

Medical Image Analysis
Professor Ganapathy Krishnamurthi
Department of Engineering Design
Indian Institute of Technology Madras
Lecture 10

Noise Reduction

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Noise Reduction	
- Model of noisy image g is given by $g = f + n$ where n is the zero mean noise - Noise removal through linear filtering consists of estimating the expected value $E(g)$ since $E(n) = 0$, also $E(g) = E(f) = f$ - Assumptions are made about f in order to reconstruct $E(g)$	- Linear Filtering $\rightarrow f$ is locally constant, E is estimated by neighbourhood averaging - Median filtering $\rightarrow$ Normal distribution, locally constant except at edges and edges are straight

$$g = f + n$$
$$E(g) = E(f) + E(n) \rightarrow 0$$

Hello and welcome back. So, in this video we are going to look at noise reduction before we get into noise reduction we have to first talk about the model of a noisy image. So, given that we made g there is a given image g and we assume that it is actually the sum of the noise free ideal image f plus n plus n where the n indicates the noise.

So, this is called an additive model of the noise where and it is also stationary in the sense that the noise is the same distribution everywhere. So, that is the typical assumption that you would make. So, it is called a stationary additive noise n 0 mean noise and the idea of noise removal is through linear filtering the first step we will consider is linear filters the idea of noise removal through linear filtering is basically estimating the expected value.

So, basically what is the average g at every point g is an image. So, at the image as a whole we are expect we are kind of estimating and we can do that because since the expectation if we take the expectation on both sides of this expression. So, $g=f+n$. So, if you take the expectation on both sides of this expression then E of f plus rotation of n so, it is expectation is distributive or

plus addition then we can just say expectation of g is the same as expectation of f expectation of n being 0.

So, how do we do this expectation this that is basically how do you do this expectation of g ? So, we make some assumptions about f . So, what are these assumptions about f ? f is locally constant or smooth locally constant is the better in which case e is estimated by neighborhood average and if you look at something called median filtering which you look at the subsequent lecture it is basically if you take a large enough neighborhood then you are assuming that the pixels are gaussian distributed or normal distribution which is what we call it.

And there is also some assumptions about the edges and they assume that edges are straight but that is median filtering in general linear filter big assumption is f is locally constant. So, that you can estimate the expectation value of g by a neighborhood averaging. So, we will see what that means in the latest slide.

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Linear Filtering

- E is estimated by averaging over a neighbourhood of every pixel.
- Operation can be accomplished as a convolution with a mean filter also known as box car averaging -> large regions lead to blurring
- Taking into account that the likelihood of f having the same value as the center pixel decreases as we move away, we can weight the pixels using a Gaussian distribution
- Gaussian distribution has infinite support, for implementation it has to be cut off in a neighborhood
- Implementation can be done in Fourier domain -> Butterworth filter, Hamming and Hanning windows

So, one thing that you are all familiar is called the box car filtering. So, if you have if you have an image we take a very small patch of that image and we assume that in that patch all the pixels should be of the same value. So, then we replace the center pixel of that patch by the mean of all the elements in the patch.

So, one other thing that we can also do is to take into account that the likelihood of the image f having the same value at the center pixel decreases as we move away from the center pixel. So,

we can weight the pixels using a gaussian distribution as birth distribution as function of distance.

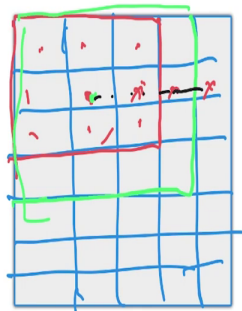
So, but then gaussian distributions are infinite support because the sense infinite support because gaussian distribution if you take the continuous distribution it is defined for all values of the input x let us say and so, since it has that is why it is called infinite support. So, it is there for all values of x .

So, for implementation purposes we have to cut it off in neighborhood and implementation of all of these linear filters can be done in a fourier domain. So, you take the free transform of the image free transform of your filter function which is in this case for instance box car averaging it is all just a bunch of ones with a normalizing factor or gaussian function, gaussian function which you can gaussian weighting function which of course it is fluid transforms also gaussian.

So, there are different filters in the fourier domain for filtering and for noise for example denoising many of them are just low pass filters that is you assume that the fluctuation sudden fluctuations in a small neighborhood that is occurring only due to noise and so, that conforms to the high frequency components in every image. So, you cut them off by using low pass filter.

So, for instance butter worth hamming and handing windows not hand windows sorry hamming windows they are some of the more commonly used things for image denoising but I am just going to have to understand make sure you understand what we mean by this linear filtering block color everything etcetera. So, I am going to do some hand drawn examples.

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Blur the image $\rightarrow$ edges might be lost

So, for instance you can consider some box let me see here whether we can draw something reasonably this is the image. So, we can draw a bunch of lines just to get the. So, we will use this pen to get some lines in. So, what we mean by expectation basically the boxcar averaging filter is let us say you take this pixel what you do is you assume that in a small neighborhood this is the assumption when you do this this kind of filtering to remove noise the small neighborhood that I got with this red box you assume that in that small neighborhood all the pixels are the same value. So, how you might ask how is this valid.

So, for instance if you are looking at particular type of tissue or in the sense let us just take natural images let us say you have taken a picture of a sky and if it is a clear sky that is going to be uniformly blue. So, wherever you move across if you in fact the pixels that corresponding to the sky you expect all of them to be the blue color which is the same rgb values.

But similarly in gray scale images also let us say for medical images also there are regions which correspond to similar tissue and you would expect that a pixel in which corresponds to which lie on the similar tissue should have similar values or same values that is their logic. So, but then you cannot do this take this too far because there are going to be some heterogeneities and they get worse as you move away from the central pixel that is what we are going to look now.

So, you can see that in this 3 by 3 neighborhood we can assume that this pixel is going to be similar to all the others. So, then instead of. So, one thing we can do is we can just easiest way

would be to replace this by a neighbor but in order to get some more statistics we replace it with the average of all the things all the pixels inside this region and because we can move this box around center it at every pixel center this box red box at every pixel and do the same. So, that you get very good averaging.

So, you can see that that this this will remove noise because it just smooths over the small fluctuations in the image. But it does not do something bad because you can imagine that this will actually blur the image it is called. So, sharpness will go away edges might lose you might lose edges or edges might be might be lost. So, this is the danger of that happening with this kind of a smoothing operation.

So, that is what people do would be they do this gaussian weighting what you do is instead of weighting it similarly so what you are basically doing is you are weighting each one of these pixels the same way that is why you are replacing an average but rather you can just put a gaussian on top of the 2d gaussian on top of this. So, that each pixel will be weighted slightly differently because as you move away from the central pixel you want to weight differently.

So, you can actually choose a larger region but you would weigh the for instance into a different color just to highlight let us say you choose a slightly larger green region and it is just here for let us say this pixel. So, as you move away as you move towards the edge of that region then you would weight it considerably lessor.

So, that that weighting can be obtained from a gaussian from the gaussian and you have to discretize the gaussian etcetera in order to get that weight this is typically what you would do if you look at this red box it has 9 pixels inside it. So, what picture if there is an edge running through it will preserve the edge because it is going to sort it and it is going to come kind of there will be some lowering of the intensity but overall it is going to be there.

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Median Filtering

- Selects center pixel value from a sorted list of pixel values in a neighborhood
- Removes intensity outliers since no averaging is done
- Preserves edges- might sometimes remove corners

Salt and pepper noise

So, before we go the next filter which is not a linear filter the previous one was a linear filter this is not linear this is a nonlinear filter it is called median filtering what it does is we saw this this selects a central pixel value from a sorted list of pixel values and neighborhood. So, it is the same as before except that in a 3 by 3 neighborhood. So, in this case we still do is you take 9 pixels you sort them from increasing in increasing order and you choose the middle one among it.

So, what this does is it removes intensity outliers and so since no averaging is done it removes the intensity outside because you are choosing the median it will preserve edges that is that you will see if you can if you are near if you are near if your box actually straddles an edge that the red box that I showed in. So, the median filter preserves edges it might sometimes remove so called corners.

There are some examples in several textbooks I will put them out there in the forum or maybe in a later lecture. So, when we summarize these few weeks I will have a look at what median filter does in detail, but middle filtering the implementation very simple you select a region you sort the pixels in that region in increasing order you select the middle one from there since you are not averaging very large outliers are easily removed and this actually is very good at removing something called salt and pepper noise.

So, some random high pixel values or random low pixel values are easily removed and you do not lose the edge as much since you are doing the sorting and then choosing the middle value.

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Diffusion Filtering

- Uses the diffusion process as a model for noise reduction
- Image intensity is considered as material density and noise increases the variability of the density
- Diffusion is carried out as an iterative process. If diffusion is homogenous then this process is accomplished by Gaussian filter
- Inhomogenous and anisotropic diffusion filtering prevent smoothing across edges to varying degrees

Praveen Nelli

So, the next couple of topics they are slightly more complicated than what you have seen so far. So, what is seen so far are just one short techniques you choose a region you average it replace the center pixel in that region or you replace it with the median have we looked at how to take gradients etc those are very fairly simple methods. So, the next couple of methods one is the diffusion filtering the other one is the bayesian estimation they are both slightly more complicated they are they are what are they are what we what might call iterative techniques. So, we will look at them in the next few lectures. Thank you.