Medical Image Analysis
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Lecture 34

Snake Tutorial

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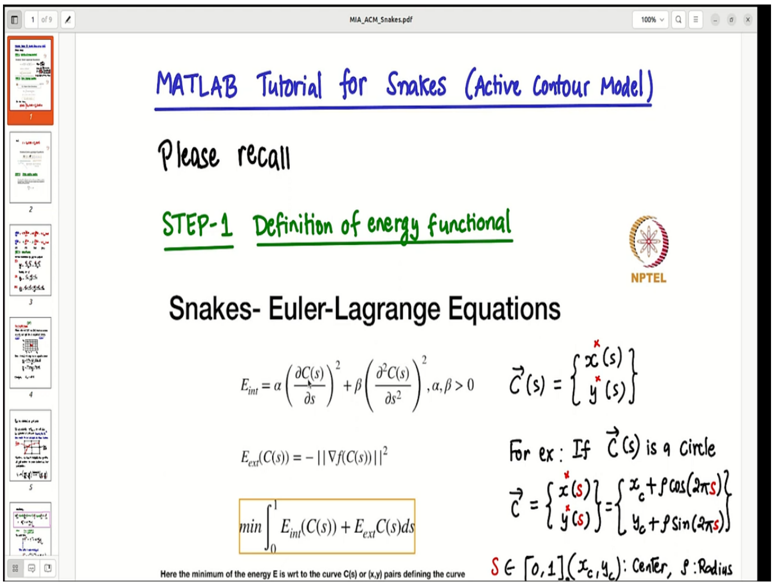

Welcome to another MATLAB tutorial video for the Medical Image Analysis course taught by Professor Ganapathy Krishnamurthi. So, today we will discuss 3 Active Contour Models for image segmentation. These models are already taught by the professor and please go through them before starting this tutorial. And the 3 models are snakes, geodesic active contours and Chan-Vese segmentation.

So, in this tutorial, first of all we will discuss the mathematical formulation. Of course, we will not go deep into the theory. For theory, you can always refer to professor's lecture but we will just touch the essential steps for mathematical formulation. Then we will discuss the discretization and then finally we will show some MATLAB codes to see these implementations in action. So, let us start with snakes.

So, snake, as you have read in the theory, this is one of the techniques. So, indeed, what we do? For example, if I have an object O and I want to segment it from its surroundings. So, what I do is I throw a snake around it, as I must say. So, snake is basically a curve. In this in your screen, we have this C. This C represents the curve and this curve C has various points.

So, these are called snake coordinates or snake points. So, we throw something outside the object and then we have to make sure that this snake evolves in such a way that it finally captures the boundary of the object that we want to segment. So, for that what we do is first we define an energy functional and then we use Euler-Lagrange equation to find the correct curve that minimizes the energy function. So, this is the overall idea.

So, for example, in this on your screen we have the definition of the energy functional that we use for snake. So, you can see it has two parts. First is internal, second is external. And the total function that we have to minimize is basically sum of these 2. Here, C represents the snake, the equation of snake and you can see C is a function of s. So, s is the contour parameter and as sir discussed this takes value between 0 to 1 and C can be any curve.

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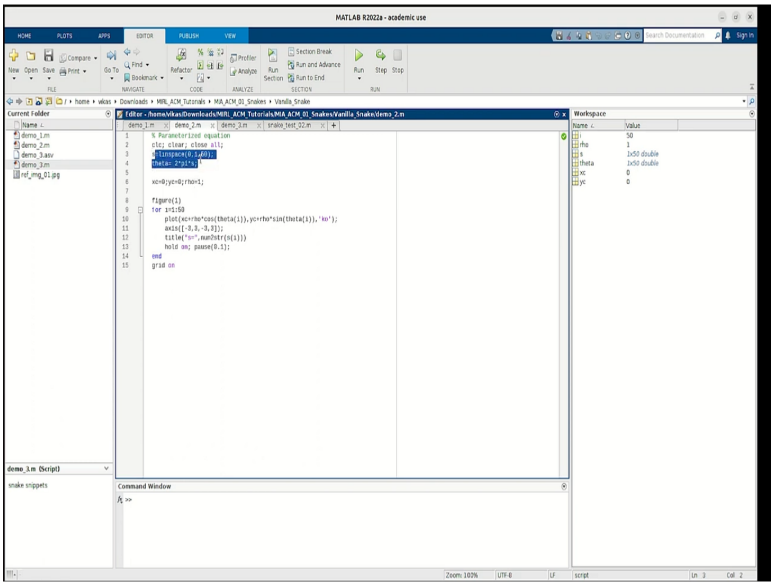

For example, in this case I am just showing a simple example of parameterized curve. For example, circle. So, here you can see s varies between 0 to 1 and here the centre of circle is $(0, 0)$, radius is 1.

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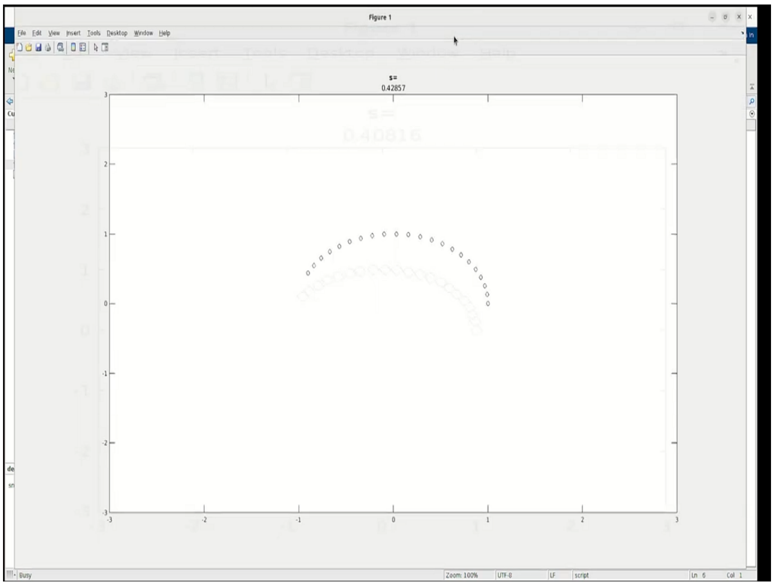

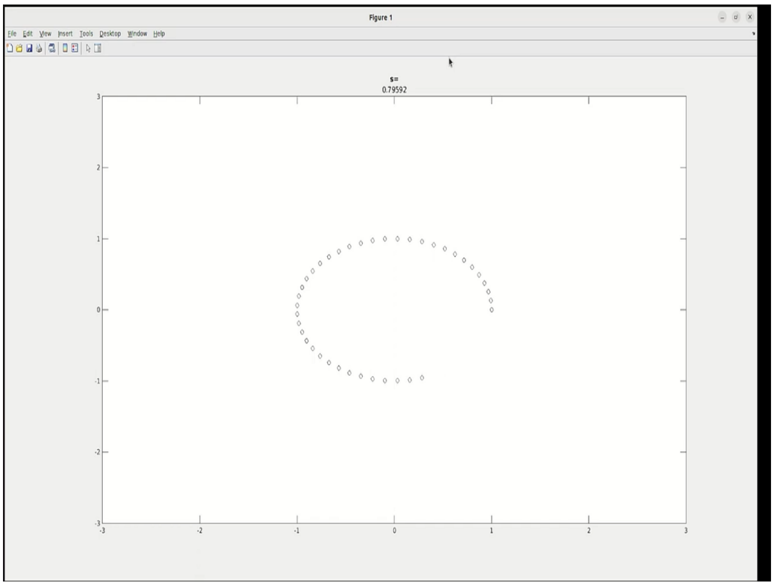

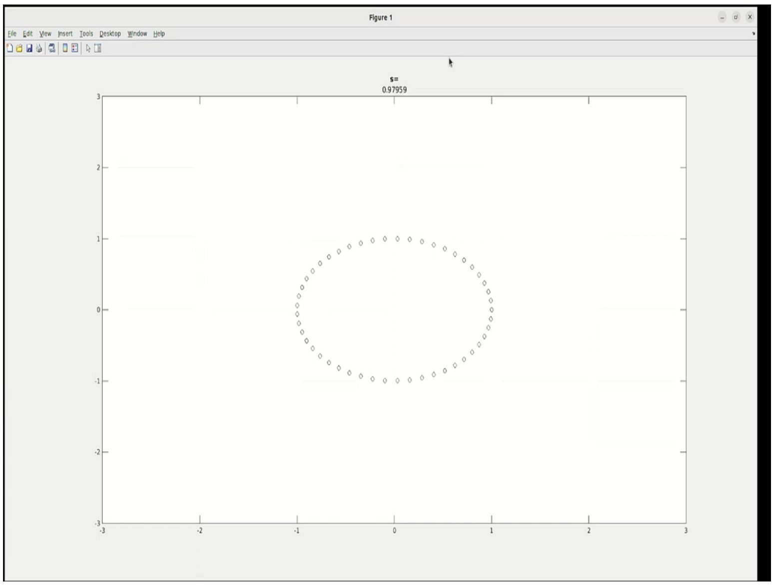

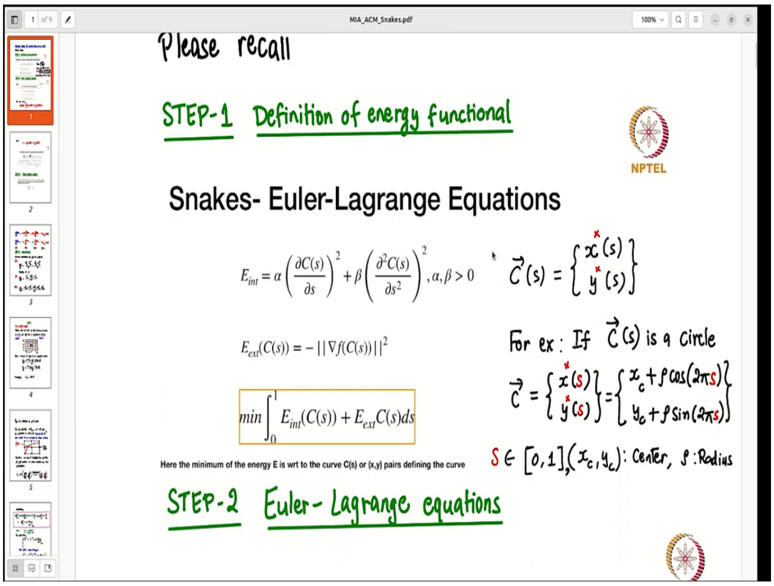

So, you can see here as the value of s goes from 0 to 1, it completes the circle. So, just see one more, one more time. The value of s starts from 0 at this point and as s goes from 0 to 1, we have a complete circle. So, similarly, in this case rho is a constant. In other case, rho can be a function of theta or s that is why here, it is written that it is a function of s. C is a function of s.

And the MATLAB code that I showed is basically, this implements this, the expressions that I have written in the right hand side here: $x_c + \rho cos(2\pi s)$ and $y_c + \rho sin(2\pi s)$. So, if we start with for example a circle, but over the time, this curve will evolve and finally it should like come closer to the boundary of the object that we want to segment.

So, this is the Euler-Lagrange equation. So, this is the energy functional. The external term, this external energy is also called a data term. Because in our case image is our data. And this internal energy is also called regularization term because it essentially represents the property of the snake, bending property and stretching property, etcetera.

For example, the first term is a representative of stretching property of the snake and second term is representative of its bending property. So, without going to the mathematical details, so, we have this functional. We can use Euler -Lagrange equation, which sir taught. I am taking this slide.

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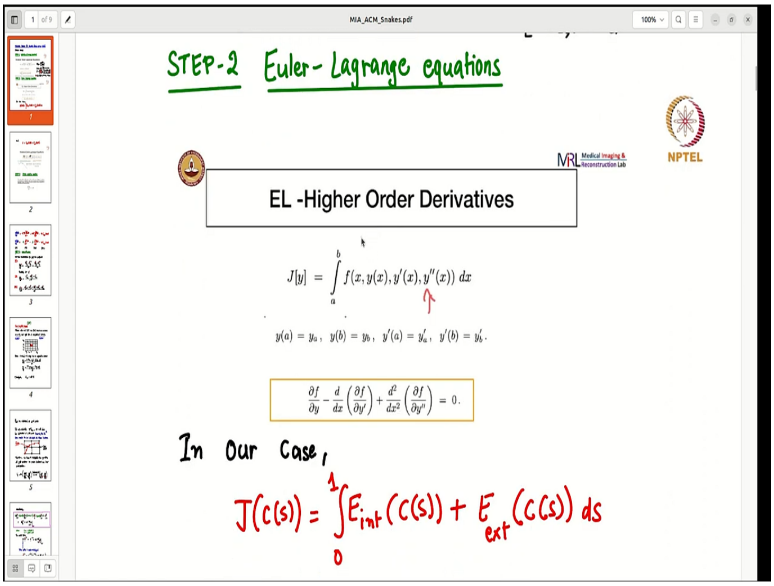

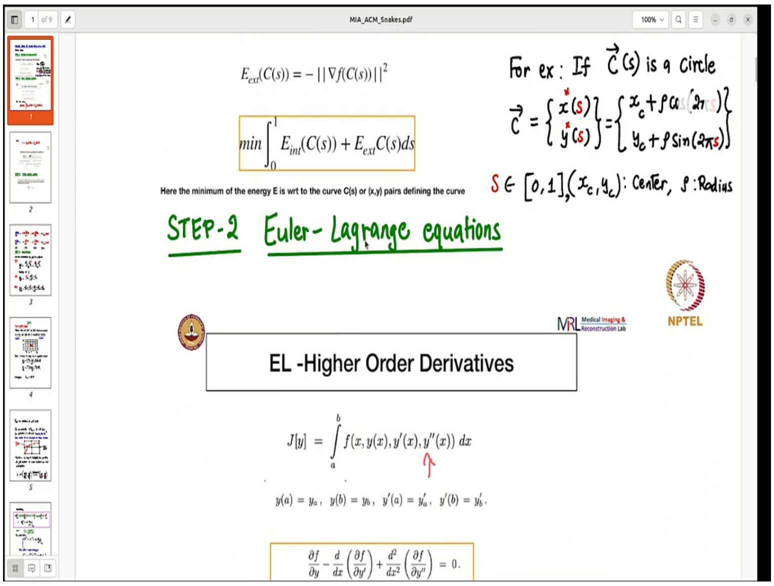

So, because it involves higher derivatives, you have to use this equation. So, what is f here? So, for using this equation, you have to know the value of f.

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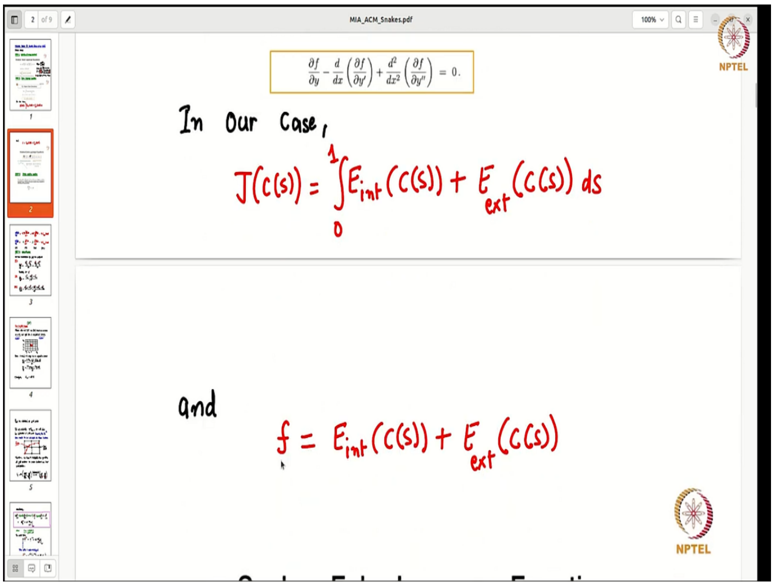

So, in our case, because j is this. That thing that we have to minimize. The value of f is this. So, if you plug this value in this equation, you will find this.

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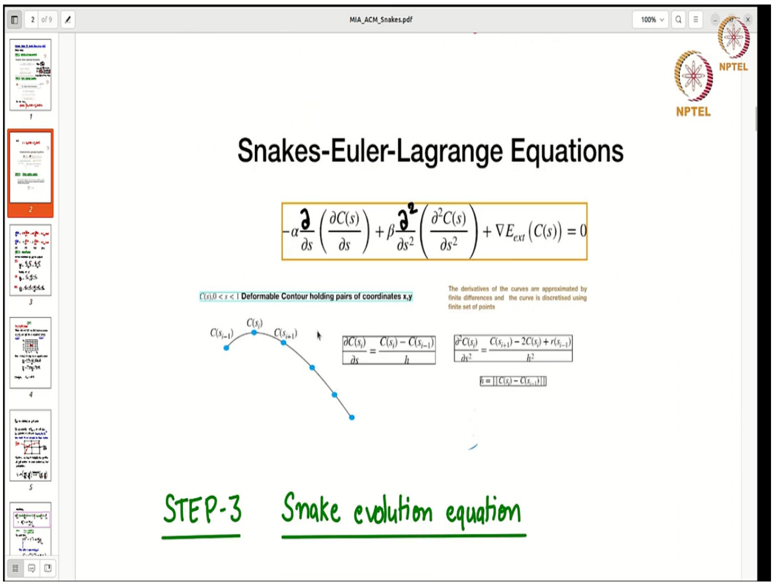

So, this is snake's Euler-Lagrange equation wherein you can see we have this. $-\alpha\partial/\partial s$. So, basically it is… you are taking derivative so, this is second order term, second order derivative. Then we have beta times a fourth order derivative plus some gradient of external energy. So, these 2, the first 2 terms are sort of coming from internal energy and second term is coming from the external energy.

So, because these are derivatives and we cannot, we do not have equations, so, we will be using numerical approximation to find these derivatives. We will explain this. So, this is the equation. This is the main snake equation. But because we have some initial condition and we have to evolve it, so, somehow we have to introduce the parameter of time. So, for that what we do?

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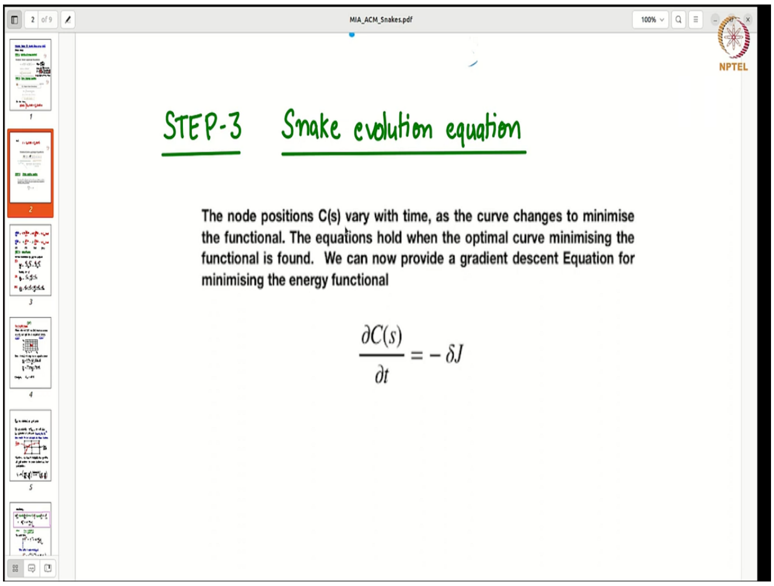

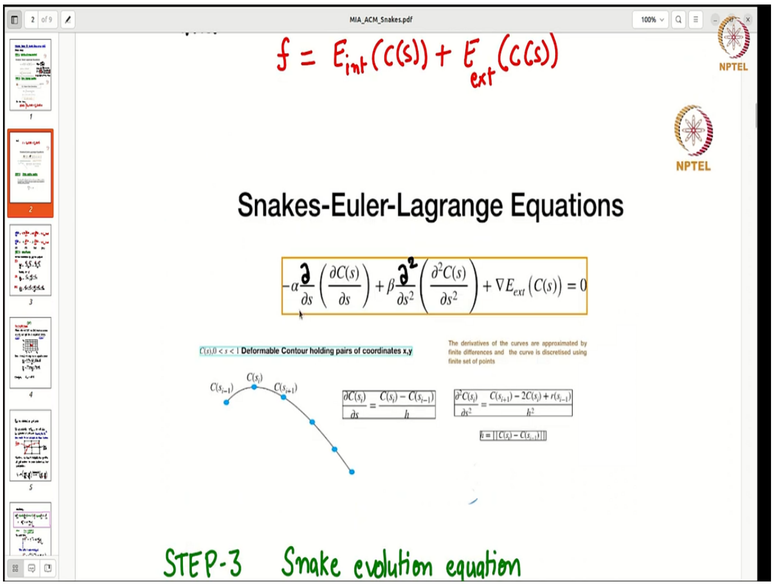

We introduce a pseudo unsteady term. So, for example this technique is common for even diffusion equation. For example, you can solve a diffusion equation by as a pseudo transient problem. So, in the same way, although we have to solve this equation, the value of C that correctly solves this equation will segment the object properly.

But we have to reformulate the problem in such a way that takes our initial snake contour to the correct one, which is solution of this one. So, that is why we introduced this term $\partial C/\partial t$. So, it looks like an initial value problem and where initial condition is specified by us. So, basically we solve this equation. We try to solve this equation. So, our equation becomes like this.

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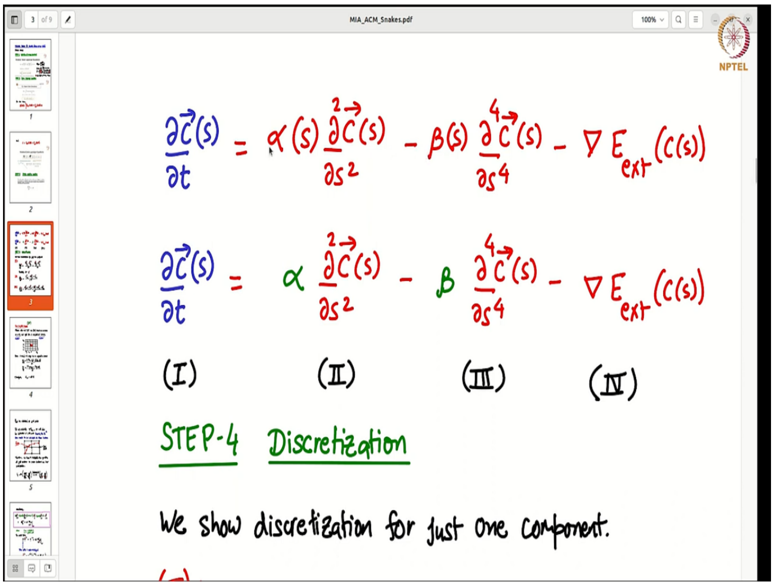

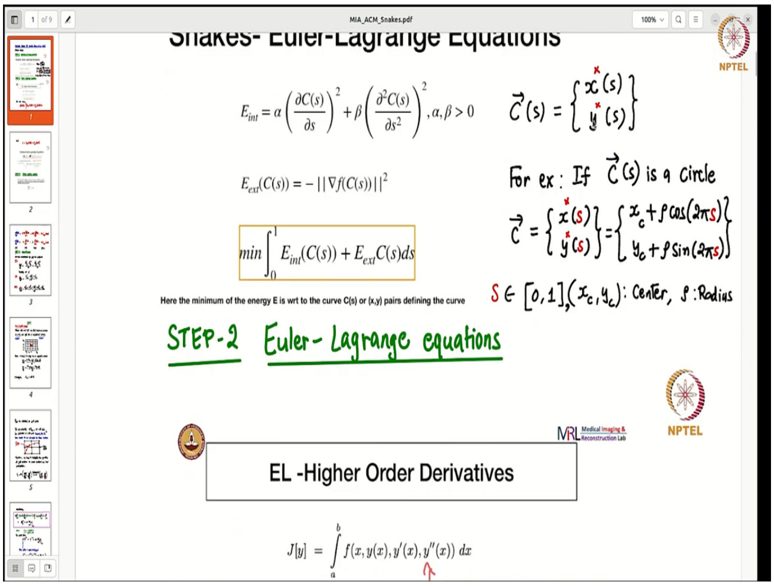

$\partial C/\partial t$ alpha times minus beta times minus this. So, here alpha and beta are the parameters kind of weights for these terms. So, they can be a parameter. They can also be treated as variables, a function of s but we approximate that they are constants. So, this is the main equation that we have to solve. $\partial C/\partial t$ is equal to alpha times this minus beta times this minus this.

Now, you can see, this equation has 4 terms. First, second, third, fourth. So, this we have to solve numerically. So, now, we come to the discretization. So, the first term $\partial C/\partial t$. So, also for example, this, because I have already shown this like C vector is x * and y *. So, whenever I write $\partial C/\partial t$ it means its derivative of it is equal to ∂x * $/\partial t$ and ∂y * $/\partial t$.

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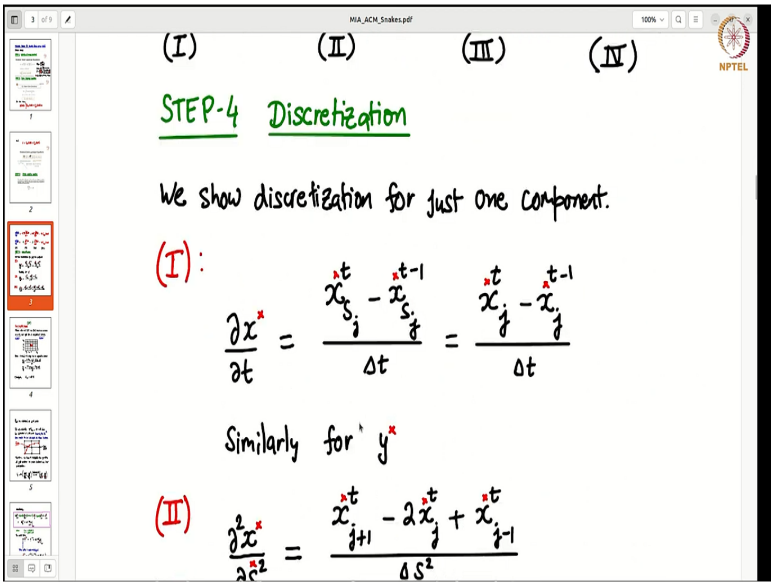

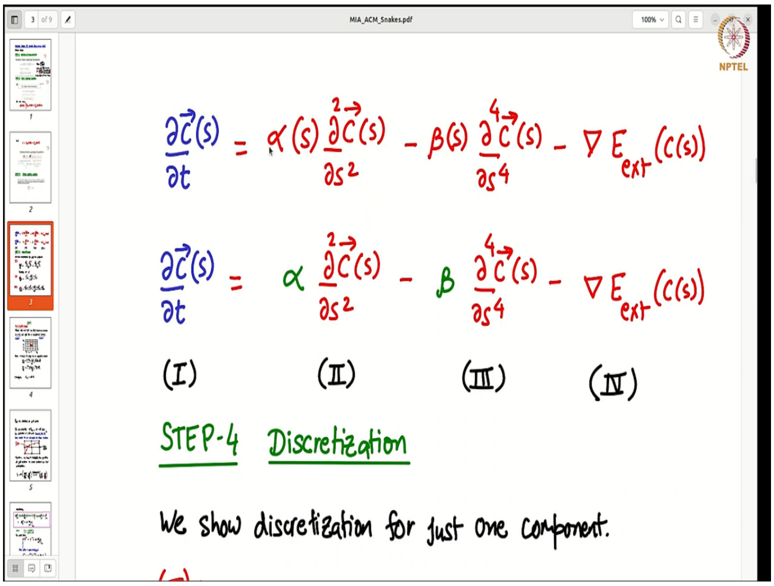

But because of the symmetry of this equation, I am just showing the discretization for one component. Other component discretization is also similar. So, for example, $\partial x/\partial t$ is, we are using backward difference for $\partial x/\partial t$. So, $(x_j^{*t} - x_j^{*t-1})/\Delta t$. And x star at s j. So, s is the contour parameter but for shorthand, we are just writing x j at t minus x j at t minus 1 upon delta t. Similarly, for y I will do. Now, for second and third term.

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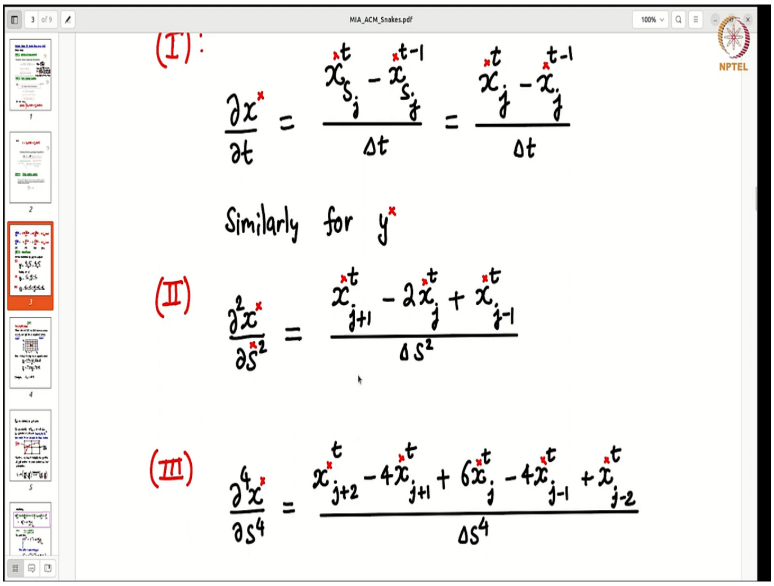

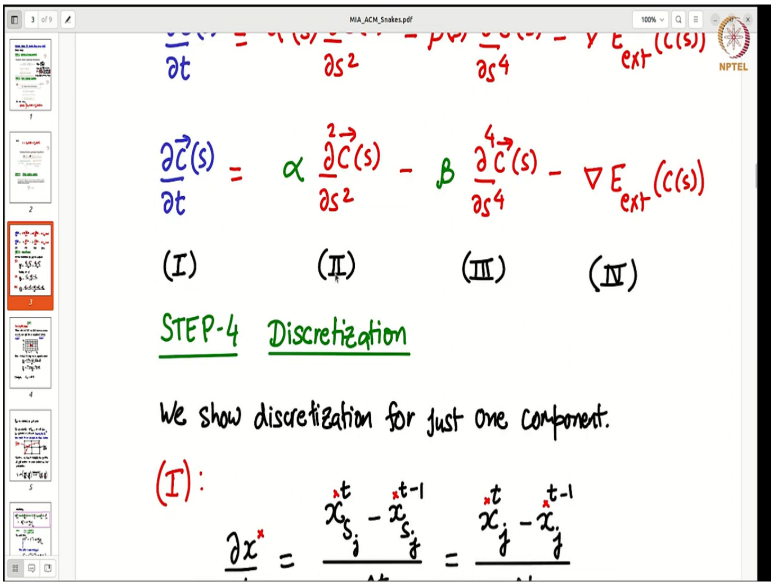

Because alpha is anyway constant, we are going to use central difference. So, you can see x j plus 1 minus 2 x j plus x j minus 1 etcetera. Similarly for fourth derivative. Please note that these second and fourth derivative or the internal energy terms, they are that we have calculated these terms at time t at the present time. You can see.

And backward difference, we have 2 times, present time t and past time t minus 1. So, in numerical methods, typically when we solve, when we use explicit methods then the for example, if we are solving some unsteady problem, so, in fully explicit method what we do? We calculate the spatial derivatives at past time and in fully implicit methods, we calculate special derivatives at present time.

So, here you can see these are the spatial derivatives, spatial derivatives and we are calculating these derivatives at present time. So, it looks like it is an implicit method but you will see that the fourth term here, although second, third are calculated at present time but the fourth term here is going to be calculated at past time. So, that is why the second and third term will implicit and the fourth term at t minus 1, so, explicit. So, this is a, this method is called semi-implicit.

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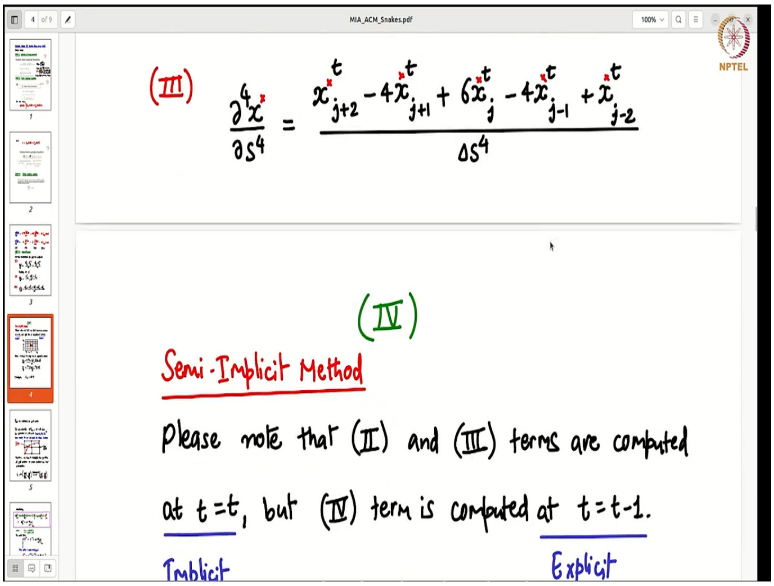

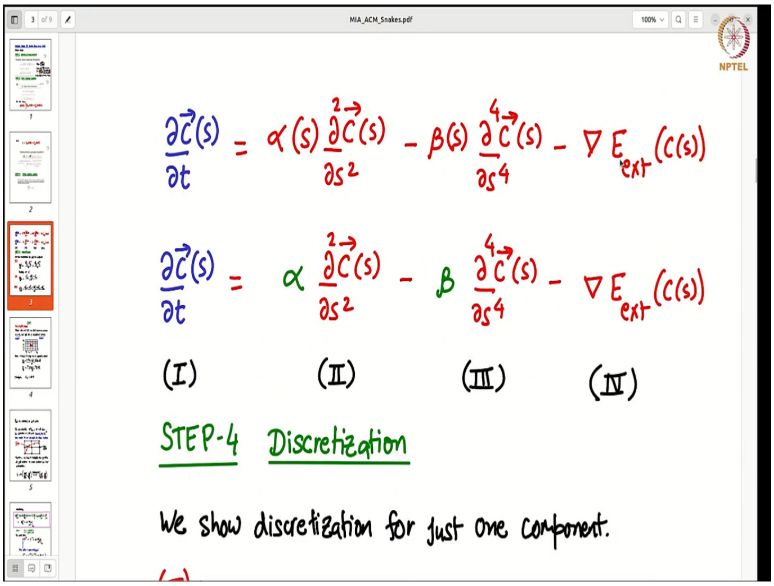

So, the fourth term is this, gradient of E external. So, how do we calculate this? So, I am just taking a small example.

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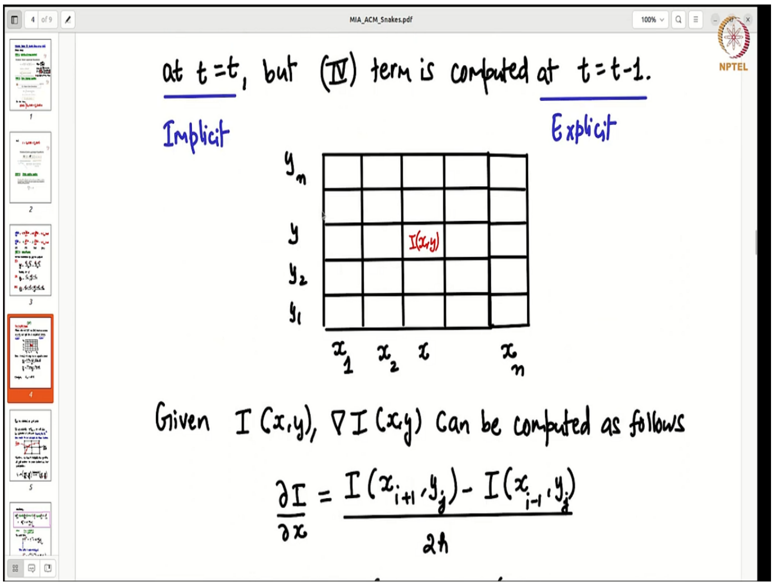

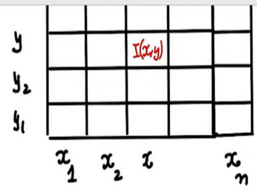

Given $I(x,y)$, $\nabla I(x,y)$ can be computed as follows

$$\frac{\partial I}{\partial x} = \frac{I(x_{i+1}, y_j) - I(x_{i-1}, y_j)}{2h}$$

$$\frac{\partial I}{\partial y} = \frac{I(x_i, y_{j+1}) - I(x_i, y_{j-1})}{2h}$$

x_1 x_2 x x_n

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Therefore, $E_{ext} = -|\nabla I|$

$$\frac{\partial \vec{c}(s)}{\partial t} = \alpha(s)\frac{\partial^2 \vec{c}(s)}{\partial s^2} - \beta(s)\frac{\partial^4 \vec{c}(s)}{\partial s^4} - \nabla E_{ext}(c(s))$$

$$\frac{\partial \vec{c}(s)}{\partial t} = \alpha\frac{\partial^2 \vec{c}(s)}{\partial s^2} - \beta\frac{\partial^4 \vec{c}(s)}{\partial s^4} - \nabla E_{ext}(c(s))$$

(I) (II) (III) (IV)

STEP-4 Discretization

We show discretization for just one component.

(I): $x_i^t - x_i^{t-1}$ $\quad x_i^t - x^{t-1}$

Suppose, I have an image which we can represent as a matrix I x y and I want to calculate its gradient because gradient of image represents edge. And so, why we are calculating this grad I, please refer to the paper and the lecture? So, grad I can be calculated like this, using central scheme.

And in our formulation, external energy is equal to minus of magnitude of this grad I, although it can have other terms also like curvature terms and then density terms but I am considering just this one. So, given an image i, I can calculate its gradient and then I can calculate its external energy. Now, the term here that we have, the fourth term, it is gradient of E external. So, if I again take the gradient of E external, what will happen?

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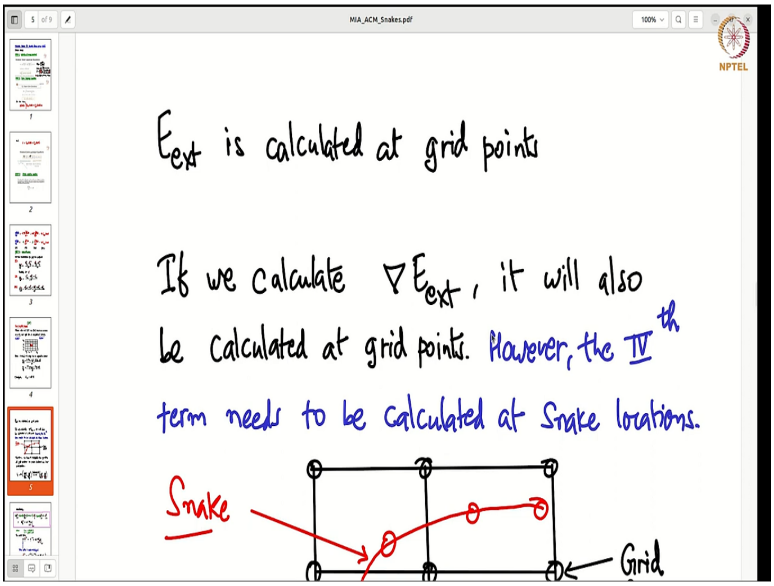

It will, because E external is calculated at grid points, if I calculate a gradient of external, E external then it will also be calculated at the grid points. However, in the formulation, the fourth sum needs to be calculated at snake locations.

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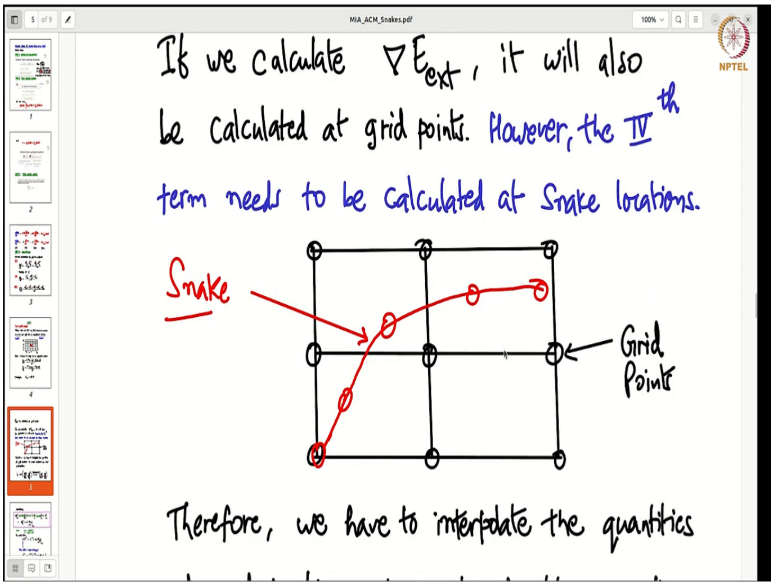

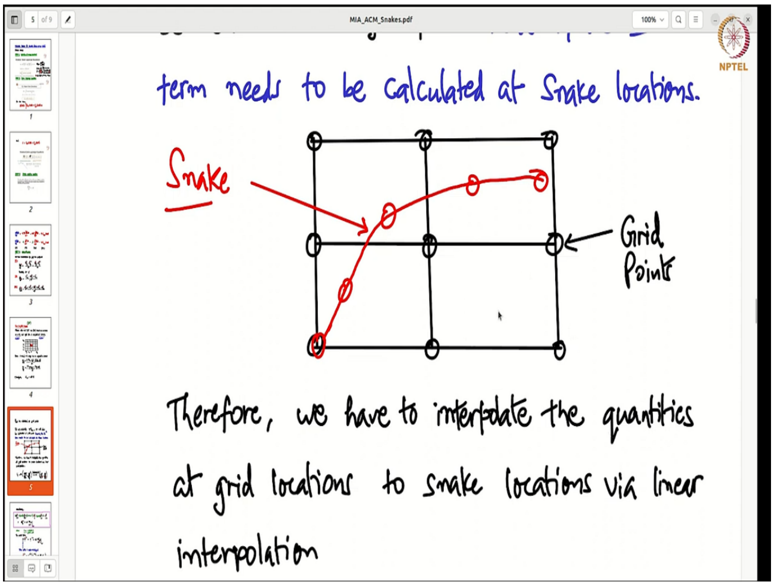

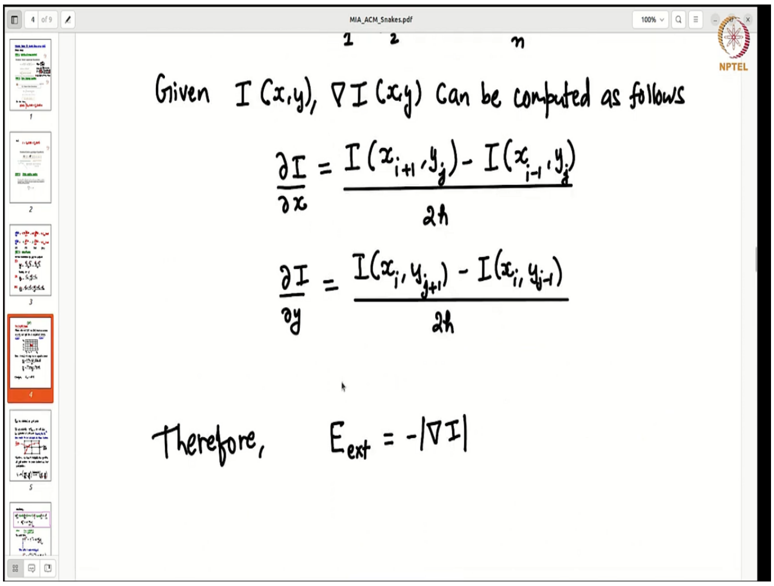

So, for example, this is my grid. So, these black circles represent grid points. So, if there is, it is not necessary that our snake points always lie on the grid points. For example, in this figure you can see, these red circles represent snake points and they are not lying on the grid points. So, what we have to do?

To calculate this grad E external, we have to interpolate the quantity that grid locations. So, we will calculate. So, first given image i, I will calculate t external and then I will again take the gradient. This gradient will give me value of grad E at these grid locations. And then finally what I will do?

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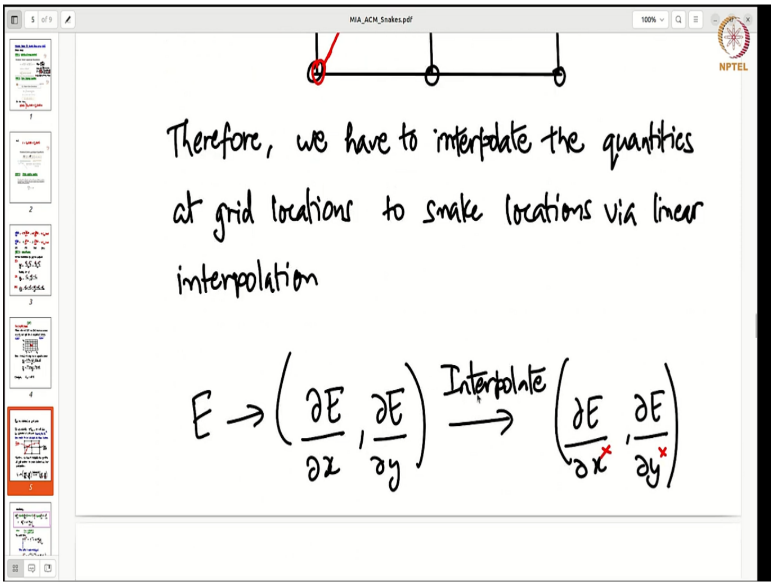

I will interpolate, use some linear interpolation or bilinear interpolation. Basically, linear interpolation. And I will interpolate these values at snake locations. So, just to show what I mean, I will show you something.

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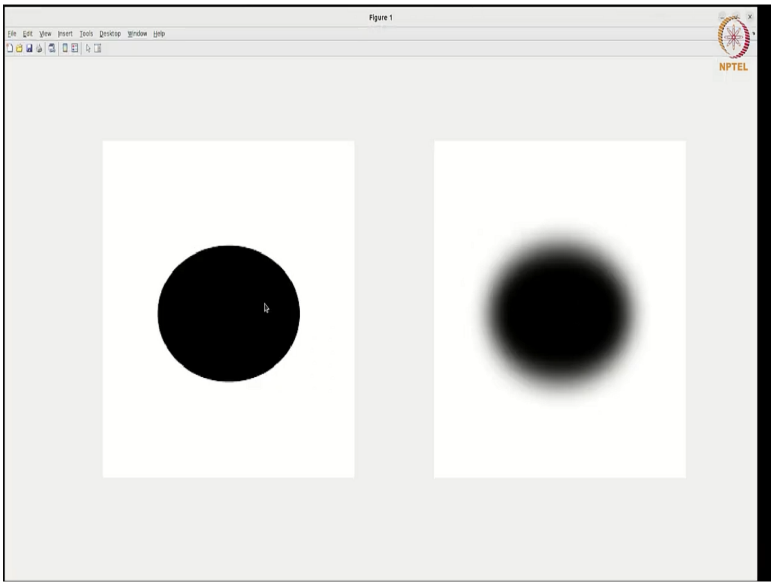

For example, I am just showing you. Suppose, I have an image like this, a black circle. Here, I have made it blur, so that its gradient, its boundary gets little bit diffused and this is done by, this is done in the paper as well.

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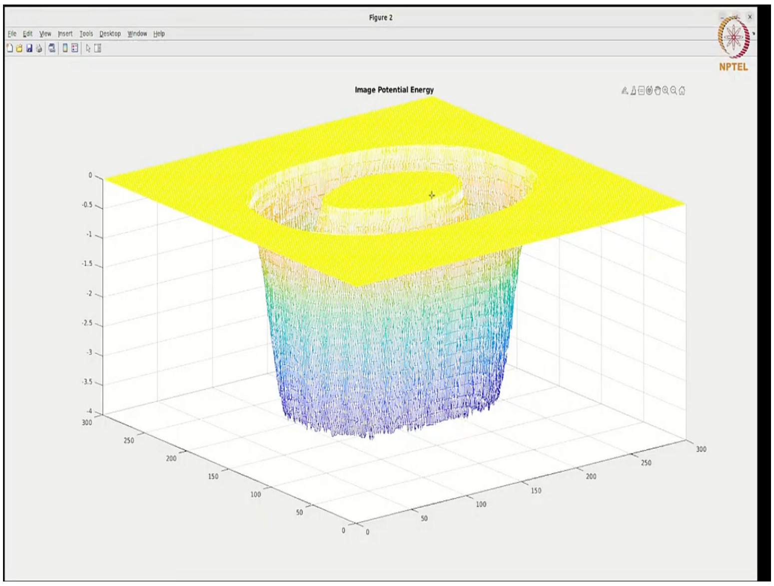

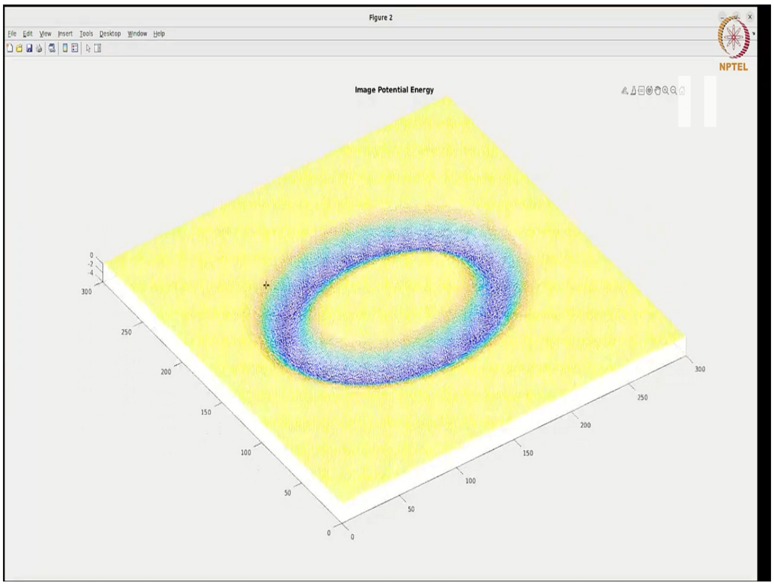

So, the effect of this blurring is this. If you will see here, it is like a well. So, if someone comes here, he will be slipped into this well. So, if we would not have used this blurring, this will be very small. But because of blurring it becomes this well becomes little wide and actually snake should come and fit into this well. This is what we want. So, this is the like plot for, this is the like surface for potential energy, E external. Now, what happens? Now, the third thing.

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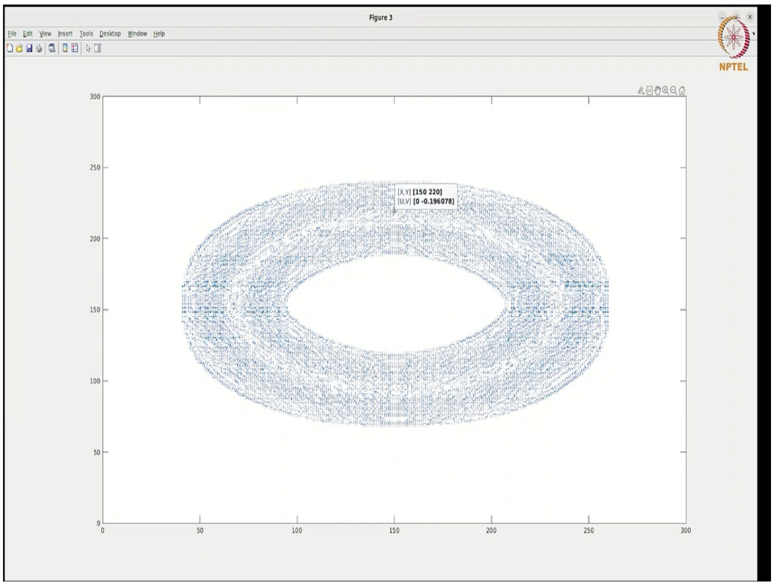

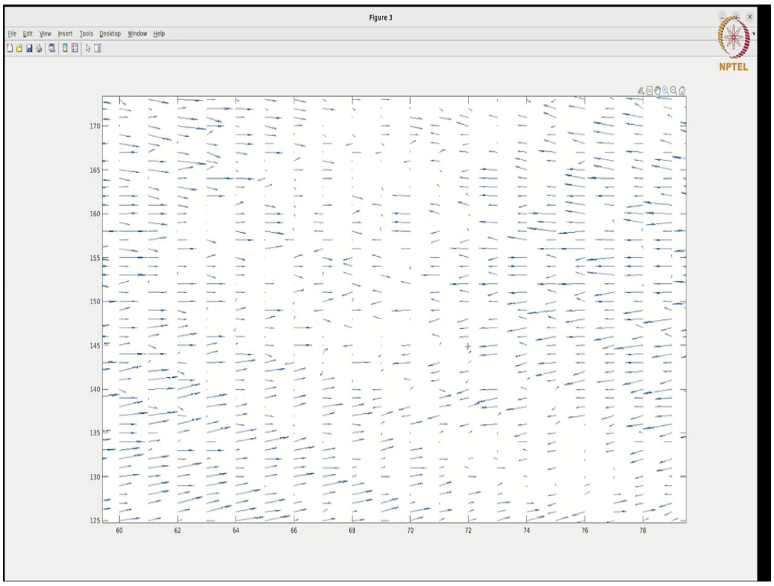

So, because of this. So, now, we have some external energy and if we take its derivative, we will find the field forces, just like we have a gravitational field. But in gravitational field, all the like if I make a gravitational field, so, at all points, we face like feel the same force and downwards. So, it will be just arrows pointing downwards of all of same length, 9.8 meter per second square.

But here you will see. So, like little bit magnify it. You see your image field like the field that you have, the force field that you have is not a constant but it has different directions also. So, for example, look here, maybe you see here. The forces are in rightward direction. It is although they are small but you can still see this is pointing towards.

Here, they are pointing towards left. So, if a particle is lying here, it will feel the force in the direction. So, it will try to come here and if it is here, it will try to come here. And the well the potential well, it is somewhere in between. For example if you can appreciate if I make it little, sorry.

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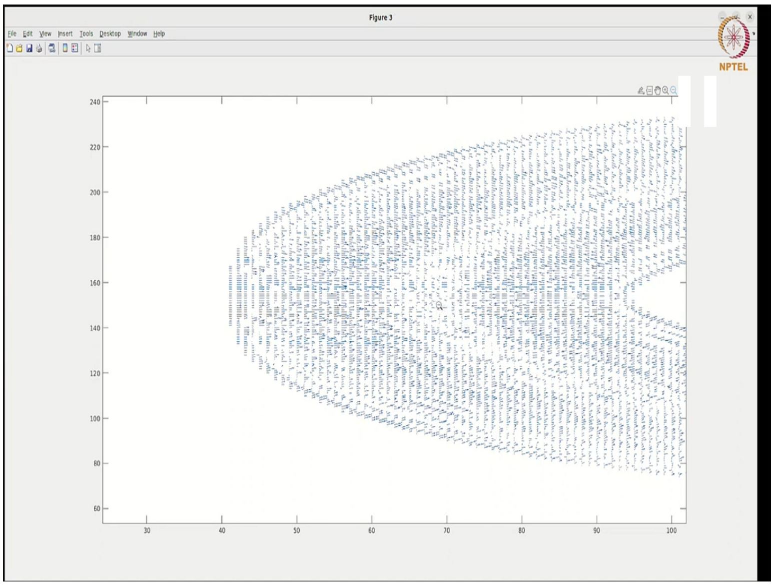

So, you see, the original thing is here and if you are here, the force will take you towards the edge and if you are here, it will again take you towards this edge. So, the minimum energy is at this point. So, this is called quiver plot. This is not very good for this image but you at least get the idea. I will show a cleaner picture now. So, our objective is to like minimize energy and we have a force field that forces, these snake points towards this potential well. So, because here, it was not very clear, I will show you something else.

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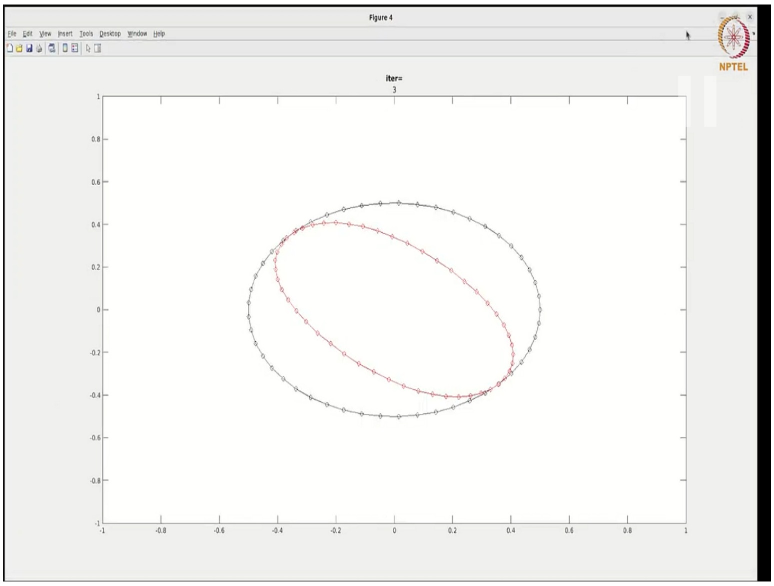

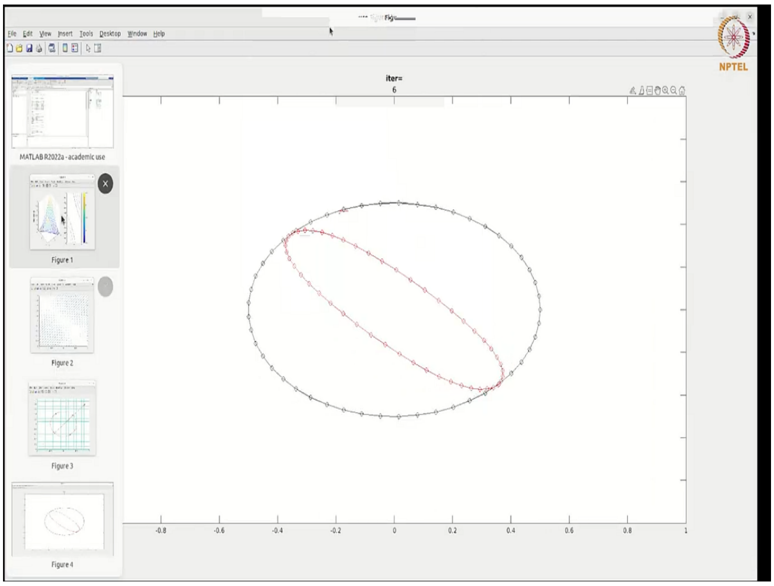

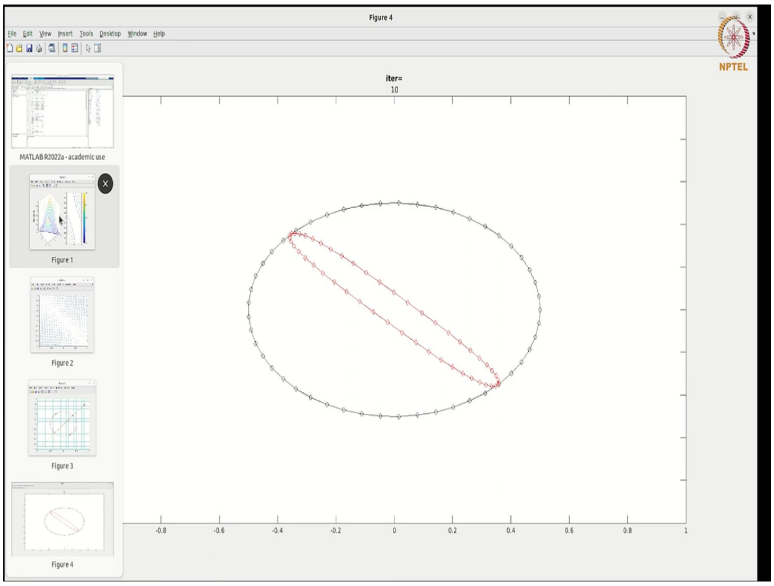

So, for example, I have created a field. This one it will show completely then I can.

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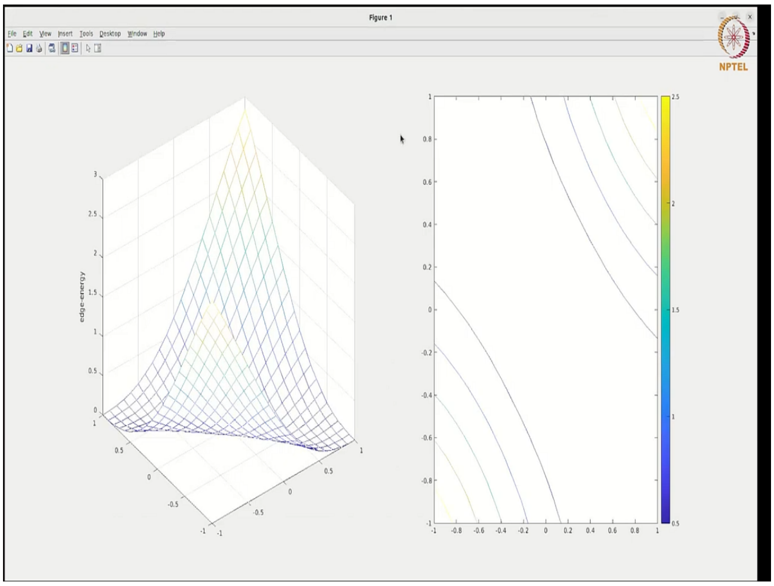

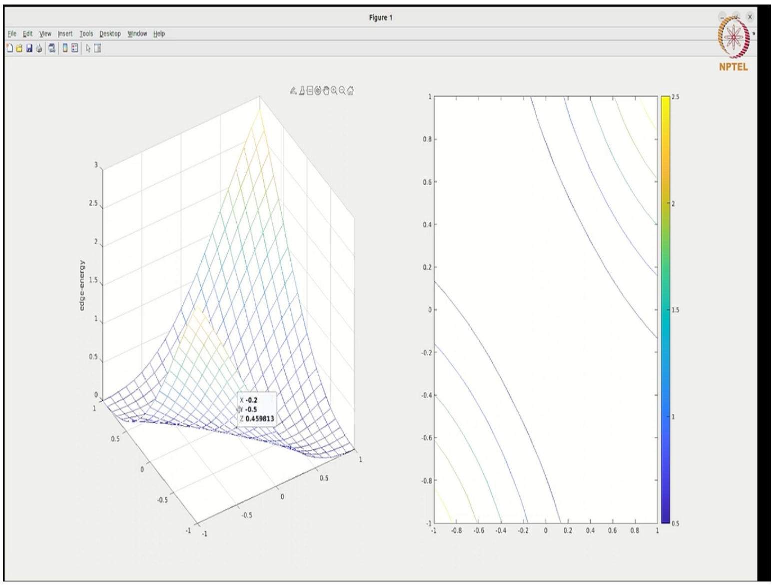

So, for example, suppose, I have some image and I calculate its gradient and find the energy due to edge and it looks like this. So, clearly you can see here along the diagonal 1, 1, along this diagonal, the energy is minimum. If you go little left, it is increasing you go little light, it is increasing.

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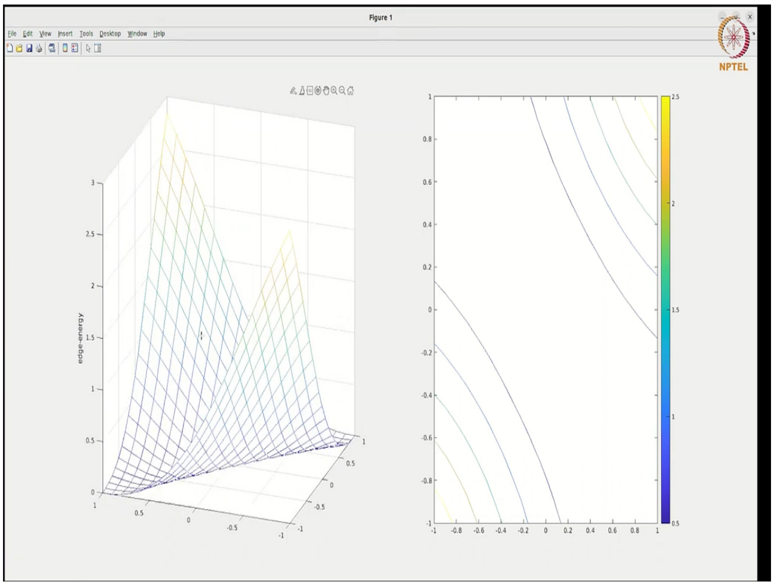

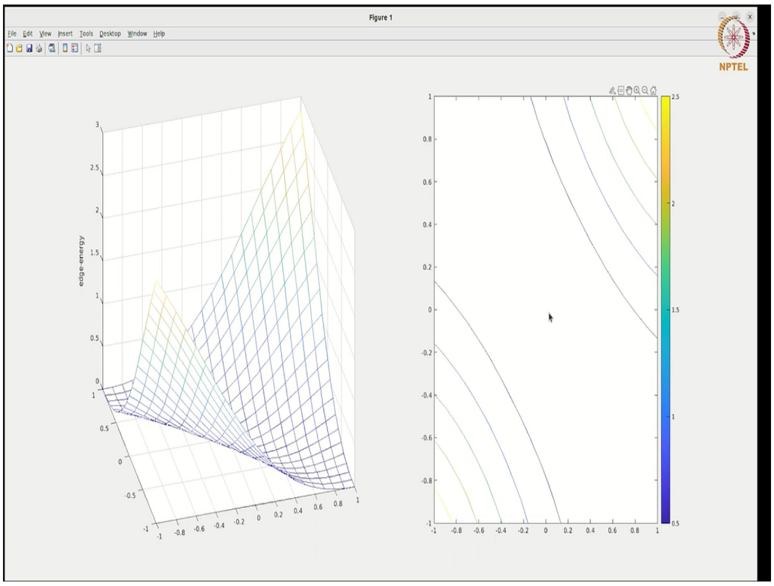

So, its minimize at this diagonal and you can also see this from the contour plot that the value is minimum 0.5 here and as you go here it is increasing and if you go towards this side, it is also increasing. So, the minimum energy is at in the diagonal. So, actually, you can think of like this diagonal represents the edge.

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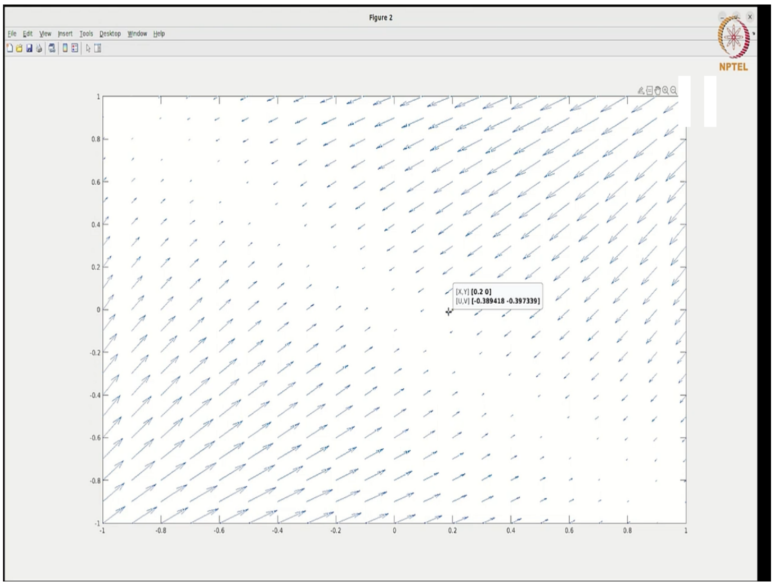

So, if you have an energy function like this then the forces will be like this, the force field. Though at that time I was not able to show the skewers properly. So, you can see, because the edge is along 1 1. So, if a particle is here. So, it will feel a force here. It will be forced towards this edge and as it becomes, as it comes closer to the edge, this force becomes less and less.

Similarly, here also, if particle is here, it feels, suppose, it comes here and it comes. And as it comes closer to this edge, it becomes, the force becomes less and less. And once it come, like settles in this edge, there is no force. Or this is the point of minimal energy where snake should actually be settled.

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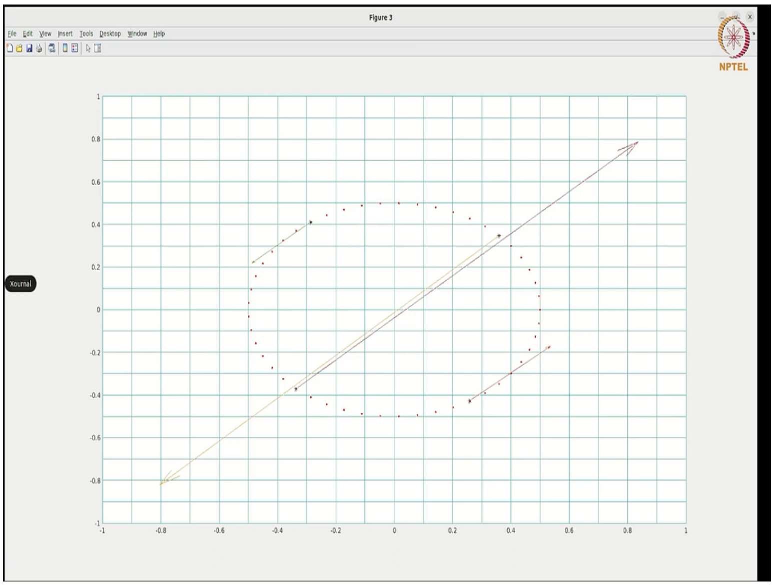

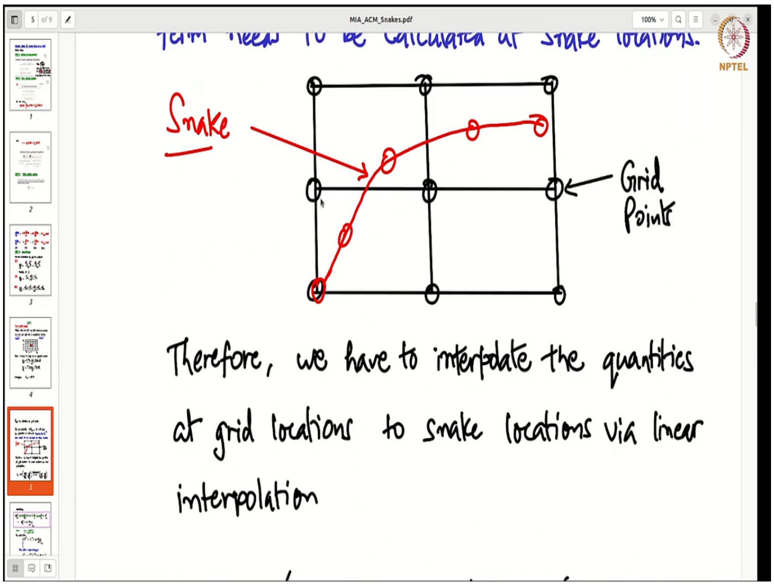

Now, as I was saying earlier like our snake points need not to be coinciding with the grid point. So, for example, in this case I have this grid, these green lines. But our snake points, these are red points, they are some of them are not falling on the grid points. So, what we have to do?

We have to interpolate. So, we calculate like gradients, basically because gradient of energy gives us the force. So, we calculate the forces at grid points and then we interpolate them in the snake locations. So, here you can see, we have calculated forces.

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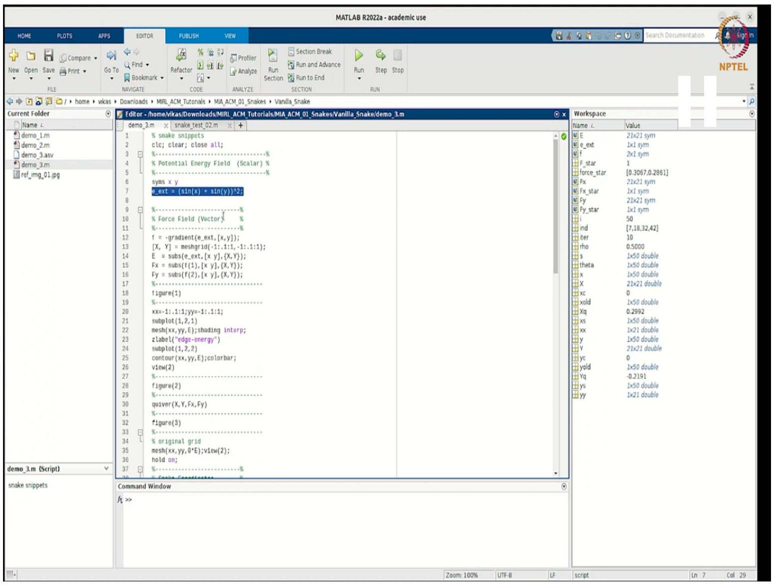

So, here you can see, we have this energy, we took the gradient of it. So, this is the force field.

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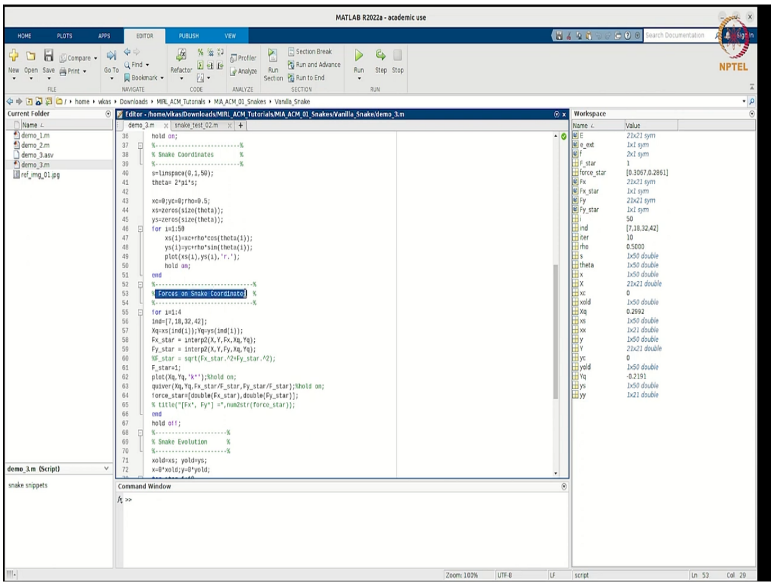

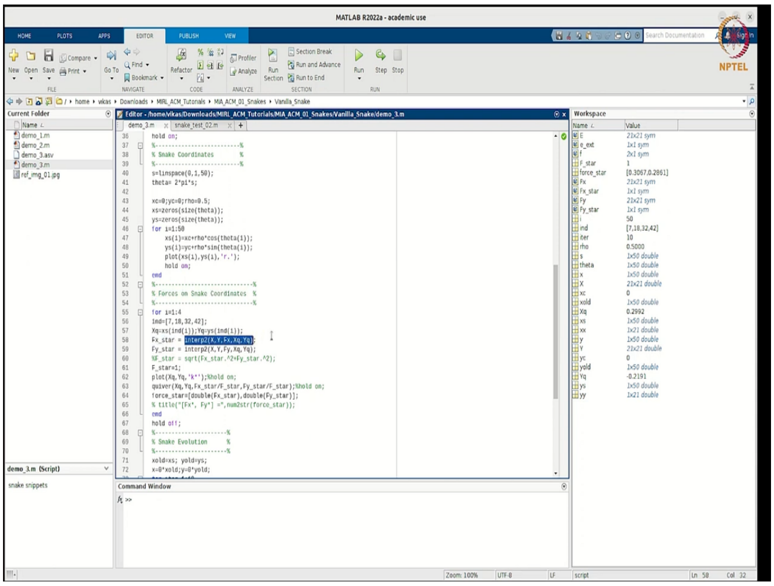

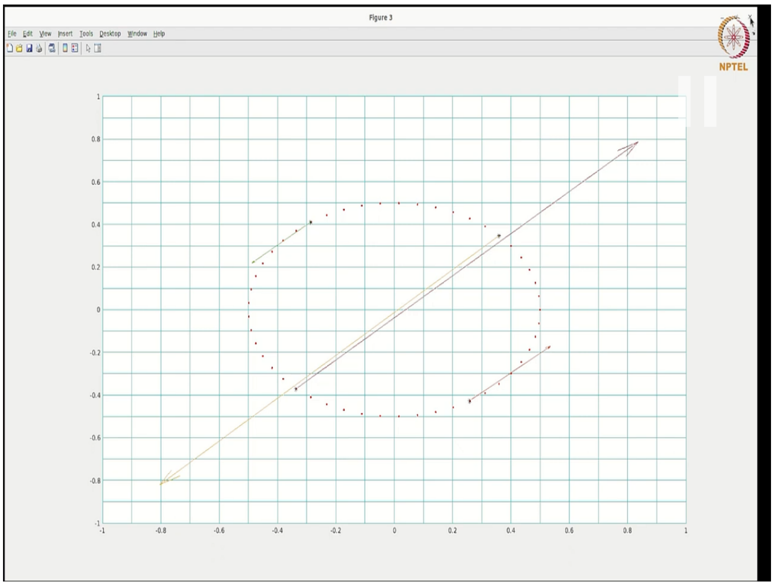

Then to find the forces on snake coordinates, we have to use interpolation. So, the default one is a bilinear interpolation in this in MATLAB. You can have other methods also. But the idea is we have to use some sort of interpolation. So, here, I am just showing that I am showing the forces, you can see like at this point the force is towards this direction.

Here the force is towards this direction. Here the force is little bit stronger than this one but it is in the same direction. So, because the edge is along 1 1. So, when the particle or the point is in this direction, it will be pulled towards the edge. And if it is on the opposite side, the left hand side of this edge, it will be pushed towards right side. You can see, this arrow and you can see because at this point is closer to the diagonal, the force is less. This is a little far. So, it is more. So, this is the interpolation idea.

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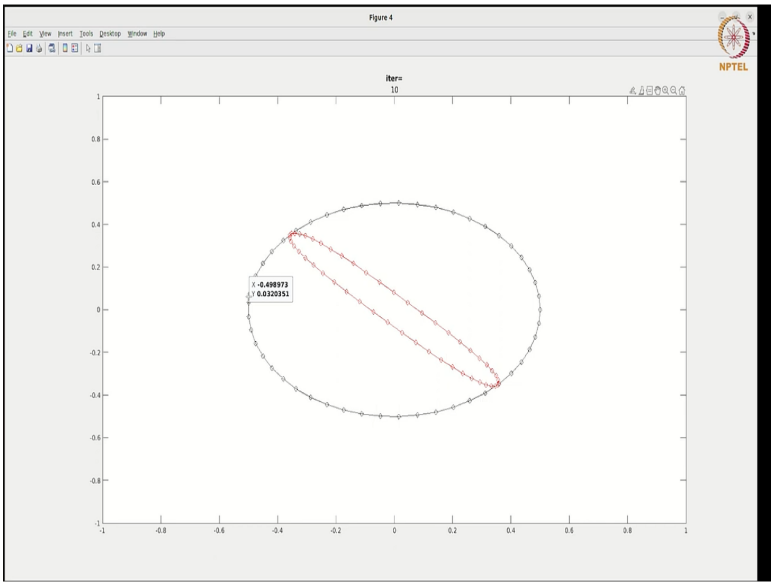

So, if I just throw some points, initial contour, if I throw some points, these black ones and this is our initial contour and I throw these points in this image field. So, where will it go? So, because of the forces that we have seen, it will try to align itself along the edge where we have it. So, you can see here, slowly due to the forces, this circle becomes an ellipse and it tries to align itself with the edge. So, I am just trying it once more. So, for example, you see here.

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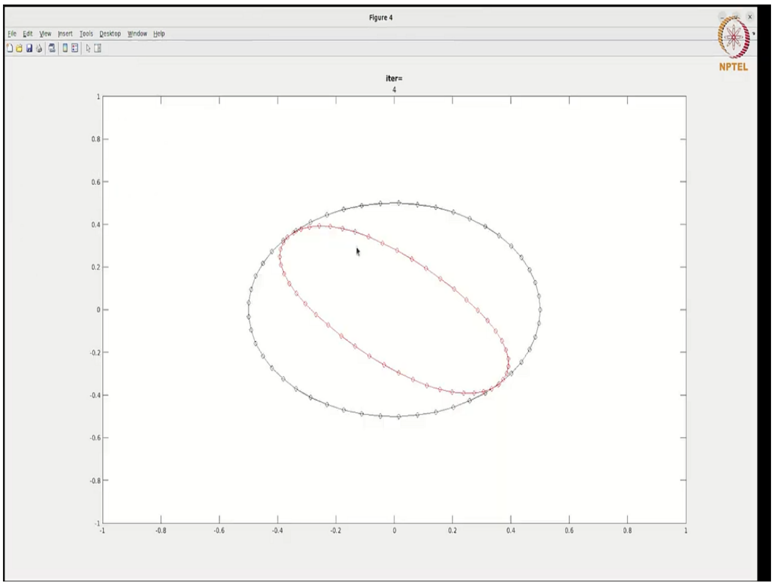

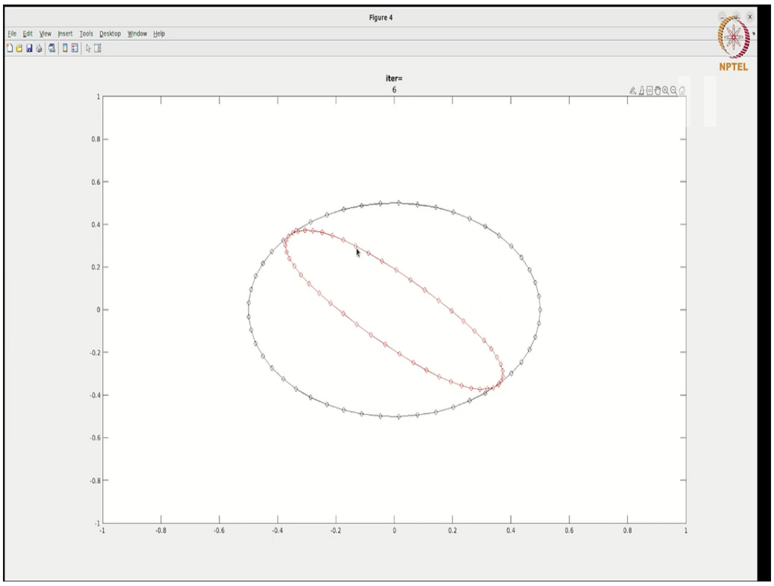
Figure 4
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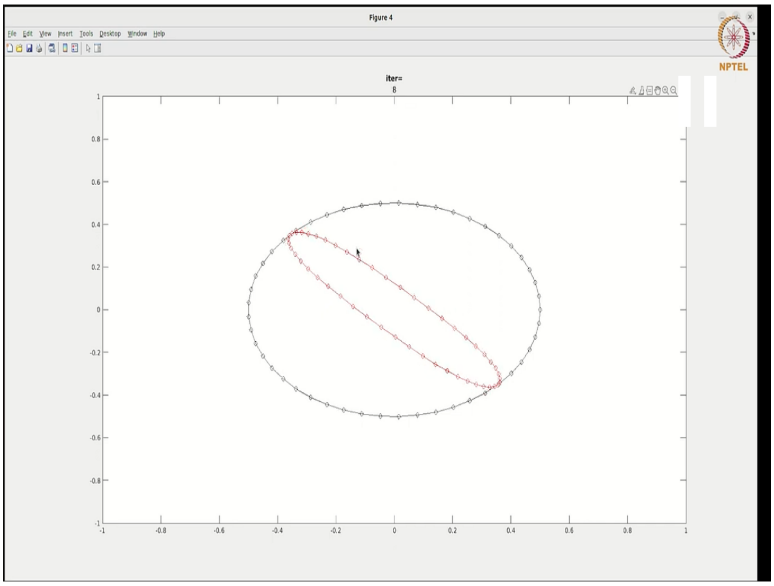
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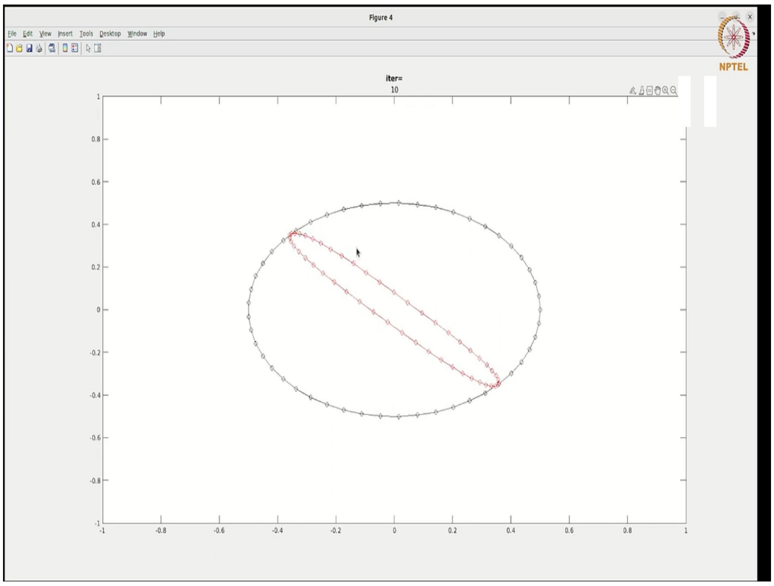
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We started with a circle, slowly, it tries to align itself with the edge. Now, here, we are completely neglecting the effect of like the influence of internal forces. So, this motion, the motion of these particles is entirely due to external force and this is an explicit computation, just like we do normally the explicit one.

But when we like also take into account the property like material properties of snakes then those alpha, beta terms also come. And then things become little tricky. Because those derivatives, alpha beta derivatives are calculated at the present time t.

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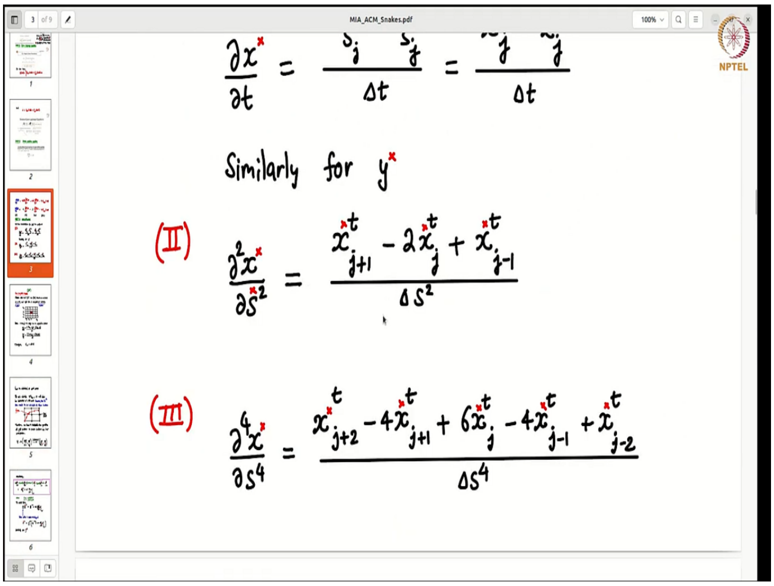

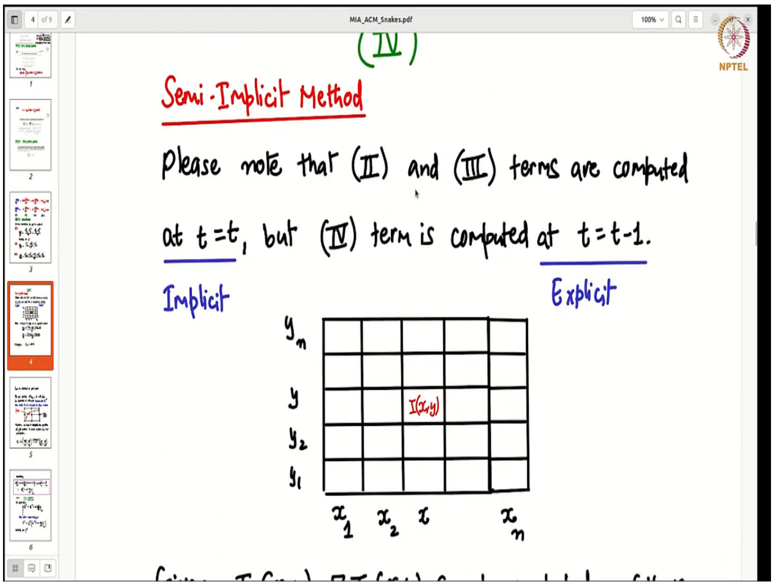

So, just remember, these are terms are calculated at t. But this term is calculated at t minus 1. I have written this also, second and third terms are t but fourth term at t minus 1 that is the combination of the 2 and it is called semi-implicit.

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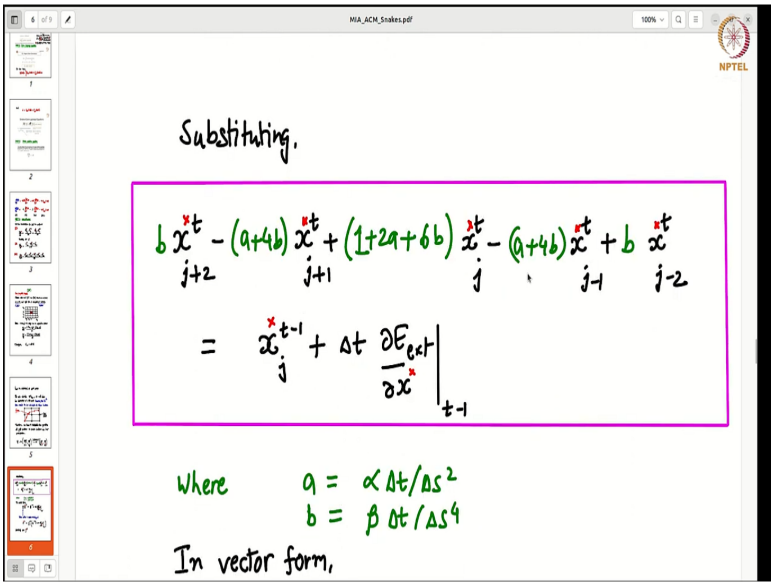

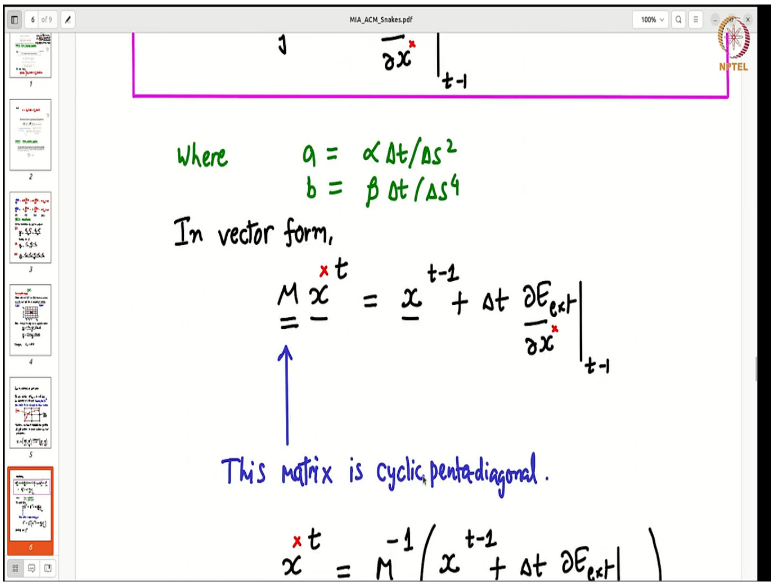

So, when you solve, when you substitute these values, you will get something like this. Here you can see, b a plus 4b, this is a standard pentadiagonal matrix structure. So, in matrix form it looks like this x star t multiplied by M is equal to x star t minus 1 plus delta t times this. Computation of this we have already seen, how we will calculate it. But here you can see, we have, if it were purely implicit, this M would be like gone there will be no M.

But because we have this semi implicit thing and we are calculating the alpha beta related terms at time t, so, we have M here. So, this M is the cyclic pentadiagonal matrix. You see, the coefficients here are a b alpha delta t by delta s square etcetera. But this is the structure. So, if I have to find x star t, given x t minus 1, should be x star actually, x star at t minus 1. If x star at t minus 1 is given and I want to find x star at t, what I have to do? I have to take its inverse.

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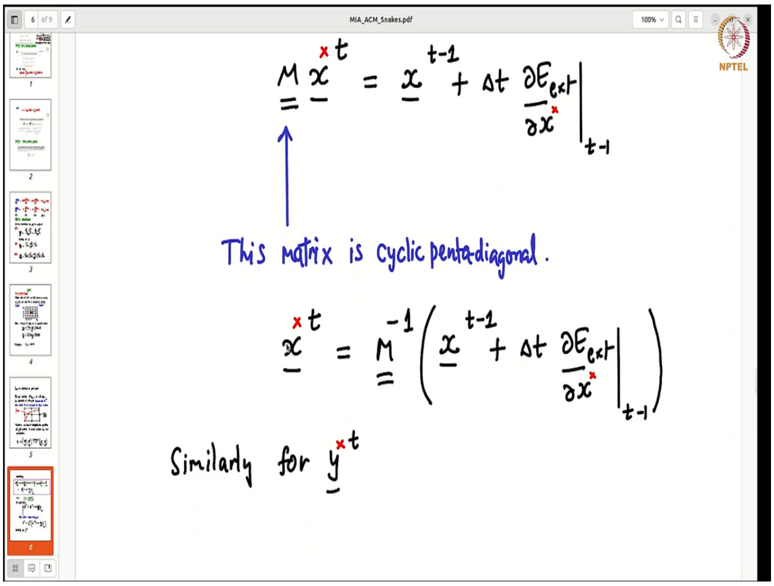

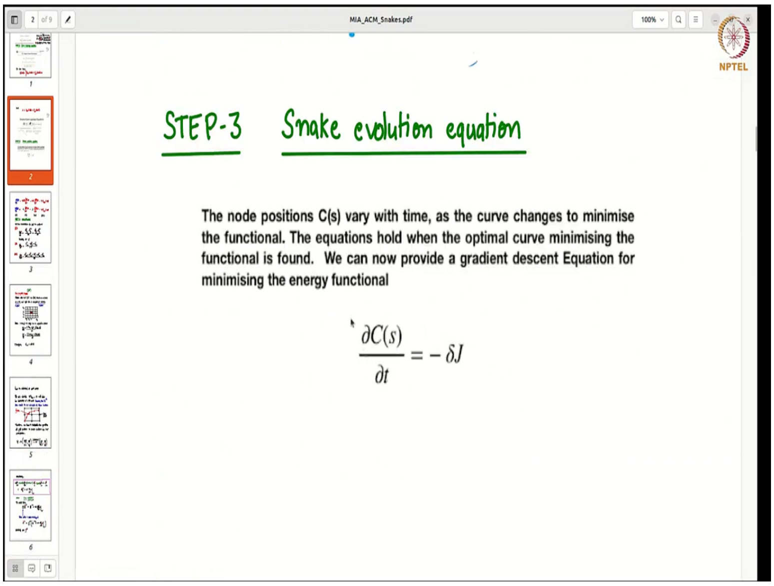

So, this is the evolution equation of snake in numerical form. So, this is the analytical form. Numerically, the same thing. $\partial C/\partial t = -\delta J$ will look like, it will reduce to this. So, we have to find this M inverse and then solve this.

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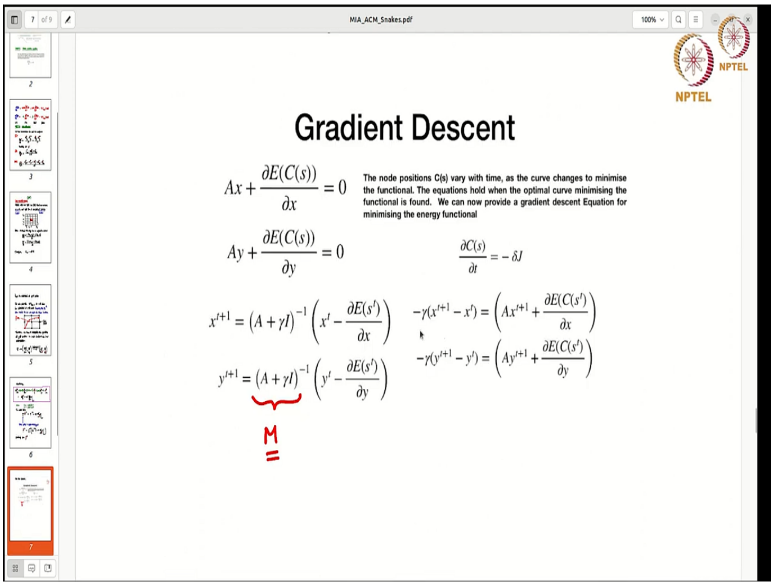

So, this is what sir has also taught in the lecture. Here a plus lambda i represents M. There will be some differences in plus minus because of the science feature sometimes we can say. So, do not worry about that but the idea is this only. So, when you like implement this thing. So, the things right now that, till now that I have shown is, they do not consider the behaviour of material, behaviour of snake but you can include that also. So, in the code that we will provide to you.

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So, this is not written by me. This is available online. It is taken from MATLAB central. So, this code includes not just the forces that I have shown here or the sir has taught there, it includes some other forces also like balloon forces and gradient vector fields. Sir has discussed these forces at the last, the last slide. So, you will find those forces also there. So, it is an advanced version of snake that is being taught to you at the class. But anyway, the structure will still be there. For example, the external forces you will find something like this.

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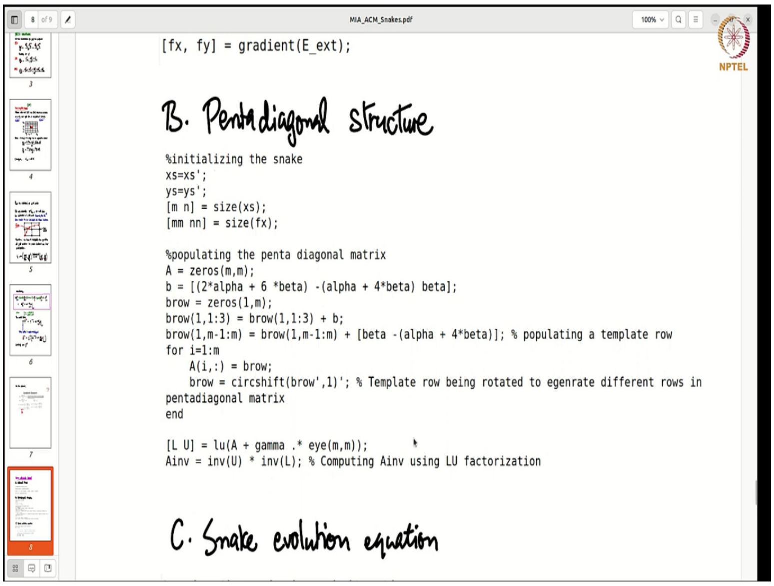

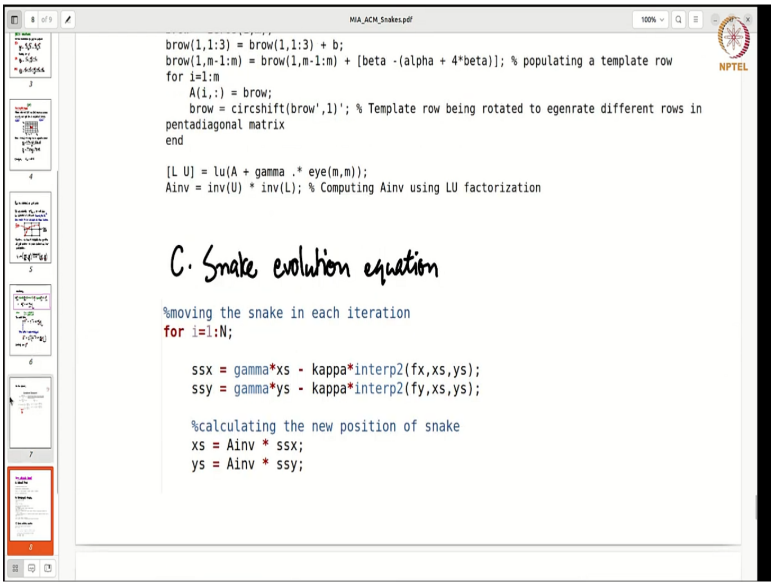

And you will also find the pentadiagonal structure there. You have to look at the code. And then you will have some evolution equation. So, when you will solve, so, if you run the code. So, with this is a full-fledged code that you will be given.

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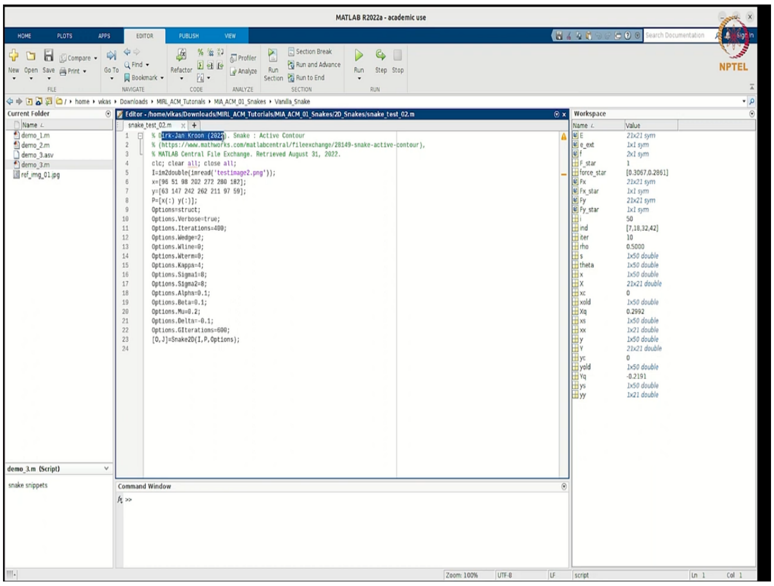

So, when you will run it, you would be able to... so, the code is written by this Dirk Jan Kroon. It is available in MATLAB central. So, you can download it. So, when you will run it, you will see the full implementation of snake with all the forces and you see here.

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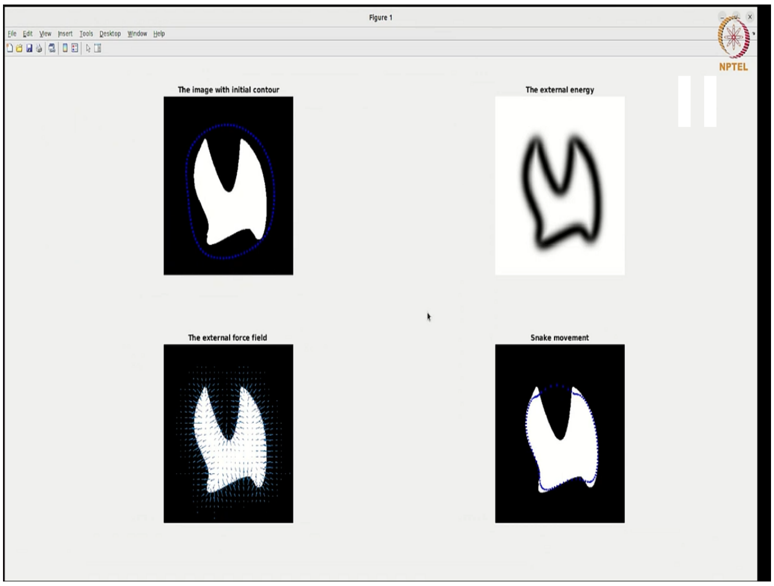

This is your initial contour and this is the object that you want to segment. And you can see, this is your external energy. Why it is looking blurred? Because this image is convolved with a Gaussian filter. So, that well becomes, the potential well becomes wide enough, so that snake can fall there. And these are the external force field, just like the quiver plot I have shown you earlier. This is the real one.

And here you can see, it is evolving over time and slowly it is like capturing the boundaries of the object. These external forces, if you want to magnify it you can zoom it also. Sorry, I do not know what happened. This also came here. But yes. So, that quiver kind of field is available here also. And you will see that if whenever you are close to well, you will find forces from both the directions throwing the particle towards this minimum potential well.

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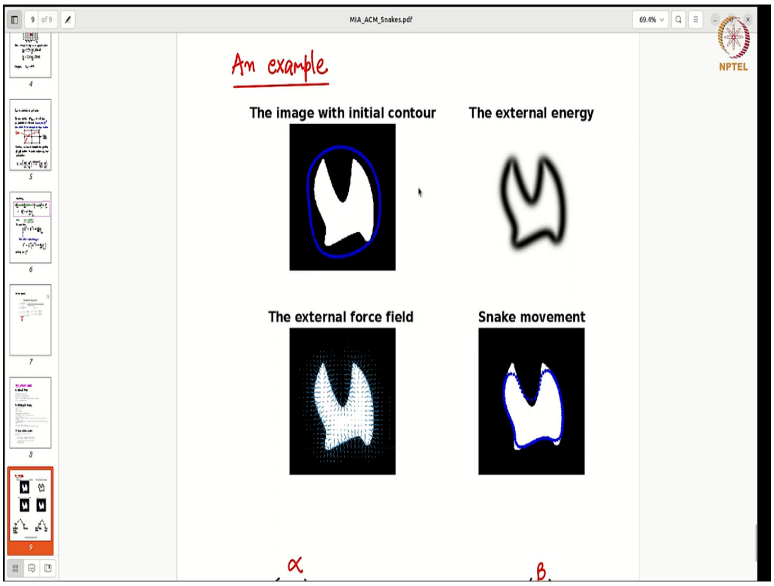

So, this is what you will get. I have just taken a snapshot of it. So, your initial contour will be like this. This will be your external energy. This is a force field that is and then finally snake will evolve in such a way that it captures the boundaries.

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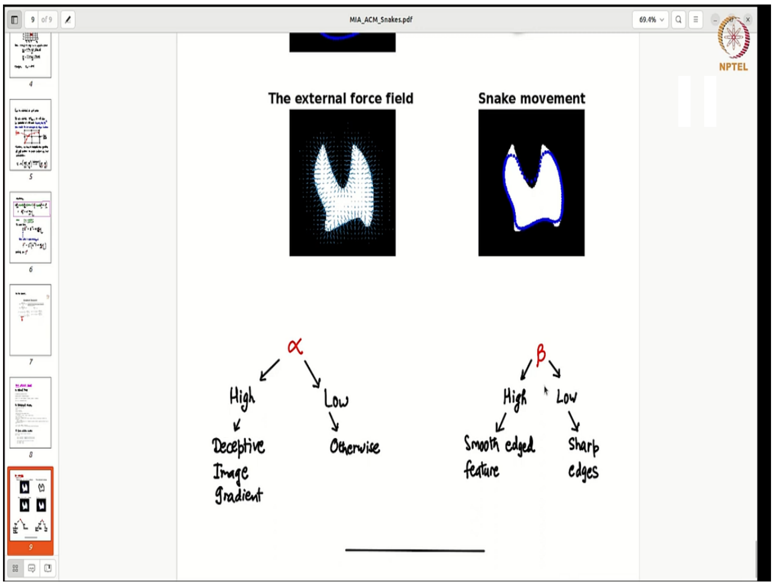

Another thing is you have 2 parameters alpha beta there, which determine the property of snake. So, how would you can play with these parameters? So, because you are given the code, you can play with the initialization, you can play with parameters like alpha beta and other things, you can play with the weights also.

So, although there are some guidelines for all these weights but I am just giving you some guidelines for playing with alpha beta. So, if your image has deceptive image gradient, if your image is deceptive gradient, you can use high value of alpha and otherwise you can use low value.

Similarly, if your object that you want to detect has some sharp features, sharp edges then the choice of beta should be low. And similarly, if it is smooth, you can choose high value of beta. Now, high and low with respect to what? With respect to like if alpha is 1, low means beta is very less.

And there are some other papers also; you can just take a look. So, this completes our first tutorial on snake. We have shown you a basic implementation of snake starting with in which we have not considered any internal force and we can see that how the snakes can evolve just with the help of external force.

And later we have seen the effect of internal forces also. The code that has been given to you is available in MATLAB central and it contains all the forces, external forces including the image force, gradient force, phototropic curvature and this balloon force and this gradient vector field force.

So, it is an advanced version of the code but you can still like go through it and see that the structure that has been taught to you is sitting there. You can also play with alpha beta and initialization to see how well or how bad this method is. So, thank you for your time. That is all for you for the snake problem. Thanks.