

Medical Image Analysis
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Lecture 2
MRI Physics

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Medical Image Analysis

MRI Physics

Hello, and welcome back. So, in this class we are going to look at magnetic resonance imaging, specifically the physics of MRI. And in the next video, we will look at some of the image acquisition techniques as well as the hardware required for magnetic resonance imaging systems.

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MRI Medical Imaging &
Reconstruction Lab




Content



- Microscopic Magnetisation
- Macroscopic Magnetisation
- Spin Precession & Larmor Frequency
- Transverse & Longitudinal Magnetization
- NMR signal-RF Excitation- Relaxation
- Bloch Equations
- Spin Echoes
- Basic Contrast Mechanisms

So, here is the outline, we are going to look at a brief introduction to the origin of magnetization. In the sample, will understand how the individual spins, so called spins are interact with the external static electric field, will understand terminologies related to transverse and longitudinal magnetization. And then, we will look at the origin of the MR signal, that one that is used for actually putting together the images that you see from an MR scanner.

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Microscopic Magnetisation

- Charge and angular momentum of nuclei
- Nuclei with odd atomic number or mass number possess a "spin angular momentum" - Φ . The spin angular momentum gives rise to magnetic properties
- The microscopic magnetic field has a magnetic moment vector $\mu = \gamma\Phi$ where γ is called the **gyromagnetic ratio**
- **Gyromagnetic ratio** has units of **radians per second per Tesla**. We can define $\gamma \equiv \gamma/2\pi$, which has units of **Hertz per Tesla** $\gamma = \gamma/2\pi$


So microscopic magnetization. So, every nuclei has both charge and the so called spin angular momentum. The spin angular momentum is actually intrinsic property of the nuclei. And it has no counterpart in classical physics in the sense that the idea of spin originates from quantum mechanics, but we will not go into the details of how it came about, just understand that in addition to charge, the nuclear also have this spin property. And this is the property that leads, gives rise to the magnetism. So, each nuclei, basically once with the atomic, atomic number or mass number, they process is something called the spin angular momentum, we denote that by Φ

The spin angular momentum gives rise to magnetic properties, that is the one that uses to interact to the external magnetic fields. The microscopic magnetic field has a magnetic moment associated with it, that primarily comes from the spin. And that is that moment vector we denote by $\mu = \gamma\Phi$ this γ is known as the gyromagnetic ratio, and it can be measured for different types of nuclei. So, the gyromagnetic ratio has units of radians per



second per Tesla, sometimes you also will also see this notation, which is nothing but $\bar{\gamma}$ which is $\frac{\gamma}{2\pi}$

Now, I am kind of going to abuse notation and use gamma and gamma bar interchangeably. But wherever required, I will try to mention that and you can put the appropriate symbol. Again, this class is just to understand some at a qualitative level, how MR images are produced, and also understand the mechanisms of contrast. So that is where we are actually headed.

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Macroscopic Magnetisation

- In the absence of any external Magnetic field, randomly oriented nuclear spins cancel out leading to a zero bulk or Macroscopic Magnetisation
- In the presence of a static magnetic field along let's say z-axis. There is a preferred orientation of spins (in a sample material) along the direction of the field (a low energy state).
- $M = \sum_{n=1}^N \mu_n$ sum of individual nuclear magnetic moments in a sample
- If the sample is left in the static B_0 field long enough, the sample's net magnetisation will reach an equilibrium value $M_0 = \frac{\gamma^2 \hbar^2}{4kT} P_D$ P_D - proton density

So, the macroscopic magnetization comes from all these individual spins. So, if you can think of a system of spins, so basically, if you look at our body, we have lot of water in our body, body is mostly water, so there is a lot of hydrogen atoms and they are mostly protons, so they are the overwhelming contribution to the spin.

So, in the absence of any external magnetic field, all these spins are randomly oriented. So, you can think of these spins like a compass needle. Something like this, that are oriented in random directions. And so, these randomly oriented spins cancel each other out, leading to a net bulk magnetization or macroscopic magnetization.

But in the presence of a static magnetic field, let us say along z axis, so, without loss of generality, we can always assume that if we apply a static magnetic field, it will be that the direction of that magnetic field is the z axis. So, the magnetization vector is actually a vector. So is the magnetic field is also a vector.

And in that case, what happens when there is a static applied magnetic field, there is a preferred orientation of spins, along the direction of the applied field. And this leads to a net magnetization, which we can write in this form, which is basically the sum of individual nuclear magnetic moments, in a sample, think of this as a vector sum. Rather discussion, treating off as a vector sum.

Now, there is actually an expression for this. So if you leave the sample a particular sample, in a static magnetic B_0 field, long enough, that for a certain period of time because it takes time for all the individual magnetic spins to orient along the direction. Or, in this case, actually, from the quantum mechanics point of view, the spins generally aligned either along, or opposite to the direction, that is a possibility, but we will not analyse that way. We will get to that analysis later. For right now, we say it is just that we have an expression for the M_0 . What is more important here is that the M_0 depends on the strength of the applied static magnetic field which is B_0 it also depends on this P D. P D is nothing but the proton density.

In this case the proton we are talking about is the hydrogen atom proton. So, basically what does the which we now know how an idea of what the kind of signal that is produced which, the signal produced depends on actually, we will see later that the signal produced depends on M_0 , which in turn depends on this applied magnetic field as well as the density of protons. So, this magnetization is what is induced by the presence of a static magnetic field. And here, considering a sample let us say in this case, the human body or a piece of tissue.

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Macroscopic Magnetisation

- The magnetisation is a function of time as well as spatially dependent, in NMR experiments - due to RF and Magnetic field gradients $M(r, t)$
- Bulk angular momentum J is related to the magnetisation of the sample
 $M = \gamma J$
- The sample in NMR or MR imaging is a volume of tissue, typically an organ of interest or lesion
- T_1 , T_2 and PD are the factors that affect the appearance of an MR image
- In addition, the pulse sequence also has an impact as it is used to manipulate M

But, this magnetization it turns out, is just not static, it is actually a function of time. And it is also need not be homogeneous also. In the sense it is needs out, so the spatially dependent. As at least in NMR experiments that is what will happen, so this M_0 or M , I will call it, will be a function of space as well as time.

Now, we will define one other quantity just for the sake of analysis. See, there is a bulk angular momentum, that we can define, remember that we have of every individual, nuclei, we said there is something called a spin angular momentum. So, we can also say for the sample itself, there is a bulk angular momentum J , and it is related to the magnetization of the sample to this expression.

But again, once again, this γ is the gyromagnetic ratio. So, in this case, now, we are like just to clarify, we are always talking about a sample. When I say sample, it is basically piece of tissue or a human body in a static magnetic field. Now, before we analyse, where the signal comes from? Just have to keep in mind there are three factors we will eventually find out, there are three factors which contribute to the contrast in an MR image, the appearance of an MR image is influenced by these three factors. One is called T1 these are relaxation times, T1, T2, and the proton density.

In addition to this, how these contrast vary? Whether the T1 contrast is higher? T2 contrast is higher? Or P D is higher is determined by so called pulse sequence. And, we will see what that is also in later slides, the pulse sequence determines which one of these contrasts is kind of contributing more to the image.

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Precession & Larmor Frequency

- $M(t)$ experiences torque in an external time varying magnetic field $B(t)$, the equation of motion is given by $\frac{dJ(t)}{dt} = M(t) \times B(t)$ $\epsilon \Sigma$
- J is the angular momentum vector associated with the magnetisation M , $M = \gamma J$
- $B(t)$ can be aligned with the positive z-direction, $B(t) = B_0$

$\leftarrow M \leftarrow B, PD, T_1, T_2$

Al right. So, what happens when you apply a static magnetic field, so, now, we know that there is a spin angular momentum for individual nuclei, and there is also this bulk angular momentum J . And so, what happens is that there is a torque induced, and the torque, torque induced on which is depend, which depends on the static magnetic field.

So, now if you have studied some high school physics, or 12th standard physics you have seen that if there is a current carrying loop, and you put that in a magnetic field, it experiences a torque it is very similar to that. The rate of change of the angular momentum which is the torque, which is dJ by dt , this is the torque that you are usually talking about is basically the cross product of the magnetization and the applied static magnetic field.

This comes from, you must have seen the same plus 1, or plus 2 physics where you see basic electromagnetism from there we can write this expression towards M cross B , dJ by dt is M cross B . Now the why and, why do we need this because this actually gives you like the equations of motion so to speak of the individual of the magnetization vector itself.

Now, so, the J we know that is a is the angle momentum vector associated with the magnetization M . So, M is γJ . So, you can always substitute them, and also we know that $B(t)$ can always be aligned with the positive z direction. So, we will always say $B(t)$ by B_0 .

So, right now this B_0 is the static magnetic field there is no this particular B_0 as it is applied in a MR scanner, is actually quite homogeneous and it does not vary with time. The B_0 field as it is called, which is typically the directional language we applied to be z axis, so that, that does not change. So, then we can always say $B(t)$ is B_0 . And we also substitute instead of γ , instead of J we will substitute M . So, why do we do this because see, we want what we want to do in when we do MR imaging is we want to map this M .

And this M in terms depends we saw it, B_0 we also saw it depends on PD. And the other two factors. The other two factor that can be controlled that can be brought into brought to bear on the contrast is also the T_1 and T_2 relaxation types. So, all of this are expressed through this M , this is what we are trying to measure. And this we want to measure as a function of space and time. And this that is why we want to write this equation, this dJ by dt in terms of the magnetization.

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Precession and Larmor Frequency

- If vector $\mathbf{M}(0)$ was at an angle α to the z-axis at time $t = 0$, then the solution to the differential equation can be written as

$$M_x(t) = M_0 \sin \alpha \cos(-\gamma B_0 t + \phi)$$

$$M_y(t) = M_0 \sin \alpha \sin(-\gamma B_0 t + \phi)$$

$$M_z(t) = M_0 \cos \alpha$$

$$\frac{d\mathbf{J}(t)}{dt} = \mathbf{M}(t) \times \mathbf{B}(t)$$

- Here $M_0 = |\mathbf{M}(0)|$, $\mathbf{M}(t) = \{M_x(t), M_y(t), M_z(t)\}$ and ϕ is an arbitrary angle

Now, if we solve this, that differential equation, this differential equation, if you solve for the magnetization remember we can always put \mathbf{M} equal to $\gamma \mathbf{J}$, and substitute here. And when we solve this equation, we see this we get the following solution. So, we will not go into the detail, so how we would solve etcetera, it is a straight forward ODE, so you can solve it.

So, we can see that there are three components. So \mathbf{M} is a vector remember, which is a vector, magnetization, induced magnetization is a vector. So, this has three, it will have three components M_x , M_y , and M_z . And we see that they evolve according to, if you see that M_x and M_y , there is a sinusoidal dependence with respect to time.

And the frequency we will see as the frequency depends this γB_0 , we will see is that is something called the Larmor frequency. I will mention that in a subsequent slides. So, from this equation, you can see that this the M_x and M_y components or in this case periodic. So, which means that there is a precession happening in the $x-y$ plane. So, we will, I will illustrate this in the later slides.

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Precession and Larmor Frequency

- The equations describe a precession of the Magnetisation $\mathbf{M}(t)$ around B_0 with a frequency $\omega_0 = \gamma B_0$ called the **Larmor frequency** has units of Radians per second, alternate definition in terms of cycles per second is $\nu_0 = \gamma B_0 / 2\pi$
- All the equations of motion can be rewritten in terms of ν_0
 - $M_x(t) = M_0 \sin\alpha \cos(-2\pi\nu_0 t + \phi)$
 - $M_y(t) = M_0 \sin\alpha \sin(-2\pi\nu_0 t + \phi)$
 - $M_z(t) = M_0 \cos\alpha$
- This motion is akin to that of a top in a gravitational field. The axis of the top is the direction of $\mathbf{M}(t)$ and the z-axis (i.e. direction of g) is the direction of the static magnetic field.

precession around \vec{B}_0

So, let us look at this. So this, based on this equation, we saw that this ω_0 is γB_0 is called the Larmor frequency. And has units of radians per second. And of course, we can always rewrite this as ν_0 is γB_0 divided 2π , and we can rewrite the equations we saw in the previous slides in this form.

So, what is this motion this is similar to that of a top or a gyroscope in a gravitational field, and this case the axis of the top is the direction of \mathbf{M} . Basically the direction of the magnetization is same as the x on the top, and z axes is a direction of that which is direction of the acceleration due to gravity, in this case is the direction of the static magnetic field.

So, this these two equations refer to means that there is a precession around z, which is the same as the direction of the B_0 field around the z axis. So, there is a precession of this magnetization in the x around the direction of the applied static electric field. Yes, applied static magnetic field sorry, I said I have to fill this out mistake. So, this is just basically this these two equations denote a precession around that direction. Because you remember if you look at this M_x and M_y are they kind of periodic for this case sinusoidal motion.

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Precession and Larmor Frequency

- Three sources of static magnetic field fluctuations
 - Magnetic field inhomogeneities
 - Magnetic Susceptibility
 - Chemical shift
- Groups of nuclei in a spin system that have the same Larmor frequency are called **ISOCROMATS**
- Hydrogen nuclei in water form an isochromat while those in fat form another isochromat

$$\omega = \gamma B_0$$
$$B_0(r,t) = B_0 \pm \Delta B$$

So, once again what is this Larmor frequency we saw is actually it dependent on the gyromagnetic ratio times B_0 , B_0 is the applied static magnetic field. Now, there are three sources of the static magnetic field fluctuations. One is there is inhomogeneities itself with hardware itself, you know exactly, have B_0 everywhere and especially, so, there are going to be some fluctuations is there. There is there are two other properties, intrinsic properties of the sample which is a magnetic susceptibility and something called chemical shift. These two affect the value of B_0 locally. Since we know B_0 is a function of, let us say this is a function of r runtime, we do not we will exclude time of r .

Now, we expect this to be B_0 everywhere, but there will be some ΔB because of these two properties. And actually, this is exploited for imaging. Because you will see later that because this B_0 varies, consequently this ω will have to be slightly different, and that is what is used for obtaining contrast. So, the groups of nuclei in a spin system that are the same Larmor frequency are called isochromats.

So, the hydrogen nuclei in water will form an isochromat, while those in for fat will form another isochromat. Because it might have a different slightly different Larmor frequency because B_0 is slightly modified because of the chemical surrounding. Sorry, I use the wiper instead of right here B_0 .

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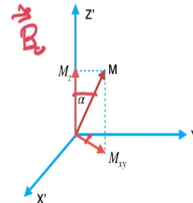


Transverse & Longitudinal Magnetization

- $M(t)$ has two components
 - Longitudinal, along the direction of the static magnetic field
 - Transverse, in a plane orthogonal to the direction of the static magnetic field

$$M_{xy}(t) = M_x(t) + jM_y(t)$$

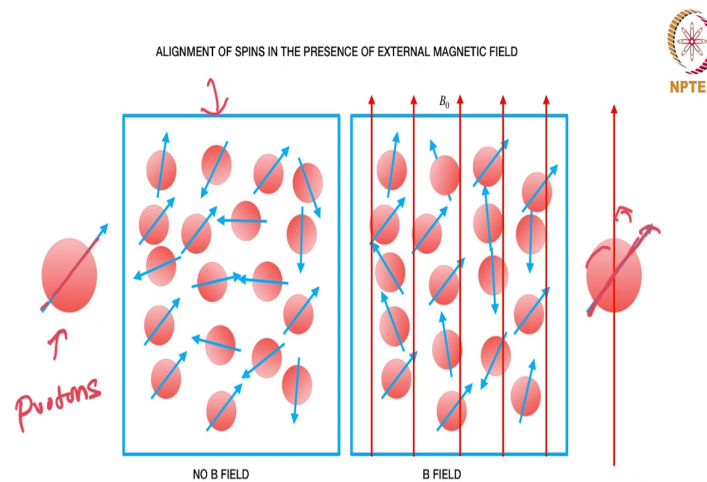
$$\phi = \tan^{-1} \left(\frac{M_y}{M_x} \right)$$



Next, so we will also now look at some nomenclature. Transverse and longitudinal magnetization. Where do this come from? So, the M as we saw has two components, we as we can think of them as having two components. One is longitudinal component, which is along the direction of the static magnetic field, remember, the direction is static magnetic field we said will be typically along B_0 . That is there, that is basically B_0 direction is also taken as the z axis, but typically, that is how it is. And M_x and M_y , which is we saw there is a equation of motion for M_x and M_y . And there is actually sinusoidal dependence. And that component in plane component as we call it, is we call it M_{xy} . M_{xy} is written in complex form like this $M_x + jM_y$ this actually simplify lot of the analysis. That is why they write it.

And then there is a direction which is basically the initial direction which is $\tan^{-1} M_y / M_x$. So, this is basically this particular angle as you can look at it that way. So this α , I have indicated here, typically there is α , you would assume that there is some tilt away from the axis, and then it is processing around it. So even with just a static magnetization, magnetic field applied in along the z axis, the spins, the individual spins, align preferentially along the direction of the static magnetic field. And they also precess, precess around the direction of the static magnetic field, there is an arbitrary phase associated with it.

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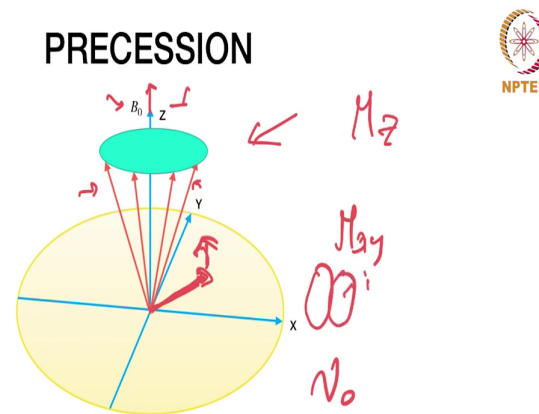
Now, this is what I was trying to show, implying from the previous slide, so you can think of individual nuclei, or, the protons in this case, these are think of these are hydrogen atom protons, because they are the most abundant in the body, and they have the spin property, and the spin is basically, think of it as contributing. And in fact, that is what gives rise to the magnetic properties of the material. So, you can think of it as like some compass pointing over a particular direction. That is what is indicated here, and it can end the direction which it points in the direction of the magnetization induced you can think of it that way. And when there is no magnetic no static magnetic field, all these are randomly aligned, leading to a net magnetization of 0.

Now, in the presence of a static magnetic field, you would have the all of them preferentially aligned along the direction. So, the net alignment is along the direction. And which is what is shown here, in this figure, you see that most of them are pointing upwards, and, therefore, if we take a component along the direction of B_0 field, there is always a component along the directional of B_0 field. And you will also see that the you can you can say that, since they are pointing in arbitrary directions, the transverse component might cancel and become 0. So, generally there is only a component pointing along the direction, and there is a precession.

So which means that if you think of this as a net magnetization, and this precesses around the direction of the magnetic field. So that is the general setup for an MR experiment, or MR imaging experiment. So, you have a sample is basically human patient, or, for that matter, any other tissue sample or biological sample, it is placed in a static magnetic field pointing along the z axis. And consequently, all the magnetic spins, or the nuclear magnetic spins are aligned

preferentially along the direction on the static magnetic field, and there is a precession also, and this is the initial setup. So then, how do we how do we make the measurement? Where does the measurement come from? So that is what we want to do?


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So, this is the picture. So, this is a precession that we talked that I was talking about. So there is a B_0 field along the z axis, these are the individual let us say, magnetic spins, which are in general precessing around the direction of the magnetic field. Now, if you think about it, then you can, you can of course, do a vector sum, have like one arrow processing, so there will be a component along z , and there is a in plane component which is the xy plane component, which is like this, if you think one component which will also be precessing in a particular direction. So, there is an xy component. The B the z direction there is an M_z comp, there is a sorry, there is a M_z component, which is along this direction, and there is a M_{xy} component here.

So, this M , this magnetization which is rapidly rotating with a frequency ω_0 proportional to ν_0 sorry, in the plane, and if you put like a coil of wire here appropriate coils of wire here, it will induce a current in this.

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NMR signal

$M_{xy}(t) = M_0 \sin \alpha e^{-j(2\pi f_d t - \phi)}$

- The rapidly rotating transverse magnetisation induces a voltage in a coil of wire outside the sample that can be measured. This is the NMR signal used in MRI
- Signal frequencies in MR imaging are in the Radio frequency range. Radio waves generated by coils are used to induce NMR.
- We can derive expressions for rapidly rotating magnetisation cutting across a coil, however this would involve integrating across the area of the coil given a net rotating magnetisation.
- However, we have a spatially varying magnetisation that we seek to map using imaging. Using "Principle of Reciprocity" a signal model can be derived for the NMR signal.

$M(\vec{r}, t)$


And that is current is your the NMR signal, what is the origin of that NMR signal? That is that is what we are looking for. So, the xy component as we saw when we go back to the previous slide, so, a component that we saw M_x and M_y have a sinusoidal dependence, and we can write it in this form. Where this μ_0 comes, this μ_0 is the Larmor frequency that we saw. This means that in the plane as I indicated earlier, the M_{xy} component is rotating rapidly. Now, in the in their assistance, there is a rotating magnetization, if we place a coil of wire outside the sample, then this rotating magnetization can be measured. And this is exactly what is used in an NMR signal. It was to use, this is the origin of the NMR signal.

So, this magnetization let me go back here signal. So the signal frequencies in MR. This μ_0 turns out very the radio frequency range, megahertz, radio frequency range. And so, then the radio waves are generated by the coils. And in fact, there is one more step that we will get to where we actually need to generate a radio wave, order to measure another radio frequency. So, there are there is how do we how do we know, how do we associate a signal we are measuring to this quantity here. That is the important part. So, we saw that, we have a net magnetization, because of the static magnetic field, there is a α , it is precessing around the direction the static magnetic field, there is a component in plane, one in plane in the sense.

There is a you can split that magnetization into two components, where it is a vector one component along direction of the magnetic field, another one perpendicular to it, the one perpendicular to it is what we can measure. And that can be measured by because it is actually rapidly changing as a function of time, we can place a coil of wire, and it will induce a current in it, and that current can be measured. Now, we want to now associate how and

relate this signal that we are measuring that current induced or voltage induced EMF as it is called, and the magnetization. And that, that relationship will then help us to map the magnetization across the volume.

So whether we will not get into the details, but what we can do is we can actually deliver an expression for a rapidly rotating magnetization cutting across a coil. So, there is a coil of wire, something like this multiple coils, and then there is a M field. M , magnetization which is rapidly every time it rotates, it cuts across the coil of wires, this will generate a signal in the wire. Now, which this will involve integrating across the area of the coil. So, you will be measuring a net magnetization. And you will have to integrate across the area of the coil.


Now, this is a problem. So just give you a qualitative reasoning. See, measuring net magnetization is pointless. Because what we want is, we want spatial localization. That is what an image is right. Remember, for CT images, we were able to localize density differences. When you reconstruct, the signal is proportional to the density, this μ is a function of energy, the linear attenuation coefficient that we reconstructed is actually a function of the energy as well as the density of the object, or the atomic number. So similarly, the net if you do this particular analysis, when we keep a coil of wire, and if you do the analysis, like you, you integrate across the area of the of the coil, that is typically what you would do.



Because you want to, you know to calculate the flux, you would end up doing that. But that does not help because you will be only able to measure one quantity, which is the net magnetization, what do you want is magnetization as a function of r time, where r is in the sample, that is as a human body. On every position, we want to be able to localize.

So, there is something called the principle of reciprocity, which lets you rewrite that integral in terms of a volume integral of M . So, the rate of the change of the flux, which is the derivative of the flux across the coil, that can be rewritten as a volumetric integral of the magnetization. And that that is called the principles of principle of reciprocity.

Once again, this is just for qualitative understanding, you do not have to get into the details, I will just show you the expression that is involved. And in the end, what is more important is, yes it understand what the MR signal that one is measuring, what is it proportional to? What influences that signal, that is what we are heading towards qualitatively understand that part.

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NMR signal

EMF Induced in coil using principle of reciprocity

$$\rightarrow M_{xy}(t) = M_0 \sin \alpha e^{-j(2\pi\nu_0 t - \phi)}$$



$$\rightarrow V(t) = \frac{\partial}{\partial t} \int_{\text{object}} \vec{M}(r, t) \cdot \vec{B}'(t) dr$$

$$\rightarrow |V| = 2\pi\nu_0 V_s M_0 \sin \alpha B^r$$

Voxel volume

$$d = \pi/2$$

So, if you look at this, for the M_{xy} is proportional to this, this is a signal we have we saw that, we solved that ODE for the magnetization or the angular momentum, then you get expressions for M_x and M_y which are sinusoidal dependence. So, you can put them together as M_{xy} in complex form, and the induced voltage in the coil of wire can be written in this form, where M is the magnetization. So, this is a vector dot product. So, both of them take the vector. And B_r is the, this is the magnetic field produced by the receive coil, the coil that you are using to measure. It is called the receive coil, and this is the magnetic field produced by the receive coil at a position, r the same position r due to a unit current.


Once again, we will not go into the details of this derivation, I will just to give you an idea of how we are going to proceed with the analysis. So, it turns out then after you act, we can actually make some simplifying assumptions. And in the end, if you look at the magnitude of the voltage induced, magnitude of the signal introduced, it is proportional to this ν_0 , which is the, it return is the Larmor frequency. This is what you call the voxel volume, in this case an infinitesimal volume. Think of this as a volume over which the magnetic field is constant, static magnetic field is constant or M is constant. M_0 , which is the induced magnetization the magnitude, and $\sin \alpha$. $\sin \alpha$ is the tilt from the axis.



Remember, we have this we showed this picture, this is the z axis, and this is the induced magnetization, and it is at an angle α . So, this is, this can be maximise. So, it depends on all of this. So, if you have a very high M_0 the signal is very high, if ν_0 is high, signal is very high, α is high, α maximum α is $\pi/2$. So, if α is $\pi/2$, which

means that you have you are actually your entire, you flip the magnetization is actually in the plane, this direction then your signal is very high.

So, the next step is you want to get a good signal, what is done is to tip this magnetization which is now precessing around the direction of the static magnetic field, tip it into the plane which is perpendicular to magnetic, the plane perpendicular to the direction of the static magnetic field. So, that is the, that is how you would actually induce the MR signal that you typically get in all these imaging experiments.

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Rotating Frame

- Determine equations of motion in a rotating frame of reference, which rotates at the Larmor Frequency
- $M_{x'y'} = M_0 \sin\alpha e^{i\phi}$ in the rotating frame, the magnetisation vector is a stationary vector with a magnitude and phase angle
- In order to generate an MR signal we have to tip this vector in to the transverse plane away from the static magnetic field

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H


$\rightarrow \gamma_0$



So, there is one other concept you will come across, I will just briefly touch upon it, it is called the rotating frame of reference. So, if you look the, the if you tip let us say the magnetic, M_{xy} component is in the plane. It is precessing at a frequency ω_0 , this is a very high frequency, radio frequency range. On the other hand, you can actually let the coordinate system rotate at this frequency, if you get the coordinate system rotate to this frequency, then your magnetization is basically, it is not it is not moving. Or you can it is not rotating or anything, it is just some vector with this magnitude. Stationary vector with the magnitude and a phase angle, that is all it reduces to.

So, this is the idea. So, this actually simplifies analysis quite a bit. So that is why we will use a rotating frames of reference, when you are working on analysing MR signals. So, in order to generate like a good MR signal, we have to tip this magnetization vector, we saw M into the plane, we saw that. It is proportional to sine alpha, alpha is $\pi/2$ is maximum. So, you make it nine. So, you flip it into the, tip that magnetization to the plane, you get very good

signal. How do you tip this magnetization? This is where this nuclear magnetic resonance aspect comes in.

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RF Excitation

- An RF signal, ie RF current through an antenna or coils around the sample, is used to tip the magnetisation into the transverse plane or away from the static magnetic field
- The rotating magnetisation induces an EMF in RF coils around the patient which is the MR signal
- To accomplish this we need a B1 field induced by a RF coils operating at the Larmor frequency. This field is modelled as $B_1(t) = B_1^e e^{-j(2\pi\nu_0 t - \phi)}$ where B_1^e is the envelope of the B1 field.
- As long as the B1 field is on it will continuously force the net magnetisation to tip away from the static magnetic field direction
- The tip angle is given by $\alpha = \gamma \int_0^{t_p} B_1^e dt$

$$= B_0 \int B_1 \leftarrow B_0$$

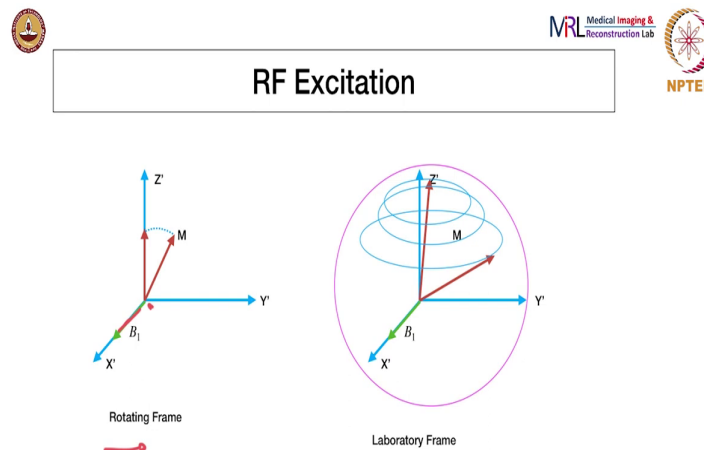
So, the way the tipping is done is by applying an RF signal, so the energy for tipping the magnetization into the plane is provided by a RF signal. So, there is another coil, which generates a magnetic field, it is called the B1 field. I have not mentioned it here, and this B1 field, incidentally also is oscillating at the Larmor frequency. Why do we need that if you think about it, let us say you apply the B1 field at an arbitrary frequency. But it is B1 field is always perpendicular to the B0 field. B0 is along the z axis, B1 let us say will be along the x, this is the B1 field.

Now, what happened we saw, we saw the $M \times B$, the torque is $M \times B$. So, when you apply this B1, this will also produce a torque, however, the B0 field is always much stronger, this B1 field is much, much lesser than B0 field. So, it will not be able to influence that magnetization that much, because the B0 field will eventually also have a torque on it, it becomes an issue. So, the way to do that is by doing something in resonance, which is basically applying this rapidly oscillating B0 field which oscillates at the same frequency ν_0 , the idea is if the B1 field also oscillates, then it will rapidly what our push the magnetization into the plane. So that is where the resonance aspect come.

The resonance, these called resonance imaging because they applied B1 field which are applied at right angles to the B0 field we will put a torque on it, which enforce the magnetization into the plane. But our it will not be enough. Because the B0 field is always

stronger. So, then you have to do it at a certain frequency ω_0 , this oscillating field has to be at frequency ω_0 in order for it to tip. And the tip angle is generally given by this expression $\theta = \gamma B_1 t$. It depends on how much time you apply. So, B_1 is not permanent, like the B_0 field, you applied for a specific period of time, that period of time determines how much away by what angle, you tip the magnetization and is given by this integral expression. See, again, this γ will be there as a prefix.

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Alright, so what is essentially happening is, if you look at it in the rotating frame of reference, this is your net magnetization, let us say initially, it is along z , just for the sake of argument. For illustration, it is along z . And again, I use x prime y prime z prime, it does not matter there. It just I just mean x , y , z . So it is along z , and then you apply this B_1 field, remember. Let us say I have applied it along B_1 , then it will tip the magnetization into the plane which is into this plane. On the other hand, or it depending on the duration. If you want to tip into the plane. You have to applied for a proportional period of time.

But if you look at the laboratory frame of reference, what you are doing is, you are it is actually spiralling, the M is actually spiralling into the plane. The magnetization is spiralling on the plane. If you look at in a rotating frame of reference, then it is actually it is just tipping into the plate. Now, from the laboratory frame of reference, you can understand this, it is spiralling because the applied B_1 field also has the same frequency as the precession frequency of the M , it is just like when your swing you are somebody on a swing, you have to time your push properly so, that next time they go or they go back and forth they get a higher displacement, very similar to that.

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Relaxation

- Once the Magnetisation is tipped into let's say the transverse plane and the B1 field is switched off, it would ideally precess there forever, inducing voltage in RF coils forever
- There are physical processes that dampen this eventually leaving to decay of the transverse magnetisation
- Transverse Relaxation or Spin-Spin relaxation
- Causes signal to decay, because of perturbations in magnetic field caused by spins nearby
- Inhomogeneities caused by spins in the magnetic field leads to momentary speed ups or slow down, leading to dephasing or loss of signal
- The resultant decaying signal is called **Free Induction Decay or FID** ←

So, now, so, this is where the actual emitting signals start to arise, correct. So, once you have tipped it into the plane what. So, once the magnetization is tipped, you would think, once you magnetization tipped, you will turn off the B1 field, B1 field is set to 0. So, ideally if you leave it that way it should precess there forever correct. Because initially you had applied a static magnetic field, and they were precessing along around the direction of the magnetic field.

Now, if you tip it then you should have a signal, there for, it does not happen unfortunately, because there are physical laws, and processes at play. So, these physical processes dampen that motion, that precession in the plane. And so, eventually the in plane the one that the magnetization you tipped in plane, will decay.

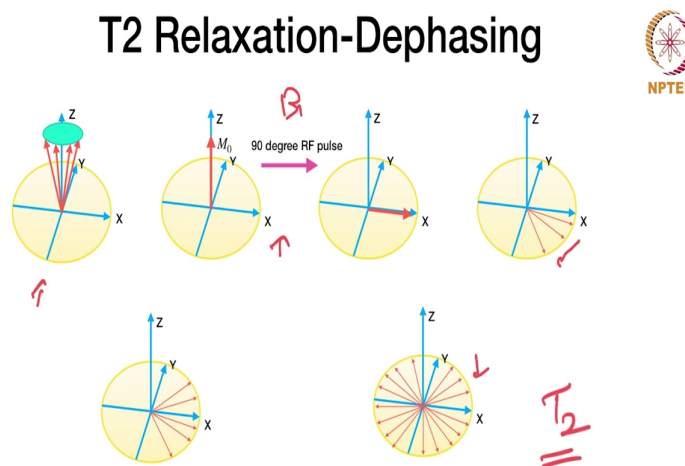
So, there are two processes that lead to that decay. So, one of them is called the transverse relaxation, or spin-spin relaxation. And this causes signal to decay and because of the perturbations in the magnetic field caused by the spins themselves, so, neighbouring spins will modify the B0 field in at a particular spin location. So, then this that leading to inhomogeneities. And the way you think of it is defacing is, if you think of a bunch of spins precessing around in plane, and each of those spins will cause some, majority somewhere else.

And that means the magnetic field there is either reduced, or increased and subsequently the spins at that location will get to either a higher frequency, or a lower frequency leading to defacing, only if they are all precessing in phase do you get, you get a strong signal, otherwise the signal goes to 0. Now, the process now once you tipped, it starts going to 0, but

still you can get a induced signal. And, that signal is called free induction decay. So, the magnetization is tipped into the plane, and it is rapidly precessing this leads to a current, or voltage that can be measured into a coil.

And, that voltage starts to, that signal started to goes to 0 rapidly because the transverse component will decay, because of the so-called transverse relaxation process or spin-spin relaxation.

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So, this is what I have shown here in this particular illustration. So, you have a bunch of spins, precessing around the direction of static magnetic field, you apply this 90 degree RF pulse, which is basically your B_1 field, and it tips the spin, you tip the magnetization into the plane, but why as soon as it is tipped, it will start to dephase. The individual spins will start to all precess with different frequencies, in getting different directions. And, over time, they will, there will be so many all of them will get acquired independent precession frequencies and phases leading to 0 magnetization.

This, however this you will see this can be measured as a signal, this process can be measured a signal, which is rapidly decay to 0. And that is called the FID, free induction decay. Mention that, this process is actually characterize by that time constant, T_2 . This is where the T_2 contrast comes from.

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Relaxation

- The second relaxation mechanism that causes a loss of signal is called the **longitudinal relaxation or Spin-Lattice relaxation**.
- Related to the recovery of the longitudinal magnetisation which recovers to its equilibrium value M_0 also as an exponential
- If an alpha pulse is applied at $T=0$ then the longitudinal magnetisation evolves according to $M_z(t) = M_0 (1 - e^{-t/T_1}) + M_z(0^+)e^{(-t/T_1)}$ $\rightarrow T_1$
- For tissues in the body relaxations times are in the range $250\text{ms} < T_1 < 2500\text{ms}$ and $25\text{ms} < T_2 < 250\text{ms}$. For all materials $T_2 \leq T_1$

T_1, T_2

$\uparrow H_z$

The second before we go any further, I just want to, the other form of relaxation is the longitudinal relaxation, which is basically the reappearance of the longitudinal magnetization. You have the M_z component, we have tipped it, to then after immediately after tipping M_z , that will go to the 0, but then over time it is totally start to reappear. So, but then that process again this is may not be, count very obvious at this time or not very intuitive, the rate at which M_x M_y goes to 0, and the rate at which M_z reappear are different. So, they have different time constants. So, that one is T_2 , this time constant is given by T_1 .

And, for different types of tissues, the T_1 , and T_2 have different ranges. So, T_1 is always higher, T_1 ranges from 250 to 2500 milliseconds, while T_2 ranges from 25 to 250. So, just to remember for the sake of understanding, T_2 is the is the time constant which determines how fast your transverse magnetization which you have tipped into the plane. The how fast that goes to 0, because this is the time constant for the processes that drive that component a 0. However, that does not mean that you recover the a magnetization along the z axis of the along the direction of static magnetic field instantaneously, that takes a slightly longer time.

And that time constant is T_1 . So, this T_1 , and T_2 , are those time constant they depend on the properties of the isochromats, Where are they made a sense? what kind of tissue or imaging, what kind of atom, or nuclei you are imaging. So, in this case in the body test, hydrogen, proton, that is a nuclei or imaging, but then the its chemical environment will determine the T_1 , and T_2 .

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Bloch Equations

- We can write the Bloch equations that describes the time dependent behaviour of bulk magnetisation under the influence of static and time varying field (B_1), along with the relaxation

$$\frac{dM}{dt} = \gamma M(t) \times B(t) - R[M(t) - M_0]$$

$$B(t) = B_0 + B_1(t)$$

$$R = \begin{pmatrix} 1/T_2 & 0 & 0 \\ 0 & 1/T_2 & 0 \\ 0 & 0 & 1/T_1 \end{pmatrix}$$

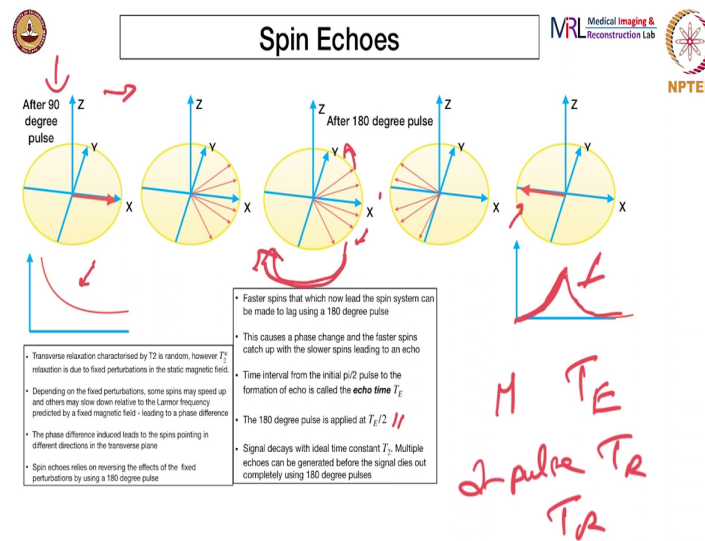
$$\left\{ \begin{aligned} \frac{dM_x}{dt} &= \gamma B_0 M_y(t) - \frac{1}{T_2} M_x(t) \\ \frac{dM_y}{dt} &= -\gamma B_0 M_x(t) - \frac{1}{T_2} M_y(t) \end{aligned} \right.$$

So, for all this together, remember, we wrote this equation, this kind of $M \times B$, equal to dM by dt is something we wrote for just a magnetization in a static magnetic field. Now, if we take into account, there is actually a bracket I left out here, so this is a bracket. Now, if you take into account all these factors that we talked about, and we talk of, and we also include B_0 and B_1 .

So then we can write down the equations of motion, these are called the Bloch equations in this fashion. So, these are two differential equations, that did show how M_x and M_y evolve in the presence of once, in the presence these equations in the presence of B_0 and B_1 . And this is after B_1 is removed, how does it behave that this determine after B_1 is removed. That is when we start the imaging process.

And that is these equations then determine your source of contrast, what, how, what do a signal depends on etcetera. So, these are used to actually figure out different types of imaging tricks, they are called sequences in magnetic resonance imaging.

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Alright, so the first sequence that we are going to talk about, again, this just for the sake of understanding what is, what exactly is happening, this is called a sequence, an imaging sequence. Which basically, what we are trying to do is manipulate this magnetization, M . One way, we saw how to manipulate this, we tipped it into the plane, that is when we have the coils there that measure the induced signal. And, we use that signal to infer M , magnetization as a function of position. And this is what we want to do. And, so one of the ways of doing that is this spin echo, just we will look at the actual 3D imaging sequence in the next video, but this is just for the understanding of how the spin echoes are done.

Spin echoes, we saw that you apply this 90-degree RF pulse, B_1 field, B_1 field, in this they are all along x axis. I think maybe, I should done the x differently. But anyway, just for the sake of argument, we will just do it this way. If you are you apply the B_1 field perpendicular to the direction of the magnetic field along one of these axes, and then it will tips let us say you do a 90 degree pulse, I call the 90 degree pulse, it will tip the RF pulse, which is the B_1 field will tip the magnetization into the plane.

However, you can always call it as alpha pulse, the alpha is any angle that you want to tip it, so the 90-degree pulse will tip it in magnetization into the plane, the red arrow is the magnetization, it will rapidly deface, but then now what you do is you apply a 180-degree pulse, so this 180-degree pulse, what it will do is it will flip the magnetization. It will flip the magnetization into this way. It is a 180-degree flip from where it is. So, what happens is they are still precessing in the same direction. So, they will once again come together here, and that resets to another signal. And, then followed by a decay.

So, this one will be a decaying signal, because you tipped it immediately. And then it starts to decay that gives rise to this signal. And, what you do is you wait up to that time called T_2 , it is called time to echo. And, after that, and when you apply 180-degree pulse. So, 90-degree pulse just to understand 90-degree pulse will flip the magnetization by 90 degrees, 180 degrees plus, will flip the magnetization by 180-degrees. In this case, when you flip by 180-degrees, you actually go right over there, you flip around, you can flip along the around the y axis here, according to this picture, but then you are still precessing in the same direction, you are all precessing in different directions.

So, then you will come together once again to form another that gives rise to another signal. So that is what is shown here. So, you go once you get this other signal, it is rising, because you come together and then you will decay again. So, this is the spin echo. And the 180-degree pulse is applied at time T , from T by 2 from the time you apply the 90-degree pulse, and then you will measure the echo at time T_E , so time to echo. This is one parameter that people manipulate all the time, time to echo, when you measure the signal T_E will determine your contrast. We will see that in the later slides.

To determine the contrast that is one more time which is called TR . Let us call repetition time. So, for instance, in all of these experiments, it is not like you just do this once and then let us say you are done. No, you will have to do these sequences in order several times the time between each time, and time between each of these very crudely speaking a time between each of these 90 degrees pulses. Every time you have to apply 90-degree pulse to flip the magnetization in plain. So, which and that time between two successive these 90-degree pulses, you can call it time to repetition. So, this T_E and TR determine contrast, so, we will see that how that happens. So, there are different forms of contrast. We saw that earlier. Also, I mentioned it, T_1 , T_2 and PD . We will see how that is.

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Basic Contrast Mechanisms

- The type and ordering of excitations and relative timing leads to different type of tissue contrasts. This is specified using a pulse sequence.
- Time interval between successive α pulses is called pulse repetition time T_R T_E
- Transverse magnetisation provides the measurable signal, larger the transverse component, higher the signal
- To see contrast in different types of tissue, the measured signal must be different in these tissue
- Contrast in MRI is provided by 3 main properties of tissue
 - Proton density - P_D : Image intensity is proportional to the number of hydrogen nuclei in that tissue
 - T1 weighted — Tissue dependent
 - T2 weighted —


So, the basic contrast mechanisms. The type and ordering of excitations and relative timing, that leads to different type of tissue contrast. And, this is what is specified using a pulse sequence. So, this pulse sequence determines what kind of contrast you are getting, the time interval like I said between two successive alpha pulses is the time to repetition, we saw what is the time to echo the last slide. See only the transverse magnetization provides the measurable signal. So larger the transverse component, higher the signal.



And, if you want to see contrast in different types of tissue, the measured signal must be different in these tissues. That is how we do, the magnetization that you measure in these different tissue types should be different, because we saw that the signal magnitude of B remembers that expression last few slides ago, it depends on M_0 . So, whether M_0 was different then you get good contrast they are the same you do not get contrast.

So, the contrast is provided by three main properties we saw proton density. Proton density understandable, because as there are more protons, you get a bigger M_0 . Because each of those protons contributes to their net magnetization, and there is a T1 and T2 weighted time. We saw what T1 was, T1 was the rate at which you recover the longitudinal magnetization, and that is a tissue dependent property, it depends on the chemical, the biochemical constitution of the particular tissue, and T2 also same thing, it is a chemical property. Because you are looking at how fast does the spins defacing a plane. And that depends on the kind of inhomogeneities introduced by other spins in there. And that depends, again, once again on the chemical composition of the tissue at hand. So, both of these are tissue

dependent, these properties, and we can capture these properties by playing with T_R and T_E . There is other property is T_2 also we measure.

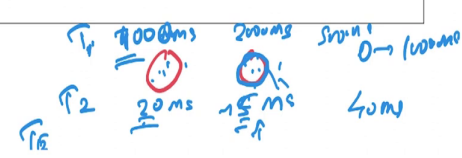
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Basic Contrast Mechanisms

- Proton Density Weighted Imaging: Accomplished by long T_R and either no echo or short T_E . Tip angle is 90 degrees
- T_2 -weighted contrast: Differences in T_2 relaxation times must be apparent. T_E should be approximately selected to be T_2 value of tissue being imaged and large T_R to reduce T_1
- T_1 -weighted contrast: The difference in longitudinal component of magnetisation has to be emphasised. T_R must be set approximately close to T_1



So, proton density weighted imaging, and this is accomplished by long T_R , or an either no echo or short T_E . Why do we do that? See, if there are if you wait long enough, so long T_R is between two successive pulse sequences, that is at time T_R . So, before you apply the second pulse, you wait for the longitudinal magnetization to recover completely. Now, different tissue will have different T_1 times, which means that the amount of magnetization that is recovered for different tissues will be, let us say there are two different types of tissue. I will do green and blue and green, blue and red. This one is T_1 this, let us say this is 100 milliseconds, or 1000 milliseconds, this is 2000 milliseconds. This is T_1 .

Let us say it is T_2 is this is 20 milliseconds. This is milliseconds and this is 40. Or in this case, 5 milliseconds. So the idea is, if we wait long enough, we do not wait long enough. Let us say we start the second experiment second pulse, alpha pulse at immediately admits the first alpha pulse at time 0 second alpha pulse let us say we apply at 1000 milliseconds. Then what happens is that in this tissue, the longitudinal magnetization has not recovered, correct. Longitudinal magnetization has not recovered here, but it has recovered there. So consequently, the M_0 that we measure will be higher from this tissue compared to that.

Similarly for the T_2 time, T_2 determines how fast it goes to 0. So, if you measure soon if you if you are time to echo is very short, is one let us say it is only five milliseconds. You will, you will not be able to capture this one, this signal because this is what dissipated by

that time. Let us say your measuring time take was 5 milliseconds. Then, all of this the contribution from this tissue would have dissipated by 5 milliseconds. So, I will only get the 20-millisecond component. So, the idea is we do not want that to happen, we want it to strictly depend on the density of the protons in each of them.

So, what you do you wait long enough, let us say you wait for 5000 milliseconds, which means that the longitudinal magnetization would have recovered in both of these tissue sets. And then once you flip, you measure immediately, which means that you do not, this both, this T₂ time will also will not influence your measurement. So that is how you get a proton estimated image.

So, like I said, T₂ weighted contrast difference in T₂ relaxation times must be apparent. So, which means that T must be selected to be T₂ value of the image of the tissue being imaged. And we should have large T_R to reduce T₁ effects. So, we need a large T_R so that the T₁ factor is not shown. So, all the magnetization in all the tissue will recover, large T_R. But if you want, if you want to have a, no just want to get T₂, then you just measure at some add the tissue, for instance should be approximately selected to be T₂ of that tissue being imaged.

So, you can either choose that say you want at T₂. See if you wait too long, all of them will go to 0. If you wait too long to measure, all of this will deface, let us say 5 and millisecond, and 20 milliseconds, if we wait 40 milliseconds, just the spins of both of them would have gone to 0. So, you will not get a, so you will not be able would have gone to almost close to 0 or very low, you will not be able to make the difference. So, you want to measure this tissue, means you take the time to echo as 5 milliseconds and measure that. T₁ weighted contrast same thing. Where do you make sure T_R must be set approximately close to T₁.

So, in this case, if want to measure this tissue, you set it to, you set T_R to 1000, which means that only that tissue would have recovered, so we will get a high signal from there. So that is the idea behind how you get different types of contrast. So, you have to think about a bit. So, understand that T₁ is the time constant for the recovery of longitudinal magnetization. T₂ is the time constant for the dissipation of the transverse magnetization. Remember that the transverse magnetization is what used raised the signal, and the transverse magnetization is actually from the longitudinal magnetization because that is what you tip into the plane. So, the larger your longitudinal magnetization, the higher your transverse after the application of the RF pulse.

So that is all for this class. So the understanding here, just to summarize, the idea is you have all these proton nuclei, protons in your body from the hydrogen atom nucleus, they all have a spin angular momentum property, which can interact with the static magnetic field, very high value static magnetic field, and which leads to a net magnetization, this magnetization precesses around the direction of the static magnetic field with a certain characteristic frequency called the Larmor frequency, which is $\mu_0 \gamma \hbar$.

Then upon the application of another rapidly oscillating magnetic field, when it say rapidly oscillating I mean, it is the same frequency as the Larmor frequency that you are trying to of the material you are trying to measure. So, the upon application of that these, the precessing magnetization is tipped into the plane, we call that the transverse magnetization. And now, once again, it is still precessing at the Larmor frequency giving rise to a signal in a coil of wire. And there are these three important factors that determine the strength of the signal, which are basically the T_1 relaxation time, T_2 relaxation time and proton density, by appropriately tuning your time to repetition, which is the time between two successive alpha pulses.

And the time to echo, time to echo is the time from the application of the alpha pulse to the measurement of the signal in your coil of wire. By changing those times, you can make sure that either the proton density is what contributes mostly to the image contrast, or it is only the T_2 times, the difference in T_2 relaxation times is what contributes to the contrast or the difference in T_1 relaxation times. Once again, before we conclude I would make the point that these are relative. So, when you say T_1 relaxation time, you are not we will not be able to absolutely measure 1000 milliseconds from the voxel values.

So, if something has a for instance just as an example if some part of the tissue has a higher relaxation time, maybe depending on the T_R it will appear dark, the lower relaxation time T_1 relaxation time might appear bright. Similarly for T_E , if you depending on how soon you do these measurements, the lower T_E might appear darker, higher T_E might appear brighter. So, this is why, and of course proton density once again is also relative the higher, you can only see relative contrast. In the sense, something is darker means you are either more or less protons than the other regions.

So, this but then if you look this kind of gives you the flexibility to do multiple different contrast mechanisms. That is why MR is one of the we can highly researched imaging modality because the modes of contrasts are huge. I only talked about two or three different

three different these are the most commonly done. But there are other contrast mechanisms also that can be done, and that is a lot of interesting research going on to that area. Thank you. That is all for this class.