**Medical Image Analysis**
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**Department of Engineering Design**
**Indian Institute of Technology Madras**

**Lecture 21**

# FFDBSPLINES

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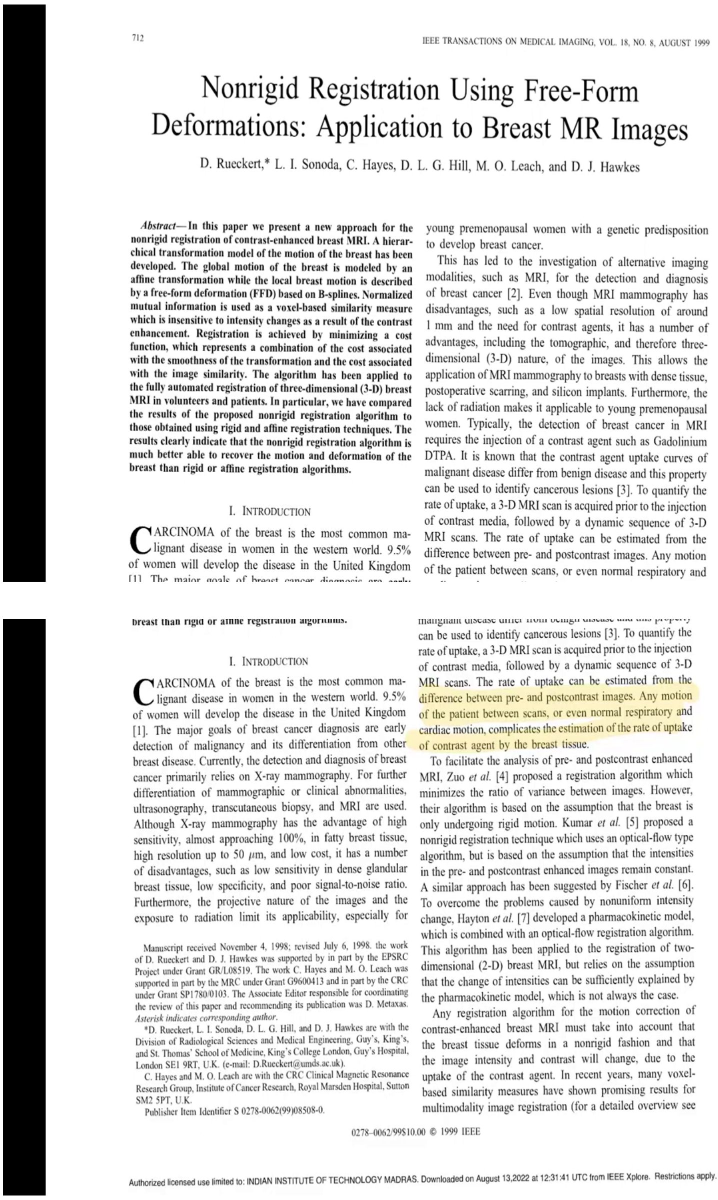



## Nonrigid Registration Using Free-Form Deformations: Application to Breast MR Images

D. Rueckert,* L. I. Sonoda, C. Hayes, D. L. G. Hill, M. O. Leach, and D. J. Hawkes

***Abstract*—In this paper we present a new approach for the nonrigid registration of contrast-enhanced breast MRI. A hierarchical transformation model of the motion of the breast has been developed. The global motion of the breast is modeled by an affine transformation while the local breast motion is described by a free-form deformation (FFD) based on B-splines. Normalized mutual information is used as a voxel-based similarity measure which is insensitive to intensity changes as a result of the contrast enhancement. Registration is achieved by minimizing a cost function, which represents a combination of the cost associated with the smoothness of the transformation and the cost associated with the image similarity. The algorithm has been applied to the fully automated registration of three-dimensional (3-D) breast MRI in volunteers and patients. In particular, we have compared the results of the proposed nonrigid registration algorithm to those obtained using rigid and affine registration techniques. The results clearly indicate that the nonrigid registration algorithm is much better able to recover the motion and deformation of the breast than rigid or affine registration algorithms.**

### I. Introduction

CARCINOMA of the breast is the most common malignant disease in women in the western world. 9.5% of women will develop the disease in the United Kingdom [1]. The major goals of breast cancer diagnosis are early detection of malignancy and its differentiation from other breast disease. Currently, the detection and diagnosis of breast cancer primarily relies on X-ray mammography. For further differentiation of mammographic or clinical abnormalities, ultrasonography, transcutaneous biopsy, and MRI are used. Although X-ray mammography has the advantage of high sensitivity, almost approaching 100%, in fatty breast tissue, high resolution up to 50 $\mu$m, and low cost, it has a number of disadvantages, such as low sensitivity in dense glandular breast tissue, low specificity, and poor signal-to-noise ratio. Furthermore, the projective nature of the images and the exposure to radiation limit its applicability, especially for young premenopausal women with a genetic predisposition to develop breast cancer.

This has led to the investigation of alternative imaging modalities, such as MRI, for the detection and diagnosis of breast cancer [2]. Even though MRI mammography has disadvantages, such as a low spatial resolution of around 1 mm and the need for contrast agents, it has a number of advantages, including the tomographic, and therefore three-dimensional (3-D) nature, of the images. This allows the application of MRI mammography to breasts with dense tissue, postoperative scarring, and silicon implants. Furthermore, the lack of radiation makes it applicable to young premenopausal women. Typically, the detection of breast cancer in MRI requires the injection of a contrast agent such as Gadolinium DTPA. It is known that the contrast agent uptake curves of malignant disease differ from benign disease and this property can be used to identify cancerous lesions [3]. To quantify the rate of uptake, a 3-D MRI scan is acquired prior to the injection of contrast media, followed by a dynamic sequence of 3-D MRI scans. The rate of uptake can be estimated from the difference between pre- and postcontrast images. Any motion of the patient between scans, or even normal respiratory and cardiac motion, complicates the estimation of the rate of uptake of contrast agent by the breast tissue.

To facilitate the analysis of pre- and postcontrast enhanced MRI, Zuo *et al.* [4] proposed a registration algorithm which minimizes the ratio of variance between images. However, their algorithm is based on the assumption that the breast is only undergoing rigid motion. Kumar *et al.* [5] proposed a nonrigid registration technique which uses an optical-flow type algorithm, but is based on the assumption that the intensities in the pre- and postcontrast enhanced images remain constant. A similar approach has been suggested by Fischer *et al.* [6]. To overcome the problems caused by nonuniform intensity change, Hayton *et al.* [7] developed a pharmacokinetic model, which is combined with an optical-flow registration algorithm. This algorithm has been applied to the registration of two-dimensional (2-D) breast MRI, but relies on the assumption that the change of intensities can be sufficiently explained by the pharmacokinetic model, which is not always the case.

Any registration algorithm for the motion correction of contrast-enhanced breast MRI must take into account that the breast tissue deforms in a nonrigid fashion and that the image intensity and contrast will change, due to the uptake of the contrast agent. In recent years, many voxel-based similarity measures have shown promising results for multimodality image registration (for a detailed overview see

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Hello and welcome back. So, in this video, we are going to look at this paper which talks about free-form deformations. Once again, displacement fields correspond to every pixel in the image. And it is non digital station, of course, and its application to breast MR images. So, why do we need this in this context of this application, basically, these breast MR images, these are 3d images of the breast taken for diagnosing and identifying breast cancer. So, the way they do it is for contrast enhanced MR.

So, basically, you inject the contrast agents and wherever there is a contrast agent there is increase in the density. But it turns out that the rate at which this contrast agent is taken up in the disease area, there is a tumor with different so it would be nice to plot this as a function of time that is you inject the contrast agent and you sequentially image the same location in the breast and look at where the contrast agent is accumulating.

Now, in order to do that, there are some issues, any motion of the patient between the scans or even, a normal respiratory like he says in the paper, if you just even talk about, there is pre and post contrast images, that is you acquire some images, which you do before you inject the contrast this can be some separate imaging session, I would like to think they are the same imaging session of course, there is patient movement between scans and there is also normal respiratory and cardiac motion.

So, which means that, if you are in this, this causes some problems because you cannot align the images as the contrast accumulates in the tissue and you are still imaging. So, this algorithm basically arises in that context. So, how do we register successive images 3d images of the breast taken, in over time and it has to be non rigid registration, because most organs are deformable. So, here we are trying to do this, this paper from this group talks about how we can use a certain form of regularization, that is smoothing using B-splines in order to reliably estimate free-form deformations. So, let us go into the paper. So, it has it comes in two steps.

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In addition, the results obtained by the nonrigid registration algorithm are compared with those of rigid and affine registration algorithms. These results demonstrate that rigid and affine transformation models often are not sufficient to model the motion of the breast adequately. Finally, Section IV summarizes the results and discusses current and future work in this area.

## II. Image Registration

The goal of image registration in contrast-enhanced breast MRI is to relate any point in the postcontrast enhanced sequence to the precontrast enhanced reference image, i.e., to find the optimal transformation $\mathbf{T}: (x, y, z) \mapsto (x', y', z')$ which maps any point in the dynamic image sequence $I(x, y, z, t)$ at time $t$ into its corresponding point in the reference image $I(x', y', z', t_0)$, taken at time $t_0$. In general, the motion of the breast is nonrigid so that rigid or affine transformations alone are not sufficient for the motion correction of breast MRI. Therefore, we develop a combined transformation $\mathbf{T}$ which consists of a global transformation and a local transformation

$$\mathbf{T}(x, y, z) = \mathbf{T}_{\text{global}}(x, y, z) + \mathbf{T}_{\text{local}}(x, y, z). \quad (1)$$

transformation.

To define a spline-based FFD, we denote the dom image volume as $\Omega = \{(x, y, z) \mid 0 \leq x < X, 0 \leq$ $0 \leq z < Z\}$. Let $\Phi$ denote a $n_x \times n_y \times n_z$ mesh points $\phi_{i,j,k}$ with uniform spacing $\delta$. Then, the FF written as the 3-D tensor product of the familiar B-splines

$$\mathbf{T}_{\text{local}}(x, y, z) = \sum_{l=0}^{3}\sum_{m=0}^{3}\sum_{n=0}^{3} B_l(u)B_m(v)B_n(w)\phi_{i+l, j+}$$

where $i = \lfloor x/n_x \rfloor - 1$, $j = \lfloor y/n_y \rfloor - 1$, $k = \lfloor z$ $u = x/n_x - \lfloor x/n_x \rfloor$, $v = y/n_y - \lfloor y/n_y \rfloor$, $w = z/n_z$ and where $B_l$ represents the $l$th basis function of th [22], [23]

$$B_0(u) = (1-u)^3/6$$
$$B_1(u) = (3u^3 - 6u^2 + 4)/6$$
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In contrast to thin-plate splines [25] or elastic-bo [26], B-splines are locally controlled, which ma computationally efficient even for a large number points. In particular, the basis functions of cubic



contrast, many nonrigid stic deformations, such as n the assumption that the remains constant. This is algorithms based on fluid xception is the registration [20], which is based on a ses mutual information as owever, due to prohibitive in-plate spline warps, the imited number of degrees r most applications, which ations.

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of transformations are affine transformations, which have six additional degrees of freedom, describing scaling and shearing. In 3-D, an affine transformation can be written as

$$\mathbf{T}_{\text{global}}(x, y, z) = \begin{pmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \theta_{14} \\ \theta_{24} \\ \theta_{34} \end{pmatrix} \quad (2)$$

where the coefficients $\Theta$ parameterize the 12 degrees of freedom of the transformation. In a similar fashion, the global motion model can be extended to higher order global transformations, such as trilinear or quadratic transformations [21].

### B. Local Motion Model

The affine transformation captures only the global motion of the breast. An additional transformation is required, which models the local deformation of the breast. The nature of the local deformation of the breast can vary significantly across patients and with age. Therefore, it is difficult to describe the local deformation via parameterized transformations. Instead, we have chosen an FFD model, based on B-splines [22], [23], which is a powerful tool for modeling 3-D deformable objects and has been previously applied to the tracking and motion analysis in cardiac images [24]. The basic idea of FFD's is to deform an object by manipulating an underlying mesh of

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To define a spline-based FFD, we denote the domain of the image volume as $\Omega = \{(x, y, z) \mid 0 \leq x < X, 0 \leq y < Y, 0 \leq z < Z\}$. Let $\Phi$ denote a $n_x \times n_y \times n_z$ mesh of control points $\phi_{i,j,k}$ with uniform spacing $\delta$. Then, the FFD can be written as the 3-D tensor product of the familiar 1-D cubic B-splines

$$\mathbf{T}_{\text{local}}(x, y, z) = \sum_{l=0}^{3}\sum_{m=0}^{3}\sum_{n=0}^{3} B_l(u)B_m(v)B_n(w)\phi_{i+l, j+m, k+n} \quad (3)$$

where $i = \lfloor x/n_x \rfloor - 1$, $j = \lfloor y/n_y \rfloor - 1$, $k = \lfloor z/n_z \rfloor - 1$, $u = x/n_x - \lfloor x/n_x \rfloor$, $v = y/n_y - \lfloor y/n_y \rfloor$, $w = z/n_z - \lfloor z/n_z \rfloor$ and where $B_l$ represents the $l$th basis function of the B-spline sed on normalized mutual ults of the application of l as clinical patient data. by the nonrigid registra- those of rigid and affine lts demonstrate that rigid often are not sufficient to uately. Finally, Section IV s current and future work

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One is that it models the registration as a combination of two registrations. One is the global and local registration. See this global registration is nothing but your rigid body registration just rotations and translations. So this is a combined transformation, they are estimating one about their estimating independently. So what is the transformation that is basically the global transformation is your familiar, it is an affine transform model. So this is an affine transform. So it is what they are estimating.

So, this affine transform model is what they split but this is the estimating independently of the according to at least this paper you can just do this first. But in order to get to, very accurate registration, you also need local deformations, the local deformation is where you try to do the non rigid registration.

So, here like we like in the demons algorithm, the local deformations or estimated pixel wise, it is a small difference, instead of looking at the displacement field of every estimate in the displacement field of every pixel, which is typically done in demons here they do it for a set of control points, these control points are then or then basically used by the B-splines these are mainly used by the B-splines control points, and then they are used to extract or interpolate deformations to the rest of the pixels.

So, the basic idea is you deform the object as it states here by manipulating and underlying mesh of control points. So this so instead of so the control points are where the deformation are estimated. And then B-splines curves are used to interpolate it everywhere. So, we did something similar for demons, we did we did do a Gaussian smoothing, so they do not do that instead, this regularization or the, the constraint is obtained using the B-splines.

So here is what the model looks like, I will show the models, the local deformation is given by the following formula here. It is basically using cubic B-splines. How does it work? So, we have a set of control points mesh or a mesh of control points and x and y and z.

And, if they are, if this actually represents $\phi_{i,j,k}$ represents the deformation at those control points, control points or uniformly spaced. And of course, you have this grid of control points, you can control the size of this grid, either make them coarse or finer, if you do very coarse grids, you get, global deformations and if you are making very fine, then you have very local deformation.

And in order to regularize this, and you are using B-splines, because then we produce a very smooth and differentiable transformation. So, how does this work? So if you look at for any, so if you have determined the deformations at the control points, $\phi_{i,j,k}$ then at any point x y z this formula tells you this formula tells you the corresponding deformation. So, how does this work? So, all it does is if you have let me just explain this first and then you can show it in the picture. So, if all it does is it takes a point x, and it calculates the nearest control point.

So, basically what is happening here, if you see this, this is calculating the nearest control indices of the nearest control point this is in 3d. So, we have x y z coordinates, so, you calculate the indices of the nearest control points. And you also look at the distance of this x y z from the nearest control points both in x y and z directions, this summation here tells you the number of neighbors has taken into account. So, l = 0 to 3, m = 0 to 3, n = 0 to 3.

So, we are looking at a large number of neighbors for a particular point x y z this many neighbors. So, because if you see your sum a summing over $\phi_{i+l,j+m,k+n}$. So, this summation tells you the number of neighbors, so, you can have any number depending on how accurate you want it or how smooth you want it.

Now, this part for the cubic place by a B-spline, this is the typical number use. So, this will translate to about 4×4×4 about 64 control points or use to estimate the deformation at one point. So, you can have these control. So, now, the control point like I said, very coarse are very fine, but they can be significantly lesser than the number of voxels in your pixels in your image. So, that is the advantage. That is one advantage.

Second is that you are since you are using this cubic baselines, you get a very smooth deformation field because you are using the cubes B-splines for the interpolation and it is also differentiable. So, what are these functions $B_0$ , $B_1$ , $B_2$ , $B_3$, they tell you the weighting of each of these control points based on the distance u. In this case u along x maybe v along y and z along, and w along z distance, distance, distance from the control points, that is what this if you look at this line of the paper that tells you how they calculate the distances.

So, for every point, you locate the nearest set of control points, in this case, there are about 64 in 3d, and you calculate the distance of these control points from, from the point of interest. And you use that to calculate the weighting for each of these control points and then subsequently estimate the value the value of the deformation field at that point. So this is the algorithm in a nutshell.

So, generally, it is of course, you can see that it is complicated to code and we also see what kind of loss function is being used. But the first step is you evaluate a global transformation which is your affine transformation. Once you have done that, then you estimate the local transformation using B-splines, B-splines interpolation.

So we have a mesh of points where you estimate the deformation and from using that mesh, you interpolate everywhere else using the B-splines. So, those control point estimates will be will be used to weight the value of deformation at different locations,. And, of course, you need the B-splines, cubic B-splines to actually do the weighting. So that is the overall idea of as far as the as far as using the cubic B-splines are concerned.

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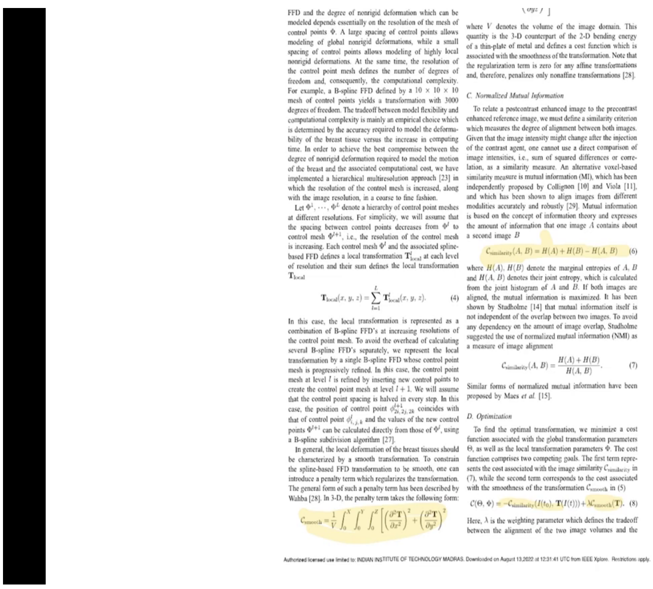

FFD and the degree of nonrigid deformation which can be modeled depends essentially on the resolution of the mesh of control points $\Phi$. A large spacing of control points allows modeling of global nonrigid deformations, while a small spacing of control points allows modeling of highly local nonrigid deformations. At the same time, the resolution of the control point mesh defines the number of degrees of freedom and, consequently, the computational complexity. For example, a B-spline FFD defined by a $10 \times 10 \times 10$ mesh of control points yields a transformation with 3000 degrees of freedom. The tradeoff between model flexibility and computational complexity is mainly an empirical choice which is determined by the accuracy required to model the deformability of the breast tissue versus the increase in computing time. In order to achieve the best compromise between the degree of nonrigid deformation required to model the motion of the breast and the associated computational cost, we have implemented a hierarchical multiresolution approach [23] in which the resolution of the control mesh is increased, along with the image resolution, in a coarse to fine fashion.

Let $\Phi^1, \cdots, \Phi^L$ denote a hierarchy of control point meshes at different resolutions. For simplicity, we will assume that the spacing between control points decreases from $\Phi^l$ to control mesh $\Phi^{l+1}$, i.e., the resolution of the control mesh is increasing. Each control mesh $\Phi^l$ and the associated spline-based FFD defines a local transformation $\mathbf{T}^l_{local}$ at each level of resolution and their sum defines the local transformation $\mathbf{T}_{local}$

$$\mathbf{T}_{local}(x, y, z) = \sum_{l=1}^{L} \mathbf{T}^l_{local}(x, y, z). \quad (4)$$

In this case, the local transformation is represented as a combination of B-spline FFD's at increasing resolutions of the control point mesh. To avoid the overhead of calculating several B-spline FFD's separately, we represent the local transformation by a single B-spline FFD whose control point mesh is progressively refined. In this case, the control point mesh at level $l$ is refined by inserting new control points to create the control point mesh at level $l+1$. We will assume that the control point spacing is halved in every step. In this case, the position of control point $\phi^{l+1}_{2i,2j,2k}$ coincides with that of control point $\phi^l_{i,j,k}$ and the values of the new control points $\Phi^{l+1}$ can be calculated directly from those of $\Phi^l$, using a B-spline subdivision algorithm [27].

In general, the local deformation of the breast tissues should be characterized by a smooth transformation. To constrain the spline-based FFD transformation to be smooth, one can introduce a penalty term which regularizes the transformation. The general form of such a penalty term has been described by Wahba [28]. In 3-D, the penalty term takes the following form:

$$C_{smooth} = \frac{1}{V} \int_0^X \int_0^Y \int_0^Z \left[ \left( \frac{\partial^2 \mathbf{T}}{\partial x^2} \right)^2 + \left( \frac{\partial^2 \mathbf{T}}{\partial y^2} \right)^2$$

where $V$ denotes the volume of the image domain. This quantity is the 3-D counterpart of the 2-D bending energy of a thin-plate of metal and defines a cost function which is associated with the smoothness of the transformation. Note that the regularization term is zero for any affine transformations and, therefore, penalizes only nonaffine transformations [28].

*C. Normalized Mutual Information*

To relate a postcontrast enhanced image to the precontrast enhanced reference image, we must define a similarity criterion which measures the degree of alignment between both images. Given that the image intensity might change after the injection of the contrast agent, one cannot use a direct comparison of image intensities, i.e., sum of squared differences or correlation, as a similarity measure. An alternative voxel-based similarity measure is mutual information (MI), which has been independently proposed by Collignon [10] and Viola [11], and which has been shown to align images from different modalities accurately and robustly [29]. Mutual information is based on the concept of information theory and expresses the amount of information that one image $A$ contains about a second image $B$

$$C_{similarity}(A, B) = H(A) + H(B) - H(A, B) \quad (6)$$

where $H(A)$, $H(B)$ denote the marginal entropies of $A$, $B$ and $H(A, B)$ denotes their joint entropy, which is calculated from the joint histogram of $A$ and $B$. If both images are aligned, the mutual information is maximized. It has been shown by Studholme [14] that mutual information itself is not independent of the overlap between two images. To avoid any dependency on the amount of image overlap, Studholme suggested the use of normalized mutual information (NMI) as a measure of image alignment

$$C_{similarity}(A, B) = \frac{H(A) + H(B)}{H(A, B)}. \quad (7)$$

Similar forms of normalized mutual information have been proposed by Maes *et al.* [15].

*D. Optimization*

To find the optimal transformation, we minimize a cost function associated with the global transformation parameters $\Theta$, as well as the local transformation parameters $\Phi$. The cost function comprises two competing goals. The first term represents the cost associated with the image similarity $C_{similarity}$ in (7), while the second term corresponds to the cost associated with the smoothness of the transformation $C_{smooth}$ in (5)

$$C(\Theta, \Phi) = -C_{similarity}(I(t_0), \mathbf{T}(I(t))) + \lambda C_{smooth}(\mathbf{T}). \quad (8)$$

Here, $\lambda$ is the weighting parameter which defines the tradeoff between the alignment of the two image volumes and the

So, on top of this there is actually a smoothness penalty which is imposed on the transformation correct. So, for the basically for the 3d penalty for the spline based transformations cannot be smooth, you have the similar smoothness constraints which are written here in integral form basically the second derivative, square of the second derivative, for us for the transformation. So, this is again this is just to make sure that you have smooth transformation.

And, of course, this is only applied to the nonrigid transformation and for the affine transformation, the revelation term is zero. So, this is the regularization term, the one we see here, basically, the smoothness constraint using the second derivative of the transformation is basically the smoothness constraint. And it is only applied for the non digital station point.

The loss function on the smoothness constraint, the loss function is your mutual information. So, if you if you remember, the demons you are mostly there, it will work for images that are from the same modality. But once again, the problem here that you are trying to handle is that we have injected contrast. So, the intensity will change. So, it is best to use something like this in this case the mutual information.

So, this is the mutual information criteria is the loss function for the optimization procedure consists of has to consist of in this case, two terms is to minimize the loss function and for that, why it is negative and the smoothness constraint and the smoothness constraint these are the two terms. So, the smoothness allows for those smoothness terms we minimized and the last confirmed negative of the mutual information has to be minimized. So, this is the total loss function because the lambda is another hyper parameter, which trades off between alignment and smoothness.

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RUECKERT *et al.*: NONRIGID REGISTRATION USING FREE-FORM DEFORMATION

**calculate** the optimal affine transformation parameters $\Theta$ by maximising eq. (7)
**initialize** the control points $\Phi$.
**repeat**
  **calculate** the gradient vector of the cost function in eq. (8) with respect to the non-rigid transformation parameters $\Phi$:

$$\nabla \mathcal{C} = \frac{\partial \mathcal{C}(\Theta, \Phi^l)}{\partial \Phi^l}$$

  **while** $||\nabla \mathcal{C}|| > \epsilon$ **do**
    **recalculate** the control points $\Phi = \Phi + \mu \frac{\nabla \mathcal{C}}{||\nabla \mathcal{C}||}$
    **recalculate** the gradient vector $\nabla \mathcal{C}$
  **increase** the control point resolution by calculating new control points $\Phi^{l+1}$ from $\Phi^l$.
  **increase** the image resolution.
**until** finest level of resolution is reached.

Fig. 1. The nonrigid registration algorithm.

smoothness of the transformation. For the purpose of this paper, we have determined the value of $\lambda$ experimentally and found that a value of $\lambda = 0.01$ provides a good compromise between the two competing terms of the cost function. We have also observed that the intrinsic smoothness properties of B-splines mean that the choice of $\lambda$ is not critical for low resolutions of the control point mesh. The regularization term is more important for high resolutions of the control point mesh. The reason for this is the fact that the ability of the FFD

as well as the correlation coefficient (CC)

$$CC = \frac{\sum (I(t_0) - \bar{I}(t_0))(\mathbf{T}(I(t)) - \mathbf{T}(\bar{I}(t)))}{\sqrt{\sum (I(t_0) - \bar{I}(t_0))^2 \sum (\mathbf{T}(I(t)) - \mathbf{T}(\bar{I}(t)))^2}}.$$

Here $\bar{I}(t_0)$, $\bar{I}(t)$ denote the average intensities of the im[...] before and after motion and the summation includes all vo[...] within the overlap of both images. In these images, the [...] and the CC provide an indirect measure of the registra[...]

So, the algorithm procedures follows as given here. The first the affine transformation parameters are obtained by maximizing the mutual information. So, we saw that once you estimated the affine parameters, first you have to apply that.

Then you initialize the control points, so initialize the control point, we need to estimate some rough estimate we can also be added to the transformer you can also assign some reasonable estimates of the deformation field. And you do this like this particular procedure that is outlined here following this class, this is for multiple resolution, so then you calculate the gradient vector of the loss function the loss function we are talking about here is equation 8 please look up the paper that equation is nothing but your smoothest imposed on the free-form deformation plus the mutual information.

So, then once you have that, once you calculate that loss function, you calculate the gradient of the loss function with respect to the nonrigid transformation parameters which are nothing but the deformations that you estimate at the control points of the mesh.

So, there is a threshold on the norm of the gradient of C, as long as it is greater than this threshold or that it is it does not go beyond the below a certain value, you do gradient descent this is gradient descent, you do this gradient descent and then after you and then of course, after you calculate your estimate this, you apply the deformation and then you recalculate the gradient vector so on and so forth.

And of course, once you have converged with for a particular resolution, you can move to a higher resolution of grid points and also the higher resolution of the image and then repeat the whole process. So this is the typical procedure for the nonrigid registration using the B-splines.

The authors have got excellent results for as we look at this demonstration of this letter class which we write for just the summary of the algorithm is what we have been trying to give you. So, for this process is like I said iterative process equaling to gradient descent, but we used it for this registration process for aligning breast tissue before pre and post contrast enhancement before and after injection of contrast in order to identify, breast lesions much more accurate.

So far, this week we have looked at two algorithm the demons registration algorithm as well as the algorithm for estimating this free-form deformation using B-splines regularization. These two are typical algorithms used this way very highly cited algorithms seemed very effective, shown to be effective in many applications, but still they still wide open lot of progress being made. People are also using deep learning to assess these kinds of deformation. So we will see that when we get to the part where we use deep learning for image analysis, that is all for this week. Thank you.