keep in mind. So of course, it is possible that the more the deviation from the equilibrium position, the more will be the force, but that relationship need not be linear. That is another possibility.

But here we restrict our attention to just linear springs, right. So that means force developed is a function of deviation from resting length, how far away from resting length you are. And that is a constant of proportionality. That is k.

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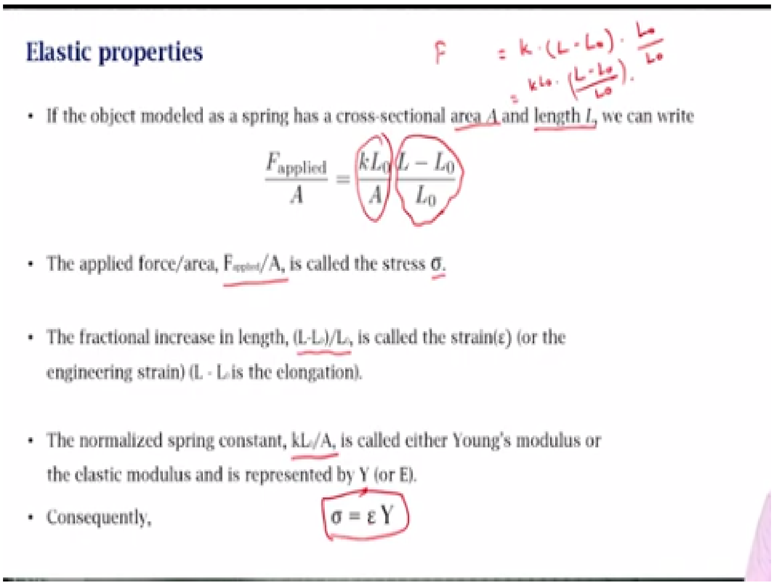

Now if the object has a cross sectional area A and a length l. Remember, let us rewrite the previous equation or the precursor to this equation. This is the restoring force. This F is, what is this? That is k times change in length. Is it not? This is what. Now let us say that I am multiplying and dividing by L naught here. You are wondering why would I do that? There is a reason why.

So I am multiplying and dividing by L naught and so that gives me you know k L naught times L minus L naught by L naught. And then I am dividing on both sides by the cross sectional area A. In RHS first I multiply and divide by L naught, having practically no other consequence. And then on both LHS and RHS I divide by area A. When I do that, I get something like this.

F applied by A is k L naught by A times L minus L naught by L naught. This force applied by A is called as the stress sigma, okay? And this quantity L minus L naught by L naught is the so called engineering strain or strain, okay. L minus L naught itself is the deformation or the elongation, okay? What is this then, k l naught by A. k L naught by A is the normalized spring constant that is called as Young's modulus or elastic modulus, right?

It is represented by Y. Because of this reason, you have this relationship between stress and strain which is sigma is epsilon Y where sigma is the stress, epsilon is the strain, and Y is the Young's modulus or the elastic modulus, okay?

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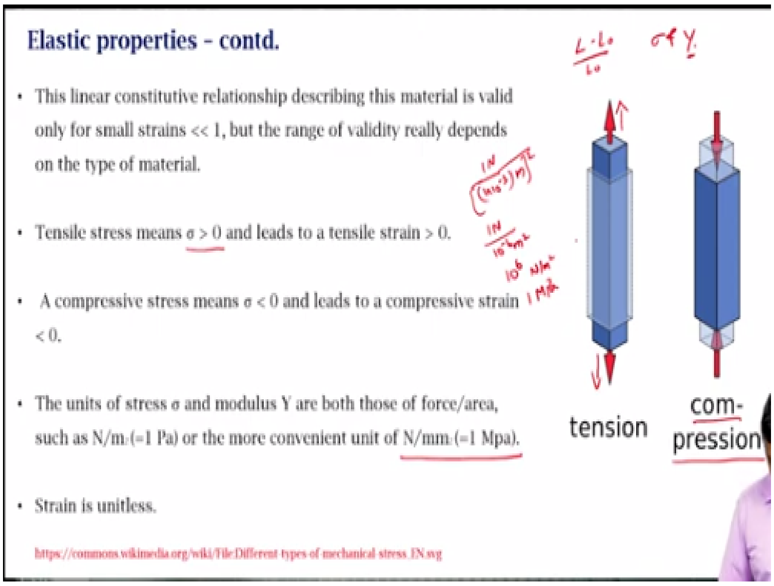

This relationship describing material property is true only when strains are much less than 1. So when L minus L naught by L naught must be very small. For very small value this is true because at other values this may not hold true, okay? But actually, this depends on the material property, the type of the material.

If the stress is applied such that the material is getting pulled in either direction, it is undergoing tension. So I am pulling the material, I am pulling the material; it is called tension. So in that case, sigma is considered to be greater than 0, which leads to a tensile strain that is greater than 0. If I am compressing the material, if I am pushing on both sides that leads to sigma less than 0 and leads to a compressive strain less than 0.

It is called compression. And these are not the only two ways in which you can apply a force or a stress. There are other ways. Also something to remember L minus L naught by L naught has no dimensions. So it has it is dimensionless. So that means, sigma and Y will have same dimension or the same unit. The unit of stress sigma and the elastic model is Y or Newton per meter square, which is 1 Pascal.

But more frequently, we use this Newton per millimeter square because 1 Newton over a very large area of 1 meter square is a very small stress, but 1 Newton in 1 millimeter square is, so 1 Newton divided by 1 into 10 power -3 meter square. That would be 1 Newton divided by 10 power -6 meter square. Or rather this would be 10 power 6 Newton per meter square, is it not?

Or rather and rather 1 MPa. 1 Newton per millimeter square leads to 1 MPa. This is more frequently used unit of the stress, okay? Strain itself is unit less because the L minus L naught has unit of meter, L naught has a unit of meter. And because these are, it is a ratio of L minus L naught to L naught it will be dimensionless, okay?

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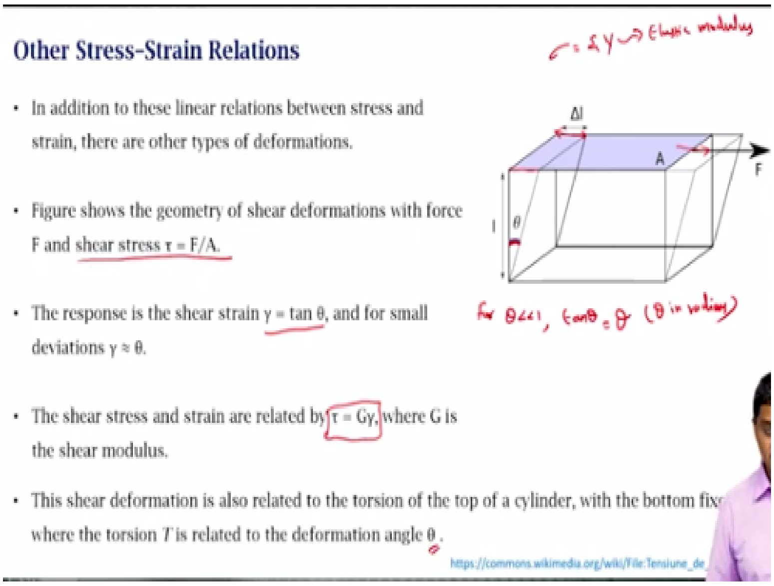

We just discussed the linear relations between stress and strain. There are other types of deformations that happen, so we just discussed what happens during tension and compression, but there is also shear deformation. So in this figure there is a geometry of shear deformation with a force F. F is applied in this direction and because of this there is a deformation that is caused in this direction.

So this delta l length is happening in this material in this direction. This is called, this so because of this force that is applied is a shear force and the stress that is applied is a shear stress tau which is F divided by A. And the response is a shear strain gamma which is tan theta for this theta.

Of course, for small deviations of theta in radians, when theta in radians for small deviations we know that for theta much less than 1, tan theta by theta is 1 or tan theta is theta, we know this right from trigonometry, for theta much less than 1 and theta in, of course when theta is in radians this is true. So for small deviations, the shear strain gamma is actually theta.

But for deviations that are comparable to 1 then gamma is tan theta, okay? And the shear stress and shear strain are related by this relationship. So you had sigma this in the previous equation in the previous formulation. Here you have tau is equal to G gamma where G is the shear modulus. Here we defined Y as the elastic modulus. Remember, this is the elastic modulus.

Note that these two are independent of each other. It is likely that one material will have a high elastic modulus and not necessarily a high shear modulus and one material may have a high shear modulus and not necessarily a high elastic modulus. So there is some relationship between these two. And that is not something that is trivial or a simple understand, that is not something that we can understand very simply.

So it is a property of the material. This deformation is also related to the torsion of a top of a cylinder when the bottom is fixed. So when the torsion is related to that deformation angle theta, when the deformation angle is theta.

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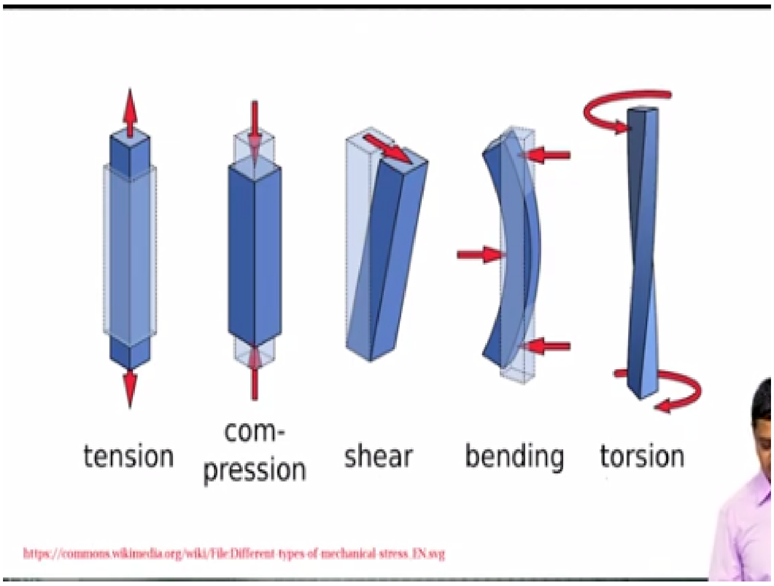

So what are the various loading that are possible on the material? Tension, pulling on the material. You are pushing on both sides. Compression. Of course, that is an exaggeration in the amount of deformation that will happen. These kind of deformations are actually large deformations, but just to show the difference, because what you can actually observe is a much smaller deformation.

And those are the ones that are usually studied and experimented upon. But unless we show a finite deformation, you cannot see it on the slide, which is why. And then a material can undergo shear and then you can bend the material, and then you can twist or cause torsion in this material.

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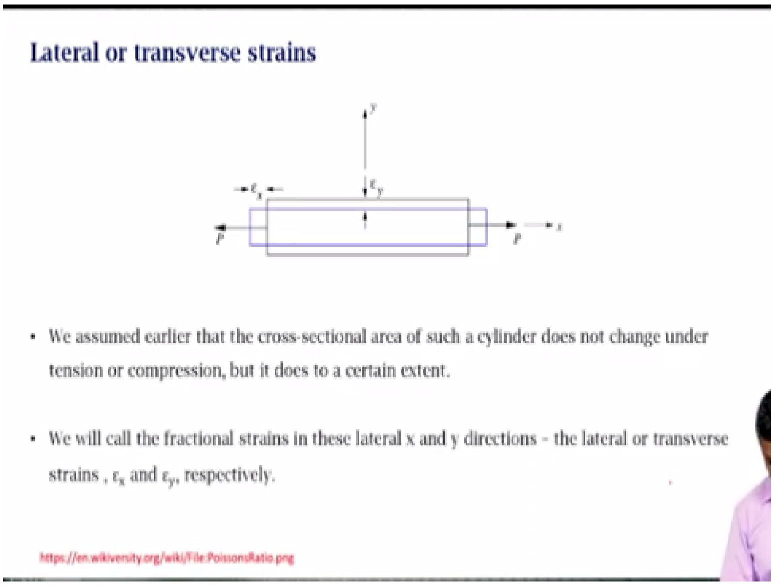

What we have also assumed is that the cross sectional area of the material, for example a cylinder does not change under tension or compression. But actually in reality, it does change. It is not always a uniformly cross sectioned object or material, okay? So there will be strains, differential strains or fractional strains in lateral directions. There will be slightly different you know strains in x and y directions, okay?

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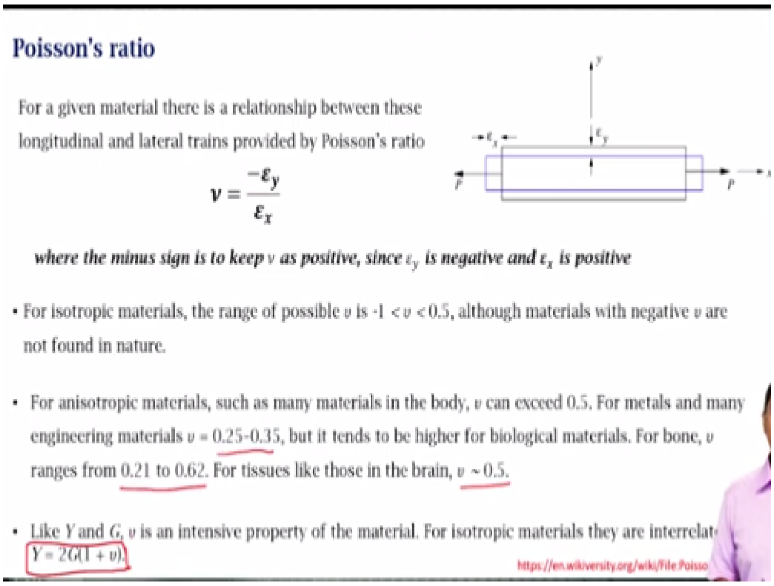

In general, you can formulate a relationship between the strain in the longitudinal direction and the strain in the lateral direction, right? These two are related by the so called Poisson's ratio. Of course, the minus sign here is kept because you want to keep nu as positive because epsilon y is negative and epsilon x is positive for this formulation.

It is known from experimentation and analysis that for isotropic materials, the range of possible values of Poisson's ratio is between -1 and 0.5, although materials with negative Poisson's ratio are not found in nature. For non-isotropic materials, for anisotropic materials, which is what are present in the human body, the Poisson's ratio can exceed 0.5.

For engineering materials such as metals right, Poisson's ratio is between 0.25 and 0.35. For biological material it is much higher. For example, for bones, it may vary from 0.21 to 0.62, relatively broad range. For tissues for the like those that are found in the brain, neuronal tissues, Poisson's ratio is about 0.5. Poisson's ratio is another material properties, an intensive property of the material.

And if that material is isotropic, Poisson's ratio relates the elastic modulus and the shear modulus using this. How did I come up with this do not ask. This is known from engineering materials. Poisson's ratio for isotropic materials are related or Poisson's ratio relates the elastic modulus and the shear modulus using this relationship. This is studied in engineering materials.

Of course, this applies only for isotropic materials. Most biological materials are not isotropic or you can assume isotropy. But then assumptions are valid only for some specific region or specific regime with which you are working, okay?

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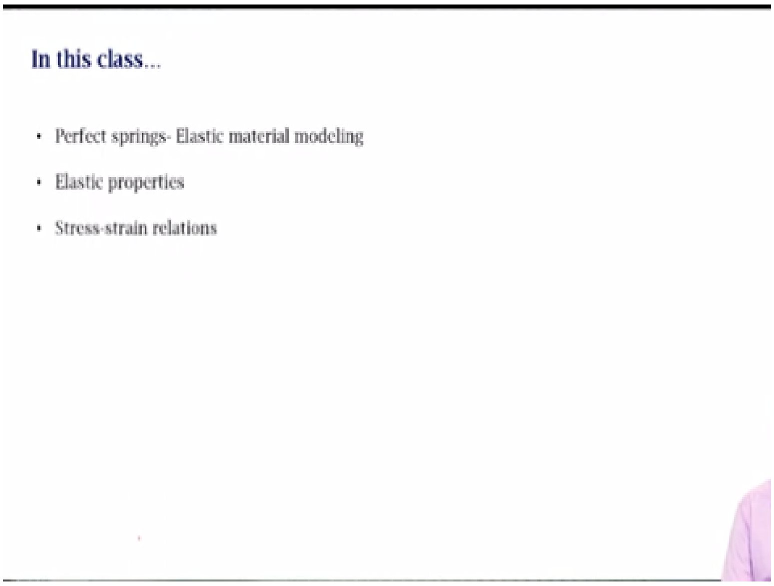

So in summary, in this video, we looked at the elastic properties of materials, how you would model an ideal spring and the stress-strain relations. We looked at the elastic modulus, the shear modulus and the Poisson's ratio, just introduced these concepts. With this, we come to the end of this lecture. Thank you very much for your attention.