Cellular Biophysics
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Lecture 60
Phyllotaxis Part 03

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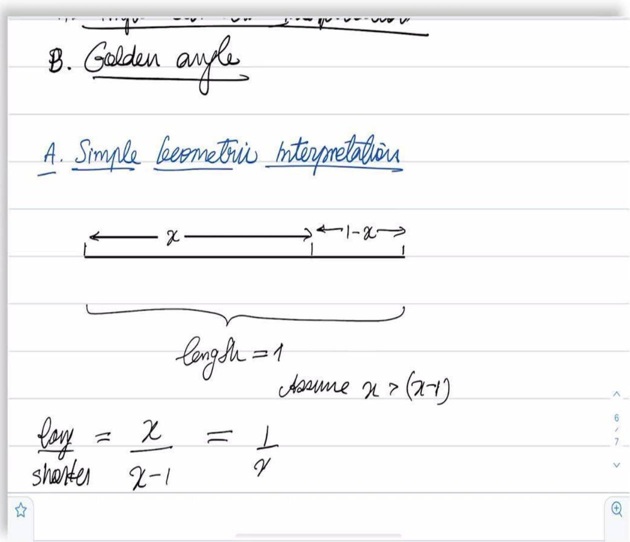

Hi, so, what do we mean by the simple geometric interpretation? Consider a segment of beam line, which is of unit length, divide it into two unequal parts x, and therefore, by definition 1-x. Then take 1 leg of the length and divide in that part that is x upon x-1. We assume x is longer than x-1, such the ratio of the longer by shorter. This can also be written as 1 by x.

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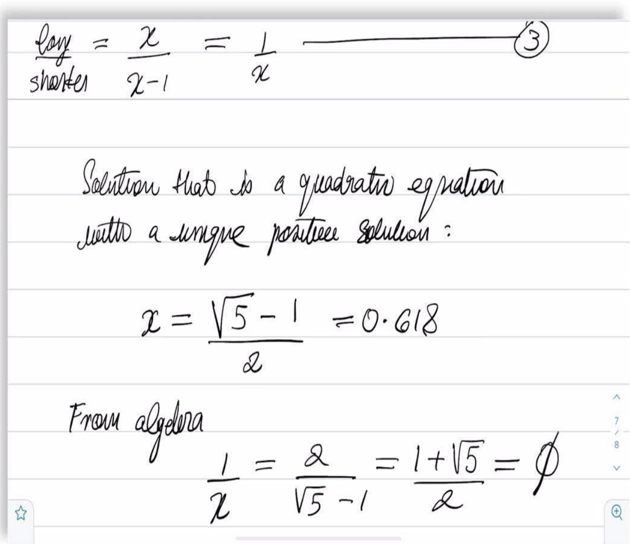

Such an equation results in a solution that is a quadratic equation with a unique positive solution which goes as this, that is

$$x = \frac{\sqrt{5}-1}{2} = 0.618$$

. From algebra, we can show that

$$\frac{1}{x} = \frac{2}{\sqrt{5}-1} = \frac{1+\sqrt{5}}{2} = \phi$$

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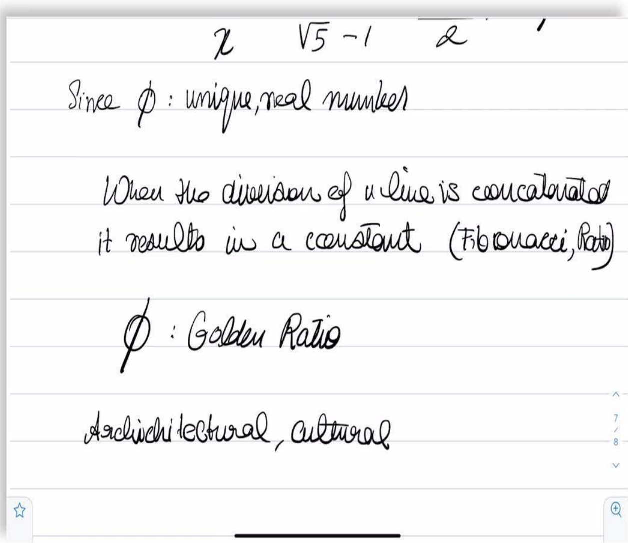

So, we come back to our phi, our golden number. Since phi is the unique real number, when the division of a line is concatenated, it results in a constant. This is nothing but what we saw at the Fibonacci series and the ratio. Therefore, phi is also then called the golden ratio, that is found in many architectural and cultural artifacts. And it is thought to have some significance. But for us, the significance is really focused on phyllotaxis.

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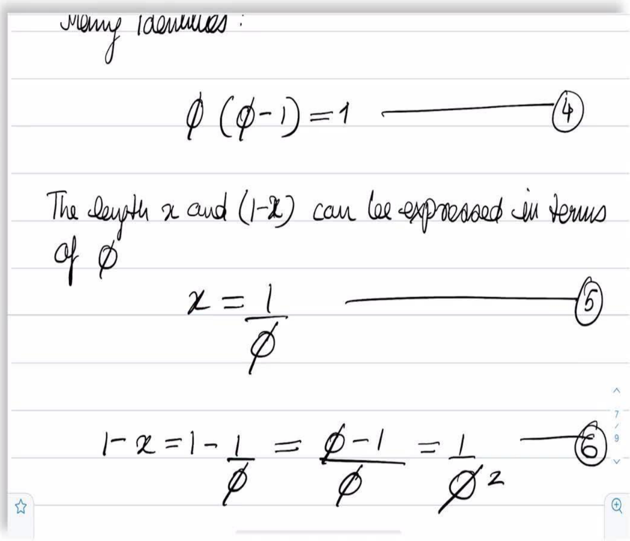

Many identities come from phi. So it can be shown that

$$\phi(\phi - 1) = 1$$

And the length x and 1-x can be expressed in term phi such as

$$x = \frac{1}{\phi}$$

x is equal to 1 by phi

$$1 - x = 1 - \frac{1}{\phi} = \frac{\phi-1}{\phi} = \frac{1}{\phi^2}$$

1-x is equal to 1-1/phi which is equal to phi-1/phi, which is equal to 1 by phi square.

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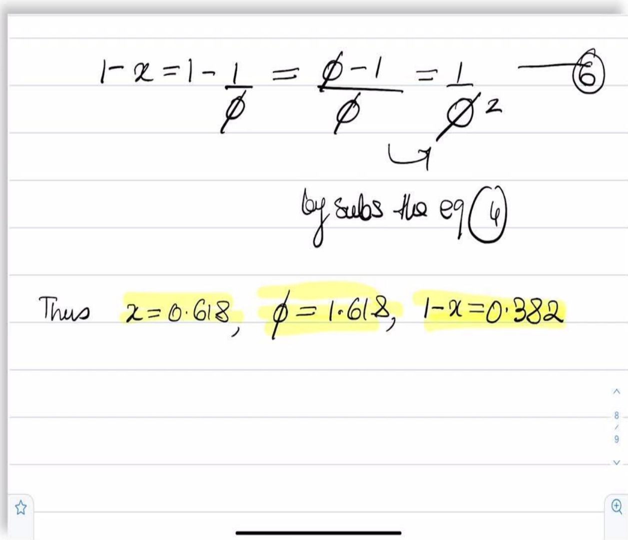

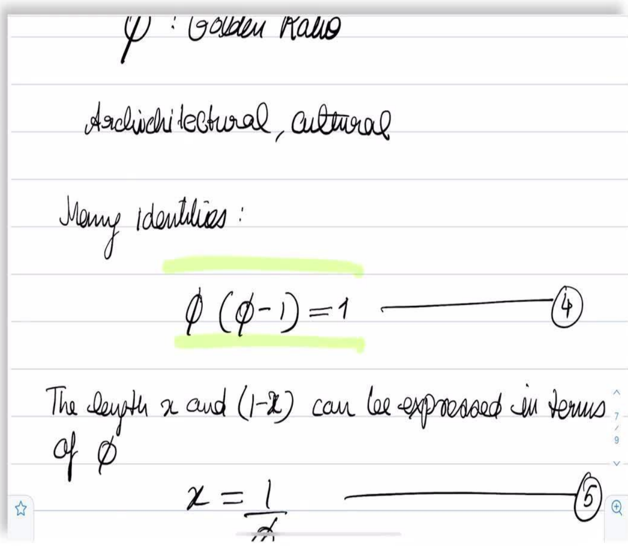

This we obtain from here to here by substituting the equation 4, which is here. Thus, x is equal to 0.618 by substitution, phi is equal to 1.618 and 1-x is equal to 0.382. These numbers show up again, so I am just going to highlight them.

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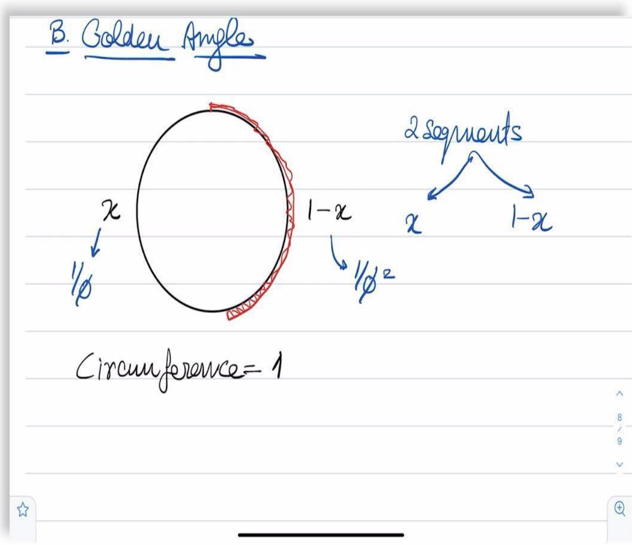

The second part is the golden angle. That, in a way builds on the geometric interpretation. We take into account a circle and divide it again into two parts. Now, I am going to draw this by hand. This segment and the other segement, the other segment is x, a longer one and 1-x, as I have shown here. The circumference is 1, that is the assumption. Fair enough. The lengths therefore, of the two segments are indeed x and 1-x. By by definition, from what we did earlier, this can be written as in terms 1 by phi square, and this as 1 by phi. So in that sense lengths are relatively by the golden number. We want to know the angles.

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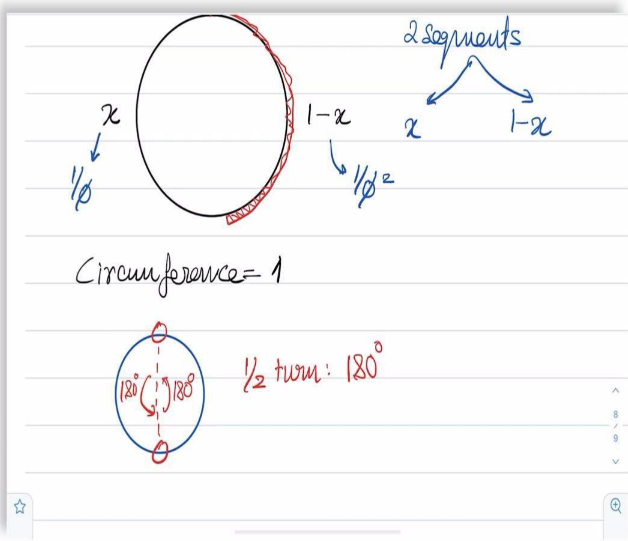

So, in case of phyllotaxis, if the organs are at two points then the angular distance is 180 degrees. We can say there is a half turn, corresponding to 180 degrees.

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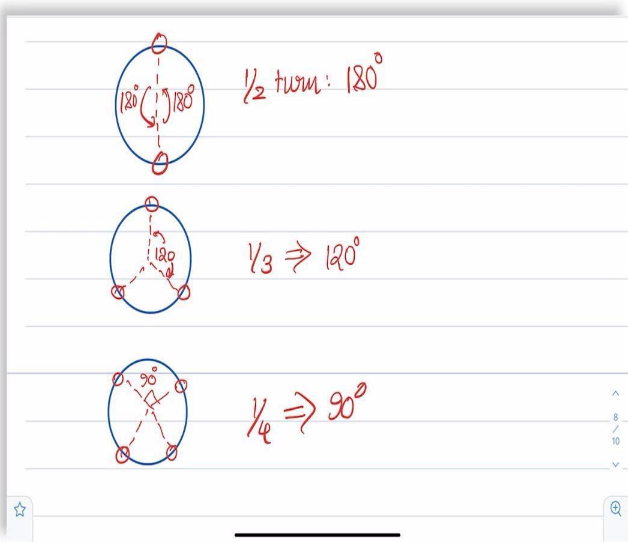

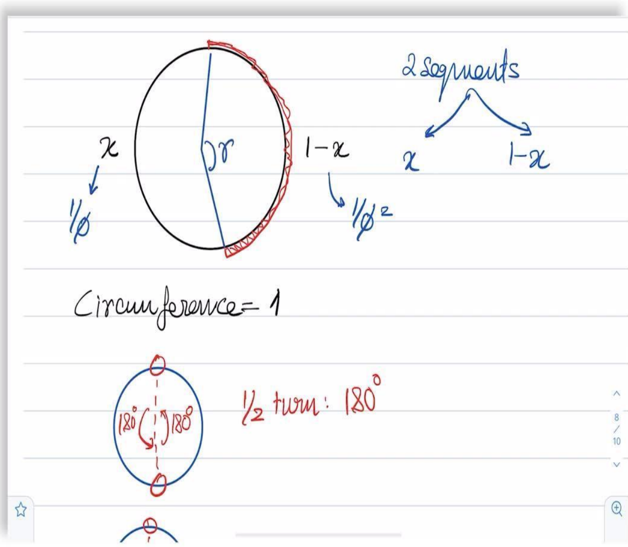

If on the other hand, we have three organs, then we are referring to an angle of 120 degrees of equal lengths, 1 by 3, 120 degrees. And similarly for a four organ structure of equally spaced angles, we get 90 degree angles. So, the fraction of the turn of a small portion of the perimeter measures the golden angles, so this angle gamma, is the one that interests us.

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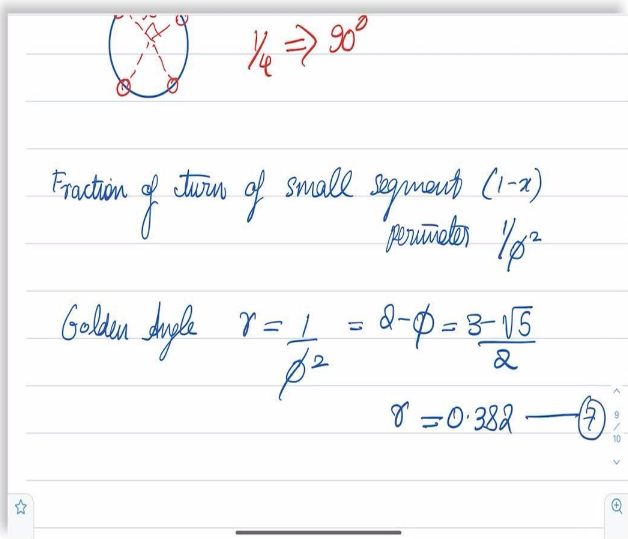

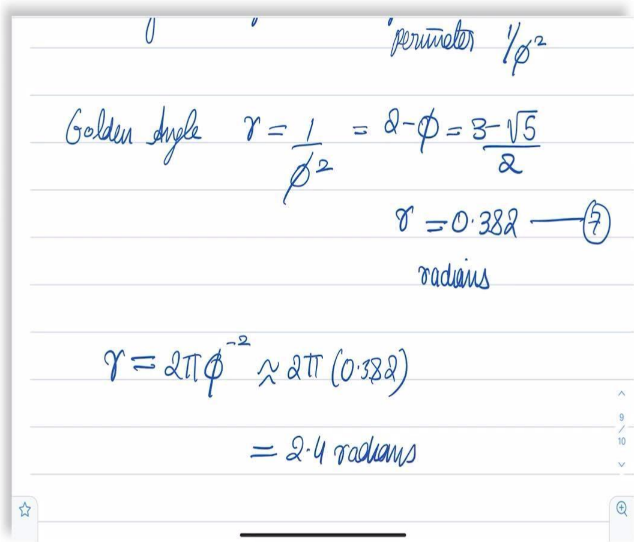

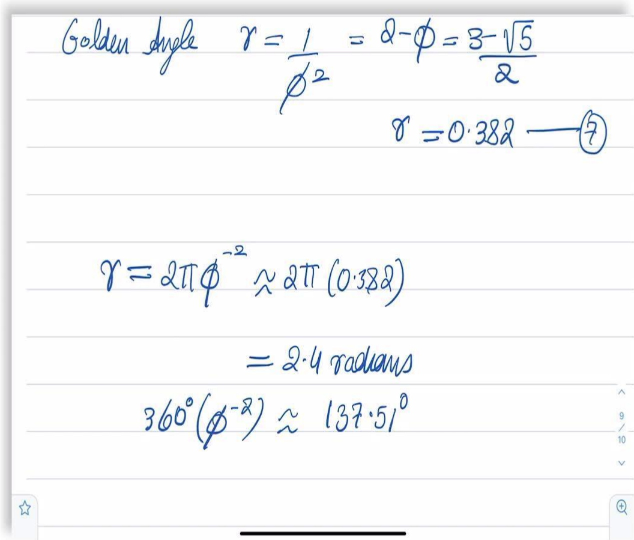

The fraction of turn of the smaller segment 1-x which also acts as perimeter therefore 1 by phi square, measures the golden angle gamma,

$$\gamma = \frac{1}{\phi^2} = 2 - \phi = \frac{3-\sqrt{5}}{2} = 0.382$$

which is equal to 1 by phi square which is equal to 2-phi equal to 3 minus root 5 by 2, and is 0.382. This is in radians units. To get it in degrees,

$$\gamma = 2\pi\phi^{-2} \approx 2\pi(0.38) = 2.4 radians$$

gamma equal to 2 pi phi inverse squared, which is approximately 2 pi into 0.382.

And the answer becomes 2.4 radians. So that was just a fraction.

We needed to convert in terms of degrees. So this means 360 degrees into phi inverse squared. That is approximately equal to 137.51 degrees. You could say about 137 degrees. This is the crux of the idea of phyllotaxis patterns.

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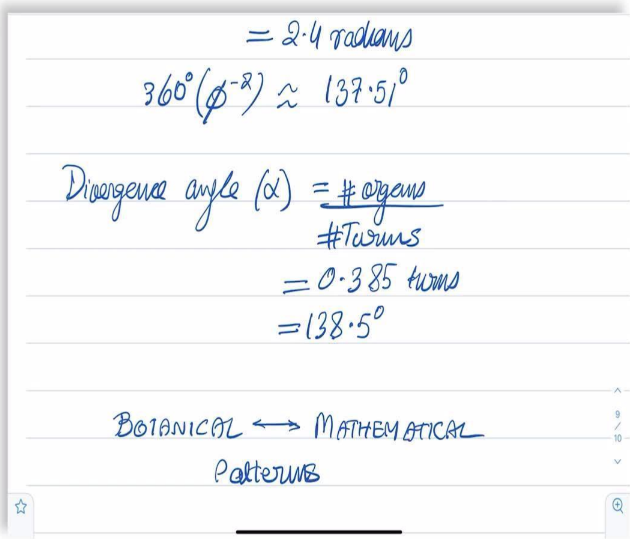

Because the divergence angle alpha that we dealt with at the very beginning, which is given a number of organs per turn, find the number of turns for most spiral phyllotaxis is 0.385 turns. And that is 138.5 degrees. So, it becomes apparent that Botanical and Mathematical patterns are intimately related.

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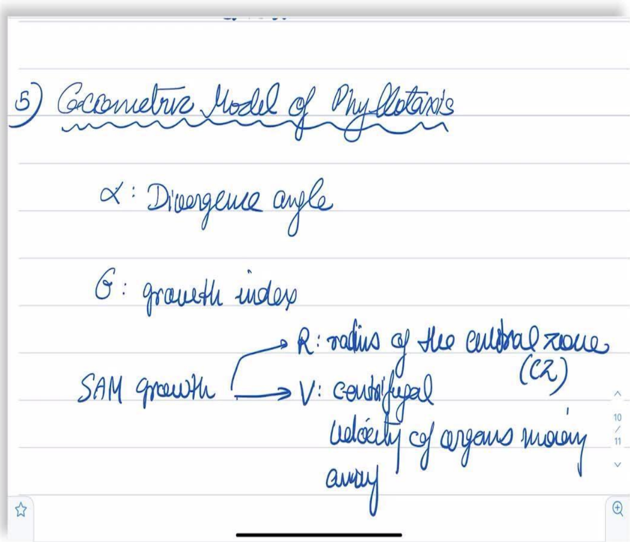

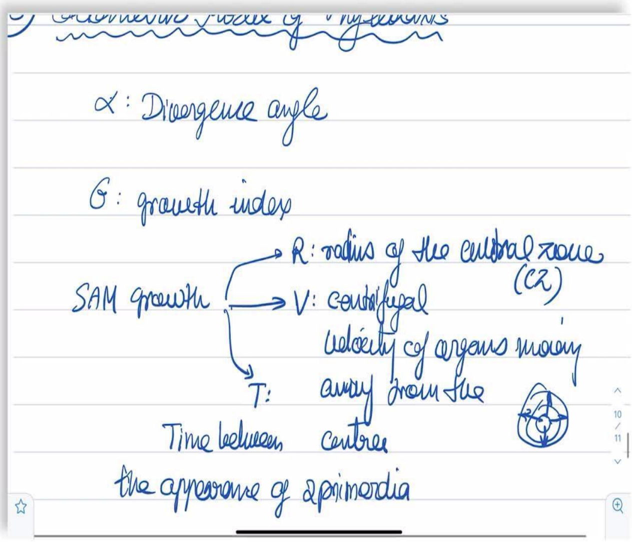

If we follow geometric model, which can be called a toy model then we can answer what is the basis of this. Now, this, we need another terms, few other terms to be defined. That we are familiar with, divergence angle, the growth index which arise this from shoot apical meristem, and its growth.

Sort of what we saw earlier, the shoot apical meristem growth depends on the R, which is radius of the central zone CZ, V, which is the centrifugal velocity of organs moving away from the centrally of growth as a growth expansion happens. This movement outwards is what is growth, centrifugal velocity. And T, which is the time between the appearance of two primordia.

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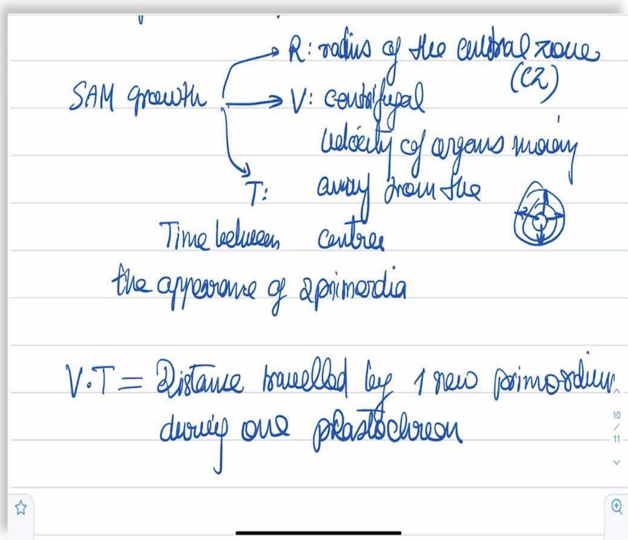

By definition, V times T is equal to the distance traveled by 1 new primordia during one plastochron. One plastochron is one expansion cycle over appearance of one new primordia.

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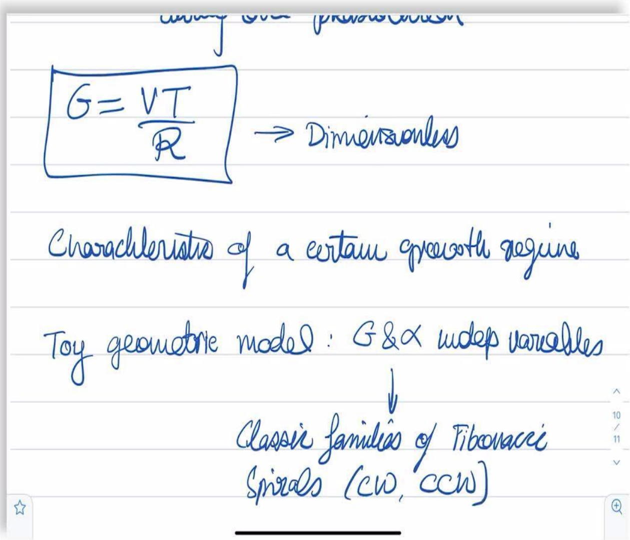

The growth index and can finally be at last G is equal to VT/R. Please note that this is dimensionless, as we are now dividing when phyllo. It is a characteristic of a certain growth regime. A toy geometric model can show that if G and α are independent variables, we can generate classic families of Fibonacci spirals, both clockwise and counterclockwise.

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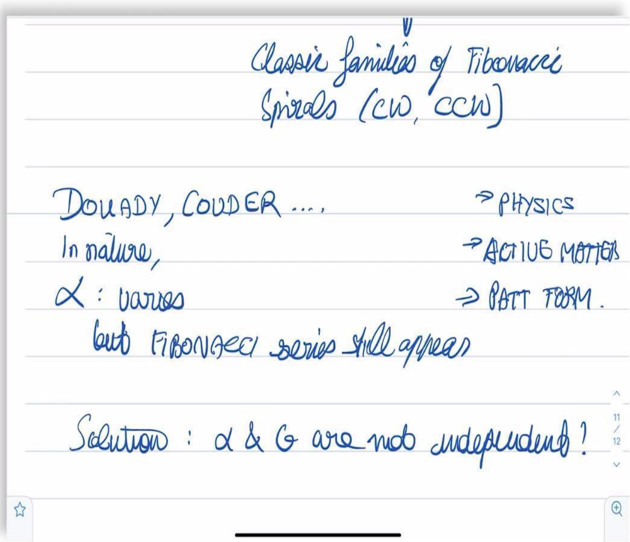

This is the insight of a group of physicists working in active matter and pattern formation. This model does not completely explain everything, because in nature while alpha varies but

Fibonacci series still appear. In other words, the patterns are robust. The solutions suggested by Couder and Douady is that α and G are not independent. And this requires more mathematical analysis, which is beyond the scope of this current study. I am only going to leave you with one quick view of the paper, because it reveals something about what we have been talking.

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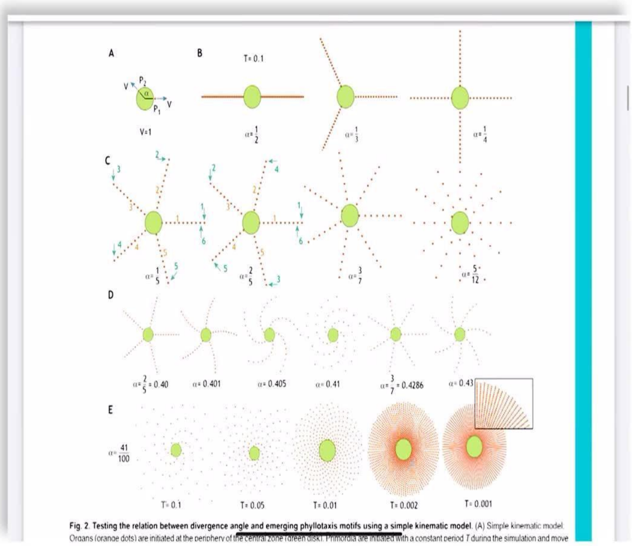

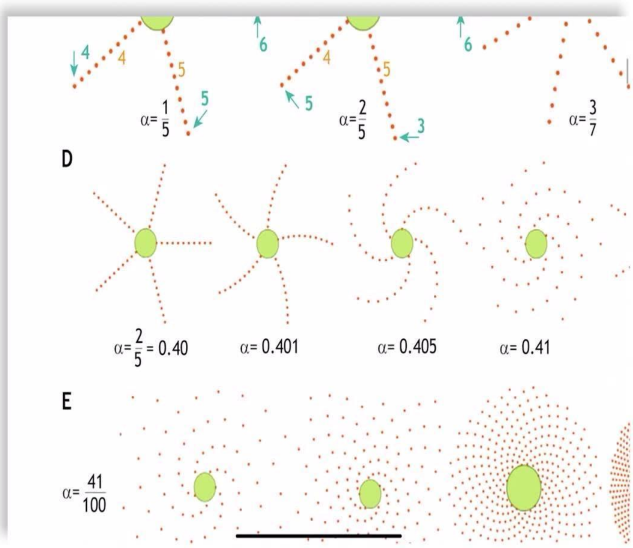

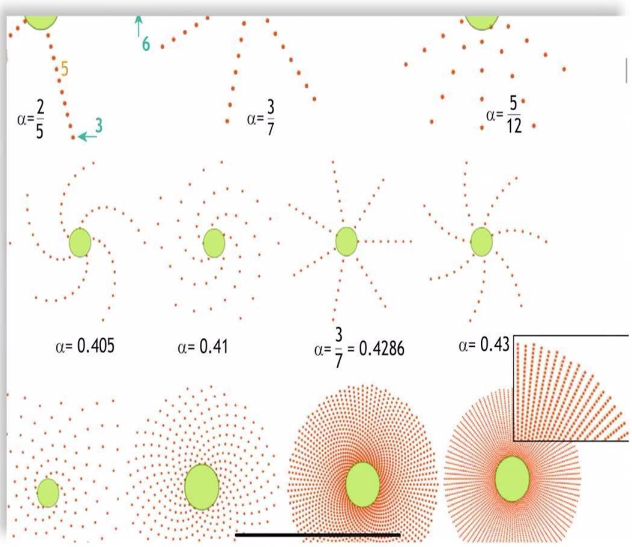

So, these are the spirals they obtained by changing the value of alpha. Now, up to here there is no spiral, just spikes. You start getting smaller and smaller alphas around 0.4 something.

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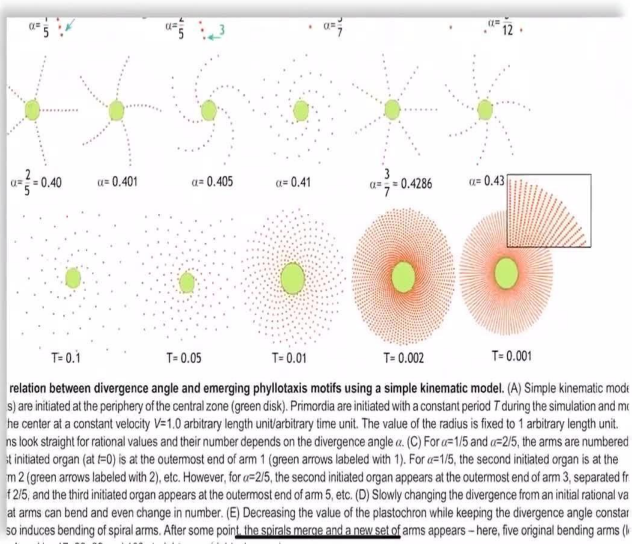

And for a fixed alpha when T is changed, you start seeing structures, that appers to be phallotaxis like.

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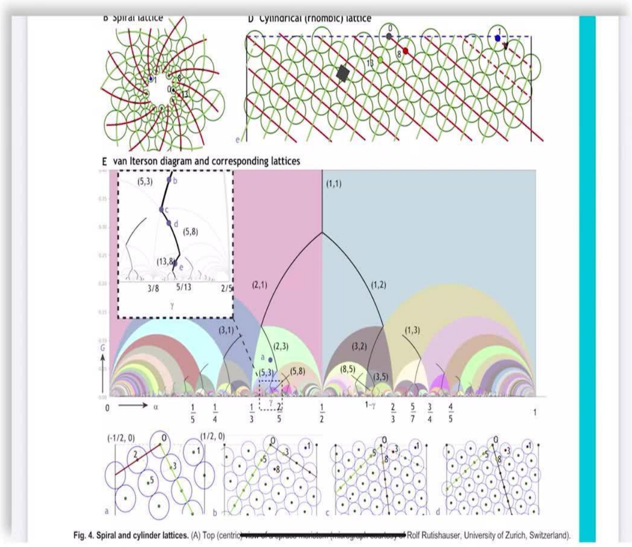

The additional analysis involves this coupled model, which we will not do. That is it for now. And we conclude the other information segment.