Cellular Biophysics
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Biology by Numbers: Bomb Yield Solved

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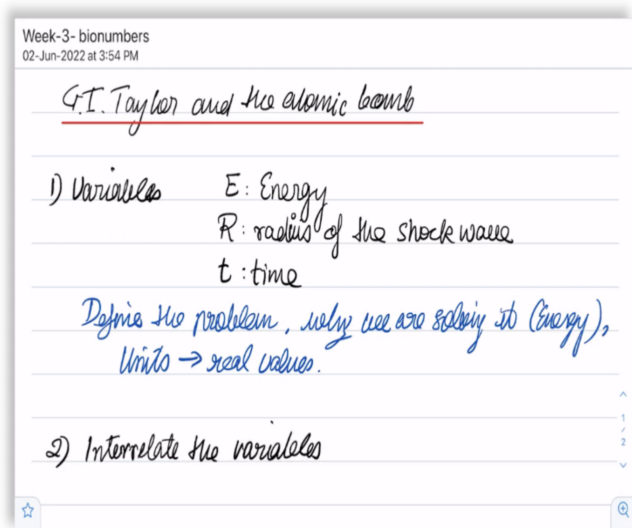

Hi, welcome back. So, we are going to discuss the method by which Geoffrey Taylor did his almost magical derivation. In order to do this, we need to identify the variables. And those are in our case, E, which is an energy, R, which is the radius of the shockwave, t, which is time because it is a time dependent process.

And we are going to see that maybe intuitively we cannot think of anything else we will see if this works. So, we have to innovate define problem, why we are solving it so you want the energy. And eventually, we will want to put units which means we need real values. So, the next step is to say that we define relations or we connect the variables. So, interrelate the variables.

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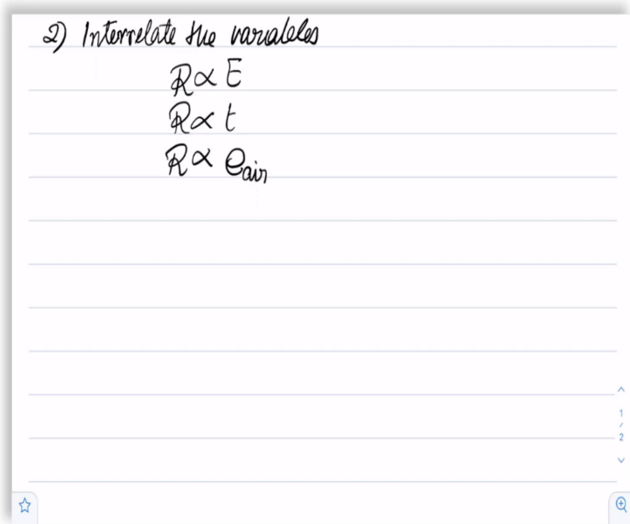

In order to do that, we are going to, note that we intuitively think the release of the fireball is proportional to the energy which increases with time we observed that already and it could have something to do with the medium in which it is happening, in this case it is air, so we will take the density of air.

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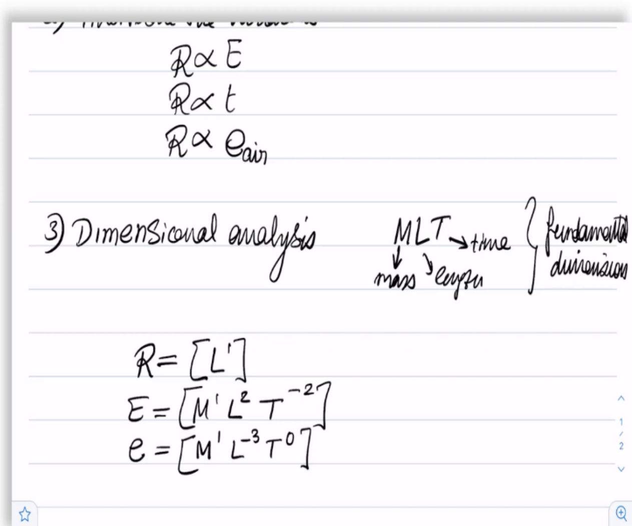

The third thing, third step we take is dimensional analysis, now this is a very important step. Those of you who remember your high school physics remember mass, length, time as the fundamental dimensions. R, therefore, in terms of dimensions is given as L to the power 1.

Energy is force into displacement, which is mass into acceleration into displacement, which means M1, acceleration is length per unit squared time and distance is length, so, you had L to the power 2 and T to the power minus 2. Density, it is mass per unit volume, so, M1 L minus 3 T0.

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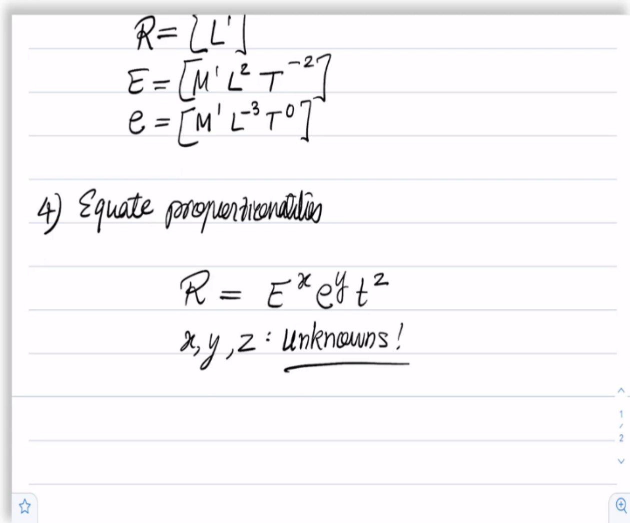

So, the fourth step is to equate proportionalities. And so, we say that R is equal to E to the power x, ρ to the power y, and t to the power z, where x, y, z are all unknowns. You want to find them out.

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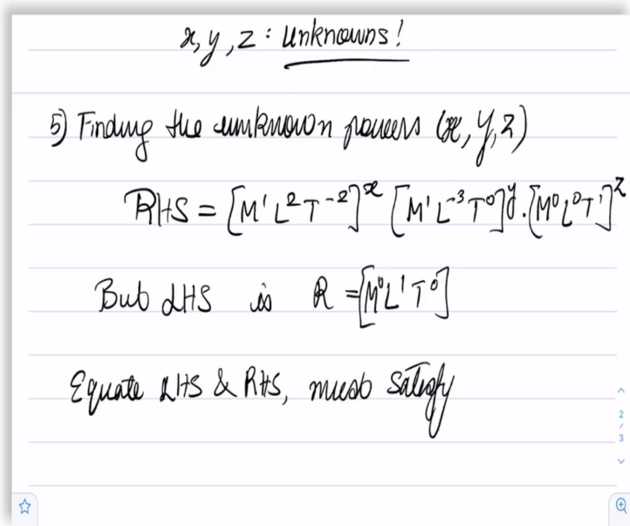

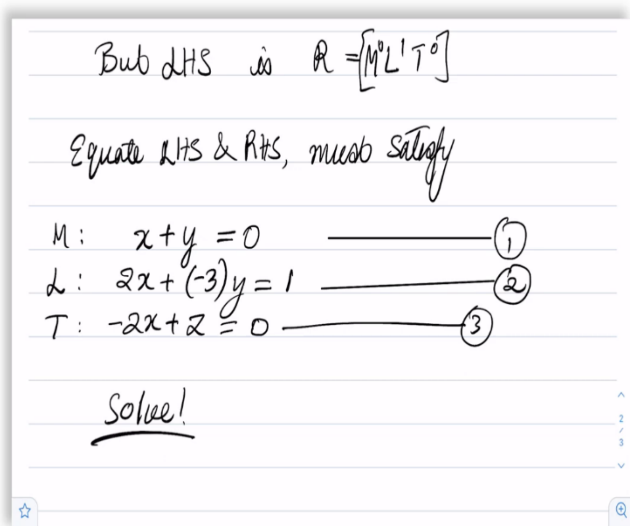

The most exciting step here is to use the dimensions to give us a sense of what these numbers are, these values of the unknowns. So, for the right-hand side, we say which is E which was M L square T minus 2, M L square T minus 2 to the power x, into ρ which is M L minus 3 T0 to the power y into M0 L0 T1 to the power z.

But LHS the left-hand side is R which is equal to L to the power 1 which means M0 T0. This means that when we equate LHS and RHS it must satisfy a series of simultaneous equations. These are x plus y is equal to 0 which is for M, for L it is 2x plus minus 3y is equal to 1, and for time which is minus 2x plus z is equal to 0. These are three simultaneous equations in terms of x, y and z. And now, our job is to solve them.

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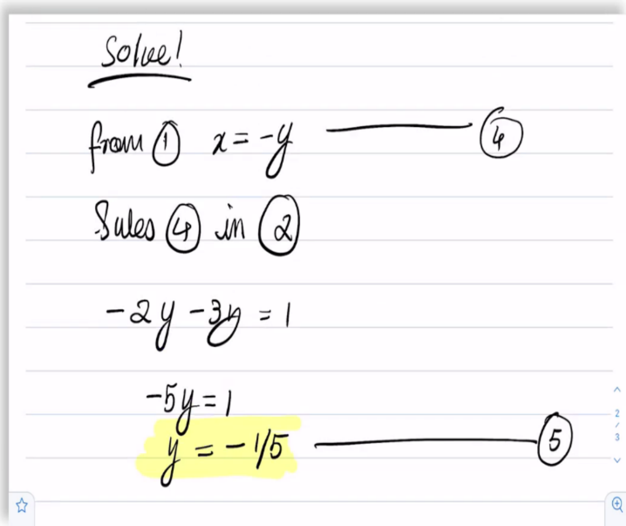

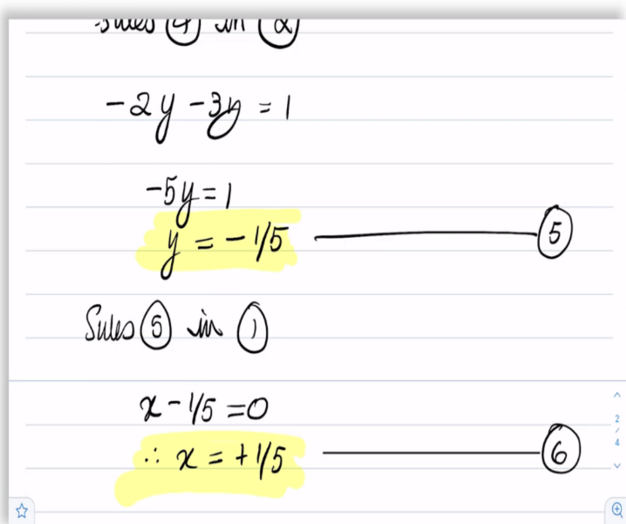

$\therefore x = +1/5$ ——— (6)

Subs (6) in (3)

$-2(1/5) + z = 0$

$\therefore z = +2/5$ ——— (7)

The way we solve them is that we can simplify this equation with x written from 1 x is equal to minus y, substituting in 4 in 2, we get 2 minus 2y minus 3y is equal to 1 minus 2 plus, minus 2 minus 3 is minus 5y is equal to 1. Therefore, y is equal to minus 1 by 5, this is our first solution.

Now, we can substitute this back into 1 and write down x minus 1 by 5 is equal to 0, therefore, x is equal to plus 1 by 5, this is a second solution. And now, we substitute 6, then the last equation, which is going to determine z, 3, we get minus 2x plus z is equal to 0 that is minus 2x is 1 by 5 plus is equal to 0, therefore, z is equal to plus 2 by 5.

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We want to know the Energy yield....

$$\frac{1}{E^{1/5}} = \frac{e^{-1/5} \cdot t^{2/5}}{R}$$

$$E^{1/5} = \frac{R}{e^{-1/5} \cdot t^{2/5}} = \frac{e^{1/5}}{t^{2/5}} \cdot R$$

$$E^{1/5} = \frac{R}{\rho^{-1/5} \cdot t^{2/5}}$$

$$= \frac{\rho^{1/5}}{t^{2/5}} \cdot R$$

$$\left(E^{1/5}\right)^5 = \left(\frac{\rho^{1/5}}{t^{2/5}} \cdot R\right)^5$$

$$E = R^5 \frac{\rho}{t^2}$$

Proportionality constant from data!

Remember these are exponents, which means substituting 5, 6, and 7 in this equation should give us our solution. So, we get to write R is equal to E to the power x which is one-fifth into ρ to the power y which is minus 1 by 5, you see a pattern here, into t to the power 2 by 5, this means that we can now get R in terms of E ρ t, rearranging because remember, we want to know the energy yield, the secret of the United States at the time in 1945, before our independence.

We get 1 by E to the power 1 by 5 is equal to ρ minus 1 by 5, t 2 by 5 upon R, inverting, we get E 1 by 5 is equal to R upon e to the power minus 1 by 5 into t 2 by 5, which means, we can write it as ρ to the power 1 by 5 upon t to the power 2 by 5 into R. We take power 5 on both sides, we end up as E, to the power 5 is equal to ρ to the power 1 by 5 upon t to the 2 by 5 into R whole to the power of 5.

And it results to E, 5 and 5 cancels here on the left-hand side, to the power, R to the 5 ρ upon t square. Now, this was missing a proportionality constant and this is the investigators estimated from the data. This was how the deep mystery was solved. Is not that exciting?