Cellular Biophysics
Professor Doctor Chaitanya Athale
Department of Biology
Indian Institute of Science Education and Research, Pune
Tutorial - Part 02

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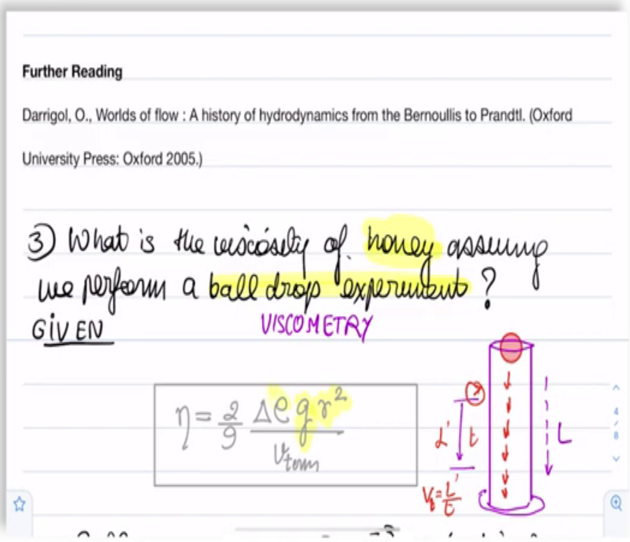


Further Reading

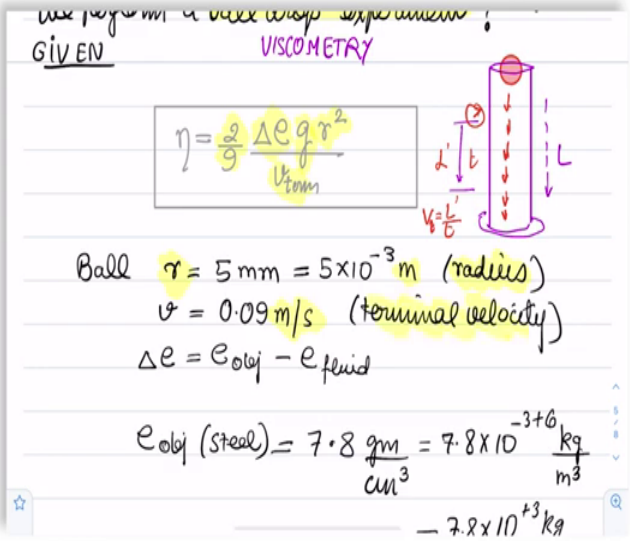
Darrigol, O., Worlds of flow : A history of hydrodynamics from the Bernoullis to Prandtl. (Oxford University Press: Oxford 2005.)

3) What is the viscosity of honey assuming we perform a ball drop experiment?

GIVEN

VISCOMETRY

$$\boxed{\eta = \frac{2}{9}\frac{\Delta \rho g r^2}{v_{term}}}$$



GIVEN

VISCOMETRY

$$\boxed{\eta = \frac{2}{9}\frac{\Delta \rho g r^2}{v_{term}}}$$

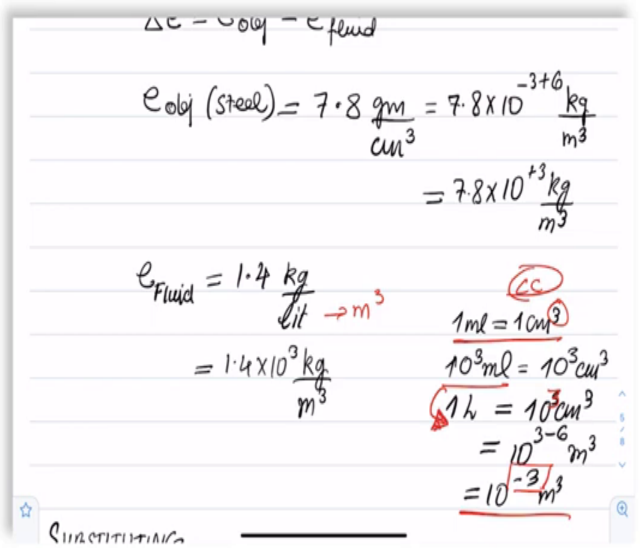
Ball $r = 5\,mm = 5 \times 10^{-3}\,m$ (radius)

$v = 0.09\,m/s$ (terminal velocity)

$\Delta \rho = \rho_{obj} - \rho_{fluid}$

ρ_{obj} (steel) $= 7.8 \frac{gm}{cm^3} = 7.8 \times 10^{-3+6} \frac{kg}{m^3}$

$= 7.8 \times 10^{+3}$ kg



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$\rho_{obj}\,(\text{steel}) = 7.8\,\frac{gm}{cm^3} = 7.8\times10^{-3+6}\,\frac{kg}{m^3}$

$= 7.8\times10^{+3}\,\frac{kg}{m^3}$

$\rho_{Fluid} = 1.4\,\frac{kg}{lit} \rightarrow m^3$

$= 1.4\times10^{3}\,\frac{kg}{m^3}$

cc

$1ml = 1cm^3$

$10^3 ml = 10^3 cm^3$

$1L = 10^3 cm^3$

$= 10^{3-6} m^3$

$= 10^{\boxed{-3}} m^3$

SUBSTITUTING

So, we talked about dynamic viscosity, we talked about kinematic viscosity and we made a small historical excursion to Stokes. Now, we are going to ask a practical question about how we would measure the viscosity of a very dense fluid, which is of biological origin, honey. In order to measure something we need an instrument, the instrument we will assume that we have used is a ball drop viscometer and the measurement is then therefore, called viscometry.

Now this is a theory course, so I am going to give you some numbers and I expect you to try to solve this and arrive at an answer. It is a test of your understanding of theory and indeed, in a laboratory you would normally conduct this experiment to validate this theory, to check whether this number is here.

So, the first thing that I have given you is something we derived as you recall in the theory lectures,

$$\eta = \frac{2\,\Delta\rho\, g\, r^2}{9\, v_{term}}$$

which is the dynamic viscosity eta is equal to 2 by 9 into the difference between density of object and density of fluid times acceleration due to gravity times squared radius of the ball drop upon v term, v term is nothing but the terminal velocity.

If you recall, the idea of the ball drop viscometer is a simple one, there is a ball that we allow to fall through a column of fluid this is typically some kind of a cylinder. We also assume that the length of the cylinder is long enough that the velocity at some point reaches the terminal velocity and it does not change, when the upward and downward forces are equated and therefore, there is no acceleration that is the derivation.

So, when we now allow this ball to fall through the fluid, we use a stopwatch, measure the time it takes to travel through some distance L prime and use velocity is equal to L prime by t and this velocity is the terminal velocity so it is not changing, so that is what I meant by terminal velocity. So, let us just look at this equation and ask ourselves what are the terms that we have and what do we need.

g is a constant, it is gravitational acceleration. If we use a ball of a known size, we know it is r, the density of the ball is something we can measure weight per unit volume. The density of the fluid also we can measure, it is weight per unit volume and v term is part of the experiment. So, in a way we know everything, this is a constant. So, let us put some numbers. The radius of the ball is 5 millimeters, which is 5 x 10^{-3} meters by converting to SI units.

v velocity as it was measured by some people in experiment is 0.09 meters per second. You could say 9 centimeters per second this is our terminal velocity and $\Delta\rho$ is the difference between the density of the object and density of fluid, so let us calculate this. What is the density of steel ball, like ball bearings? This is 7.8 grams per centimeter cube. You can find this on many engineering and physics textbook, constant pages.

Converting that to kg per meter cube gives us 7.8 into 10^3, which is 7800kg per meter cube. Density of the fluid is for honey has been reported to be 1.4 kg per liter. We want to convert from liter to meter cube. In order to do that we refer to our standard idea 1ml is 1 cc, as it is sometimes called cubic centimeter, therefore, 10^3 ml is 10^3 centimeter cube, cc.

10 to the power 3 ml is 1000ml, which is 1 liter, 1 liters 10 to the power 3 same cube, but cm 2 meter cube is 10 to the power minus 2 into 3 that is minus 6, therefore, 3 minus 6, therefore we end up with minus 3. So, with this in hand we can convert our kg per liter to kg per into cube. So, now that we have the two density values for steel, we have in terms of kg per meter cube 7800 and for honey we have 1400 kg per meter cube.

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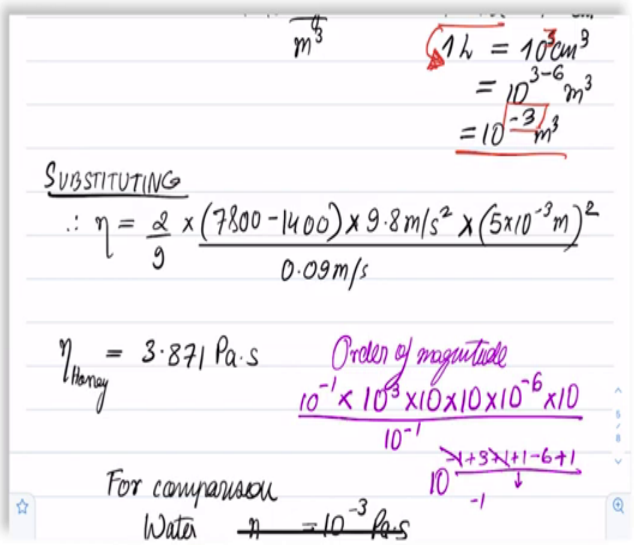

m^3

$1\,L = 10^3\,cm^3$

$= 10^{3-6}\,m^3$

$= 10^{\boxed{-3}}\,m^3$

SUBSTITUTING

$$\therefore \eta = \frac{2}{9} \times \frac{(7800-1400)\times 9.8\,m/s^2 \times (5\times10^{-3}\,m)^2}{0.09\,m/s}$$

$\eta_{Honey} = 3.871\,Pa.s$

Order of magnitude

$$\frac{10^{-1}\times 10^3\times 10\times 10\times 10^{-6}\times 10}{10^{-1}}$$

$$10^{\cancel{-1}+3\cancel{+1}+1-6+1}$$

-1

For comparison

Water ~~$\eta = 10^{-3}$ Pa.s~~

So, now we are in a position to substitute all the values. So eta becomes 2 by 9 into the difference between 7800 minus 1400 multiplied by g which is acceleration due to gravitational and 9.8 meters per second square multiplied by r whole squared, which is 5 millimeters, radius which is 5 to 10 to the power minus 3 meters whole squared, upon 0.09 meters per second. And if you do the calculations, you get 3.871 Pascal second.

Now something that I have not mentioned before that is a very useful trick, which is sometimes called order of magnitude estimates, which is to say how do we know this answer is correct. So, we can look at our numbers 2 by 9 is roughly 1 by 10, which is 0.1, 10 to the power minus 1. The difference between the densities is 7000 minus 1000 which is about 6000, so that becomes 10 to the power 3.

And 6 rounding off is 10, 9.8 meters per second squares 10, 5 to 10 to the power minus 3 is 10 to the power minus 6 and 25 which we will take as 10 and 0.09 meters per second is approximately 0.1 meters per second which is 10 to the power minus 1. And when we take the orders together we end up with minus 1 plus 3 plus 1 plus 1 minus 6 plus 1, so minus 1 plus 1 goes, plus 1 plus 1 becomes 2.

So, all this adds up to 3 minus 6 is minus 3 plus 2, 3 minus 6 minus 3, minus 3 plus 2 is minus 1. Now, it looks like we may have missed something but this could be attributed to the exact values that we have taken something like 10 to the power 0 is what we were expecting. This may have something to do with some of the approximations in the denominator that we took. So, we got a 10-fold lower value than expected.

Such orders of magnitude estimates are very good quick checks that our numbers are not wrong, but the basic check over here is, of course, do it in a calculator and check again.

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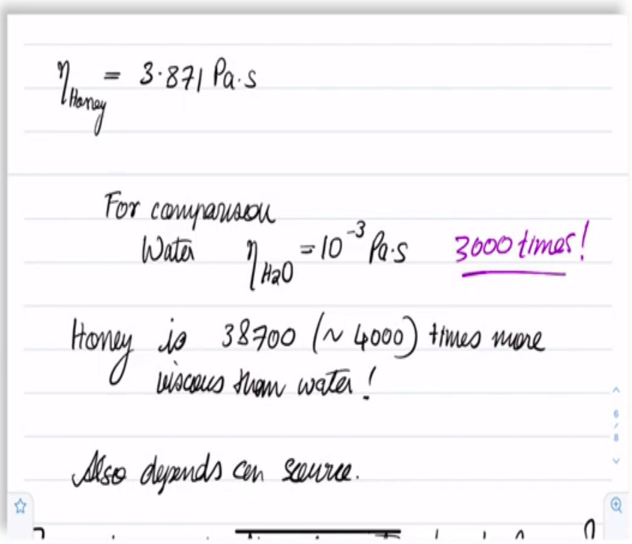

$\eta_{Honey} = 3.871\ Pa \cdot s$

For comparison

Water $\eta_{H_2O} = 10^{-3}\ Pa \cdot s$ 3000 times!

Honey is 38700 (~ 4000) times more viscous than water!

Also depends on source.

For comparison the viscosity of water is 10 to the power minus 3 Pascal second, which means that honey is 3000 times more viscous, that is a huge number. So what of it? So some of the questions that we can ask now is that, of course, honey also depends on the source, so depending on which flower, which bee collects it, from honeybee to honey bee, from beehive to beehive, they may find different numbers.

In fact, viscosity of funding can be used as a qualitative measure of honey, so this is a biotechnologically important measure because as you know honey can be adulterated, made synthetically using sugar solutions and physical constants like this can help us measure things.

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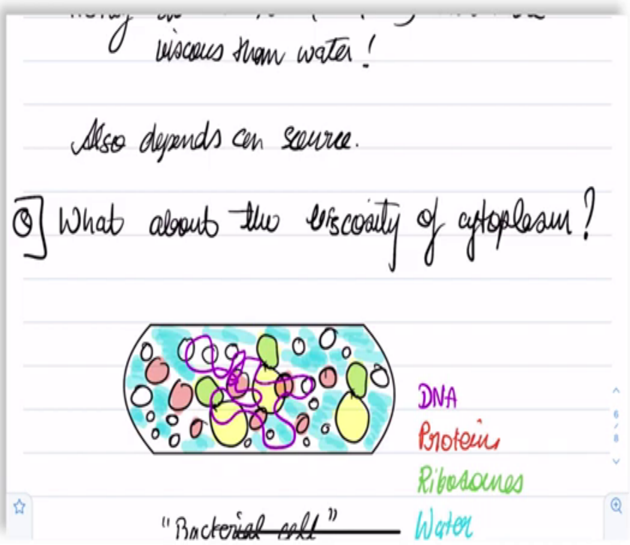
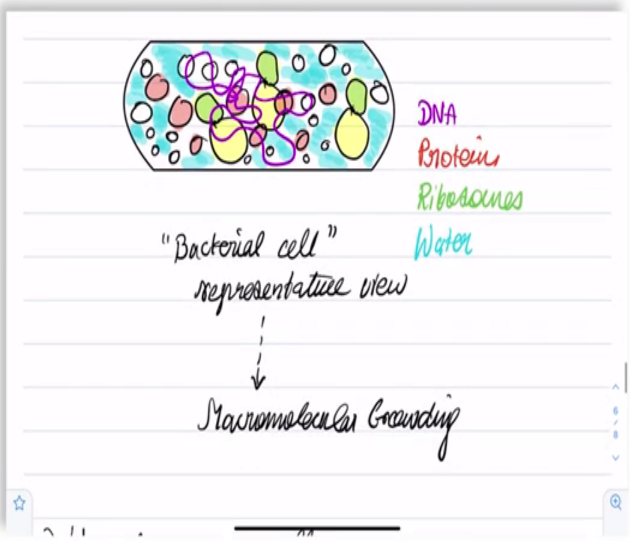

viscous than water!

Also depends on source.

Q] What about the viscosity of cytoplasm?



"Bacterial cell"
representative view
↓
Macromolecular Crowding

What about the viscosity of cytoplasm? We are in a biophysics course, you might actually want to know - what is the viscosity of cytoplasm. Well, the answer to that question is a bit complicated, because we know while the cytoplasm is 70 percent water, there are also DNA, proteins, ribosomes, small molecules; in prokaryotes no organelles, no intracellular membranes, but in eukaryotes also membranes.

So, all these structures are likely to affect the measure of viscosity and in this context we are going to come to this topic later about how crowded the cell is and whether this affects the viscosity. So, I just want you to keep in mind that the viscosity of a cytoplasm is a complex measure, there is a lot to it and you can intuit a certain number but these numbers may be off due to reasons of physiology.

So, in this case I have taken a bacterial cell and a representative a view, and I just want to say remind you that micro-molecular crowding might be an aspect that we will need to consider.

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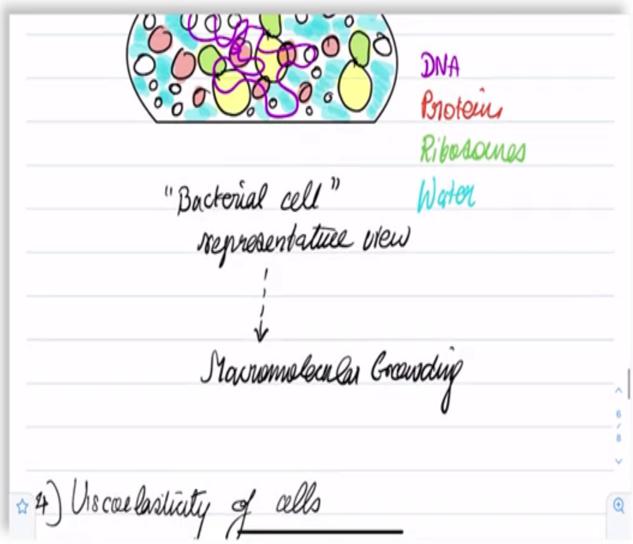

DNA
Protein
Ribosomes
Water

"Bacterial cell"
representative view

Macromolecular Crowding

4) Viscoelasticity of cells

So, for the last topic I am going to talk about viscoelasticity of cells.