Cellular Biophysics
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Tutorial 06
Fluid Mechanics and Reynolds Number

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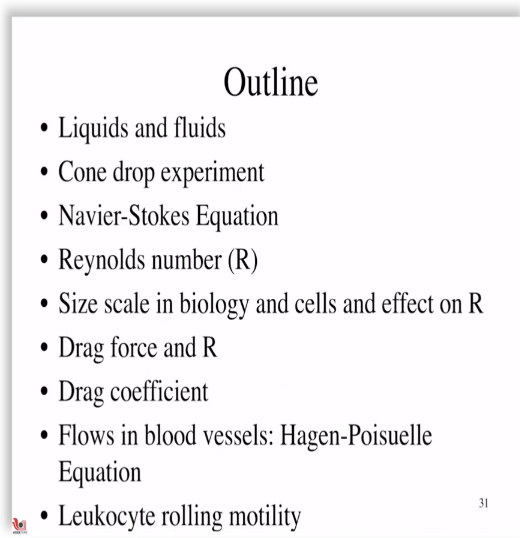

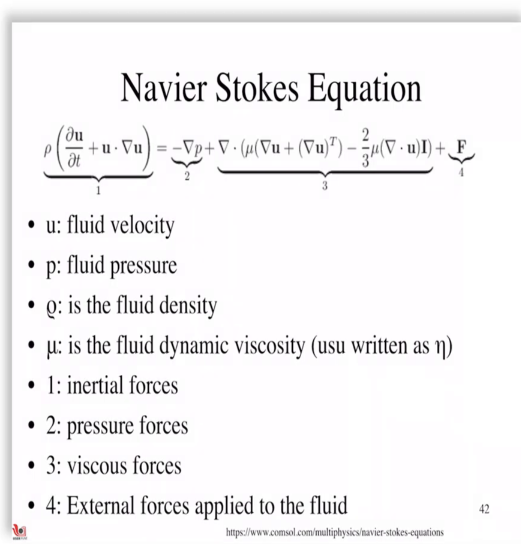

Hi, so we are going to continue on the tutorials with a discussion of the Navier-Stokes equation where we will introduce this fundamental equation to fluid mechanics which has four basic terms inertial forces, pressure forces, viscous forces and external forces. The usual symbols applied are μ for fluid flow dynamic viscosity this is an engineering way of writing it usually we write it as η and biophysics physics q is the fluid density p is the fluid pressure use the fluid velocity.

As it so happens from our discussions of Reynolds numbers, inertial forces disappear become zero at very low Reynolds numbers which means viscous forces dominate and external forces and viscous forces combined with pressure forces to define everything that we can observe.

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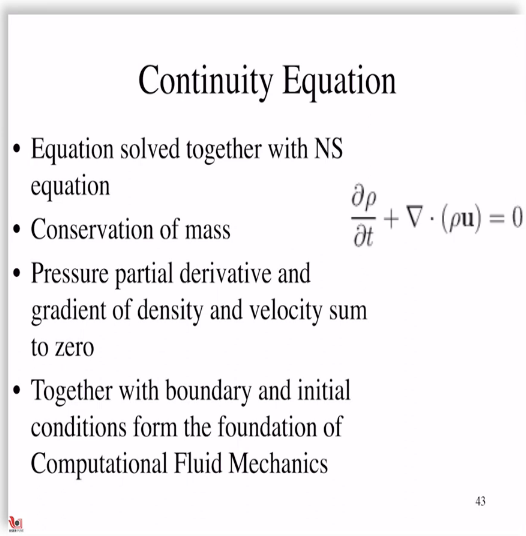

This law is derived from the conservation of momentum of fluid parcels and combined with the continuity equation that is

$$\frac{\partial \rho}{\partial t}\nabla \cdot (\rho U) = 0$$

serves to help us solve any fluid equation in a bulk sense. The partial derivative and the gradient density at velocity sum to 0 this is the meaning of this continuity equation and it is a conservation of mass indicator of conservation mass.

So, together with the boundary and initial conditions, they form the foundation of computational fluid mechanics, so CFD and you will see sometimes calculations of jet engines and flows around cheetah or flow dynamics around new cars all these are computational fluid mechanics outputs and very important for engineering and maybe not as widely explored in biology but definitely the basis of a lot.

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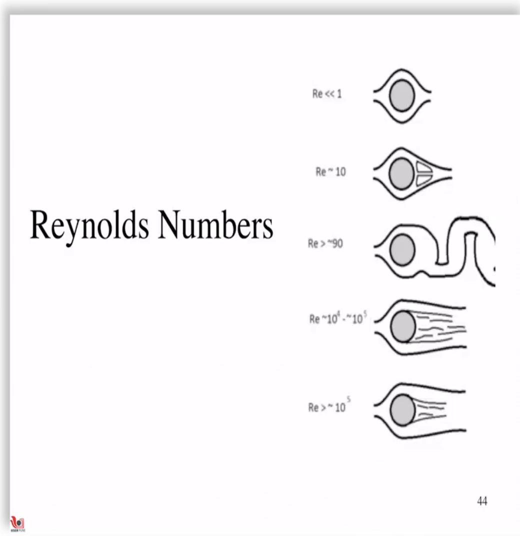

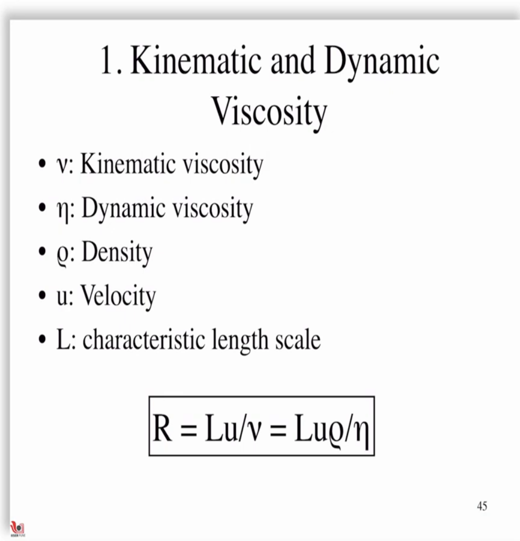

Now, for our purpose we have discussed earlier also that Reynolds number differences make a difference to the stream lines that we can observe, and to get at some answer with this we had defined kinematic and dynamic viscosities ν being the symbol for kinetic and η for dynamic, ρ density u velocity L characteristic length scale. So, Lu/ν or $Lu\rho/\eta$ because ν is η by ρ.

Now, the question that arises is what is meant by characteristic length scale? So, for a flow in a cylindrical tube along a certain length with a certain pressure drop between the start and end of the tube, l is the length of the tube for a bacterium swimming in a bath l is the size of the bacterium and we have discussed this before but the typical size scale of *E. coli* bacteria is two microns and we say microscopic organisms are of bacterial type prokaryotes, typically in the range of a few micrometers that is l, u is the velocity so how fast it moves at a microscopic scale the velocity is around tens of microns per second. Rho which is the density of water is in SI units 1000 kg per meter cube and η as we said is 1/1000 Pascal second.

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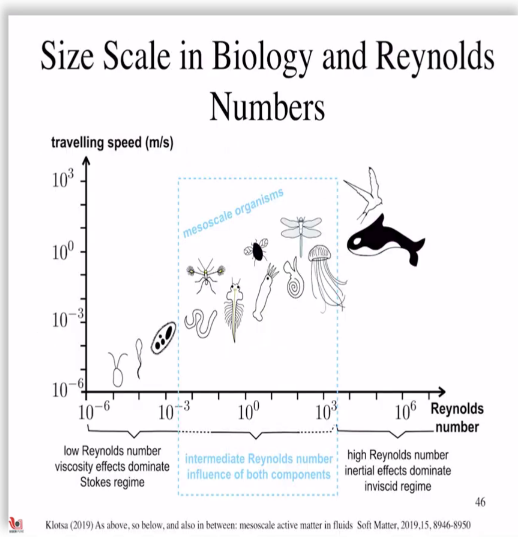

So plugging these in, we end up with low Reynolds numbers for bacteria but what about other organisms, what about large organisms not single celled but multicellular metazoans. So, as it turns out that the length scale as we said earlier or size and the traveling speeds scale evidently we expect that larger animals are able to swim faster it is typically true, for example a killer whale travels at tens if not 20 meters per second, whereas a bacterium does only 20 microns per second.

Now you can of course divide out the length scales and try to ask what the relative body length covered time is but if you are looking at absolute velocities then indeed the bigger organisms move faster birds fly faster than insects, shrimps swim faster than sperm cells and so on and so forth.

Now in this graph what you are looking at is that low Reynolds numbers and viscosity dominate at small length scales single celled at some intermediate size and velocity scale Reynolds numbers are influenced in the range of 10 to the power 0 there is 1 to 10 to 100 to 1000 both the viscous and inertial components dominate and at high Reynolds numbers that is 10^6 and so on only inertial numbers dominate viscous drag is not that important we also refer to it as inviscid regime.

This is a figure taken from klotsa and company in 2019 a review in soft matter. So, if you focus from a quantitative cell biology perspective for a cellular biophysics perspective, then of course we are looking at 10^{-3}-10^{-6} Reynolds numbers.

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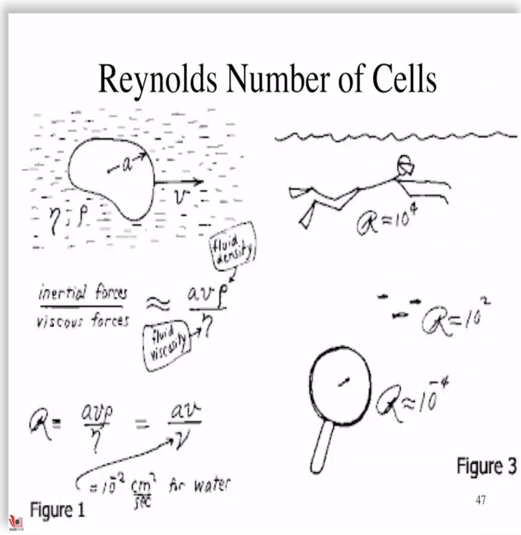

And this was what was illustrated in our previous discussion if you recall of the ratio of inertial to viscous forces being expressed as Reynolds number and with a known velocity, for example bacterial versus human swimming the Reynolds number is 10^{-4} for bacteria and 10^4, plus 4 for humans. So, humans of meter scale per 10,000 value of Reynolds number and for bacteria 1/10,000 you know you could say eat orders of magnitude difference.

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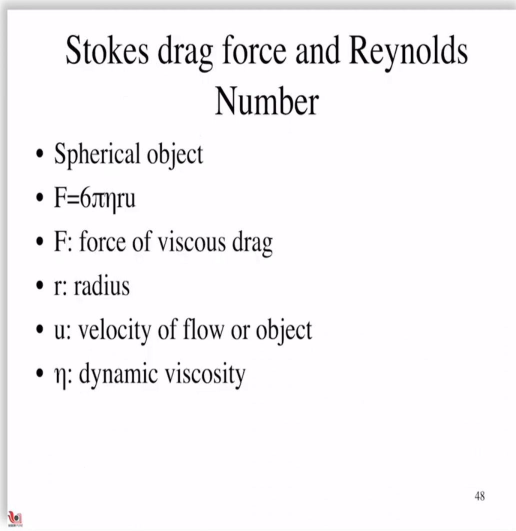

Just to remind you that the stokes drag force and Reynolds number are related to each other and we had gone over the strokes drag force for a spherical object with left being force of viscous drag are being radius u being velocity and determining dynamic viscosity.

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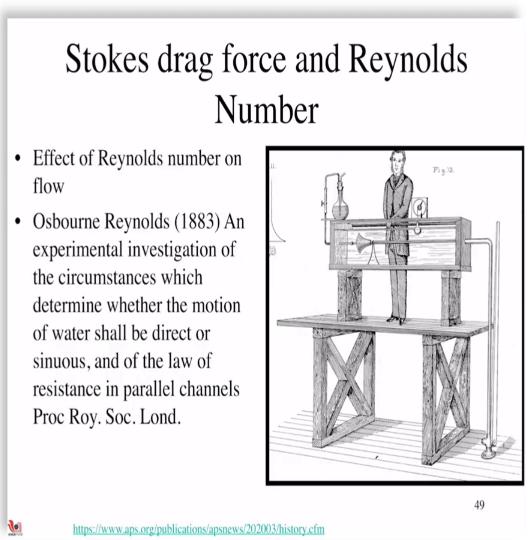

Because in fact Reynold that is Osborne Reynolds in 1883 showed experimentally that the motion of water shall be direct to sinuous in certain circumstances and these certain circumstances were the part where he discovered that high Reynolds numbers or values of are high in the high range lead to smooth flow, whereas low range lead to turbulent flow and that was due to drag. So you could argue that the drag force and Reynolds number are related to each other.

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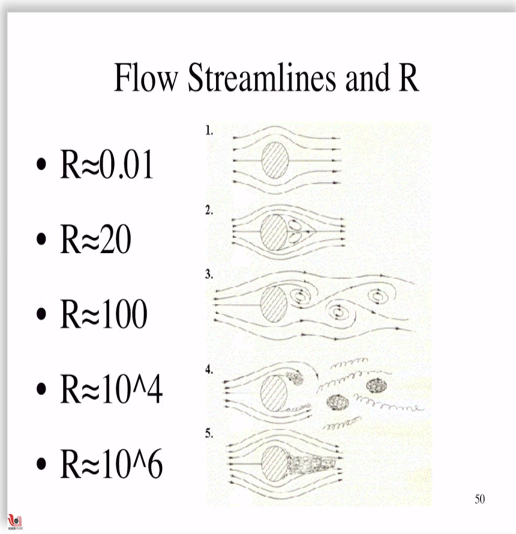

And this is illustrated by this example that we have discussed earlier too of streamlines, sorry so high Reynolds numbers are turbulent, not low Reynolds numbers I misspoke.

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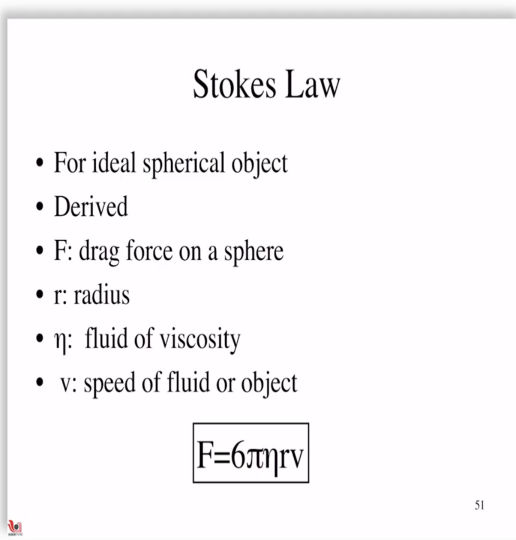

So I hope you have noticed this. So, Stokes law we have gone over this earlier and to remind you for an ideal spherical object can be derived expresses the force drag on sphere of radius r and η with viscosity v in the speed.

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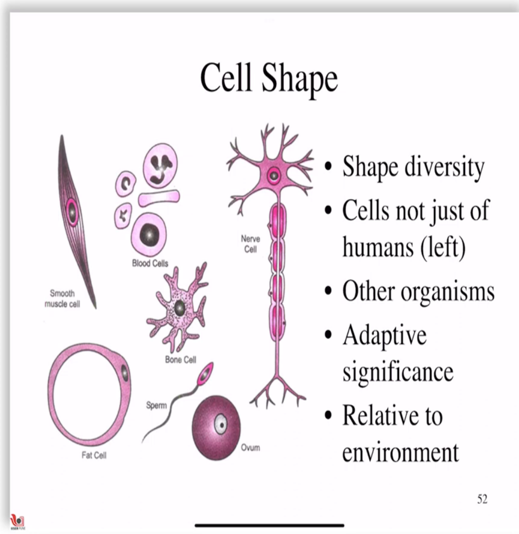

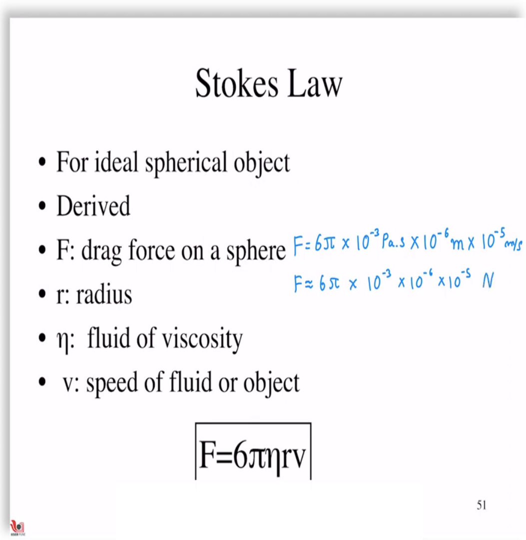

Let us try to see if we can calculate the Strokes drag force for a bacterium. So we say that f is equal to 6πηrv which we want to use SI units is 10^{-3} Pascal second r for a bacterium is micron scale, so 10^{-6} meters v is 30 microns per second we say again tens of microns per second. So, we end up with F is approximately equal to we said

$$F = 6\pi \times 10^{-3} \times 10^{-6}\ 10^{-5} \sim 10^{-13}\ N$$

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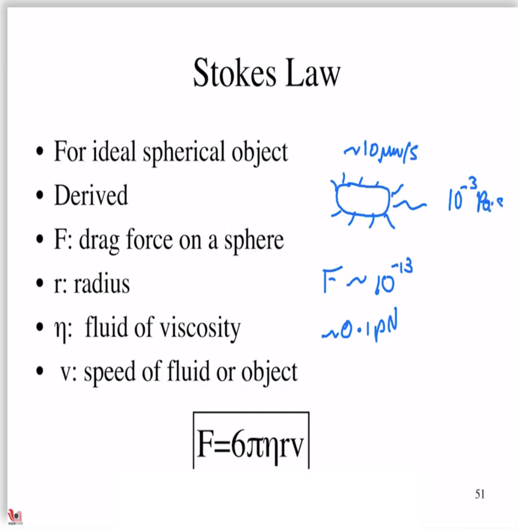

$$F \sim 10^{-13}\ N$$

$$F \sim .1\ pN$$

which is approximately 0.1 pico Newton this is the approximate force that we expect bacterium swimming in a water environment will experience moving at around 10 micrometers per second in water which has viscosity 10^{-3} Pascal second.

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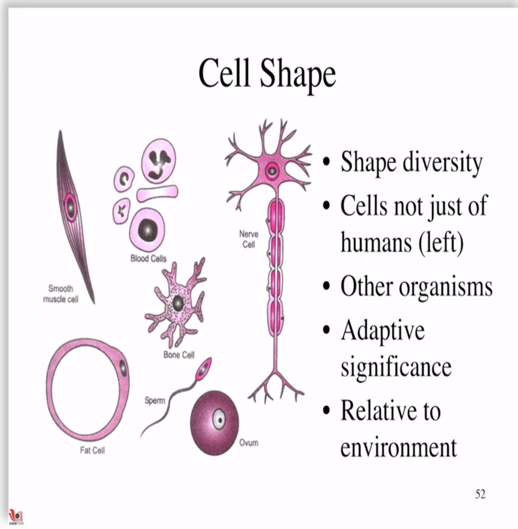

So, interestingly while for spherical objects which is what we did right now for bacteria we can calculate easily a drag force for non-spherical objects things get trickier and it is important to bear in mind that cell shape is quite diverse, in fact if you look only at human cell shape diversity over here in this image you will see a dramatic difference between smooth muscle cells, blood cells, bone cells, sperm cells, fat cells, ova and nerve cells. So the size difference so the longest axon of a nerve cell the spinal cord neuron can be a whole meter long smooth muscle cells 30 microns, 40 microns sprinkler shaped, blood cells can be in tens of micrometers. So, all this indicates that there may be a reason to think about other than shapes other than spheres alone.

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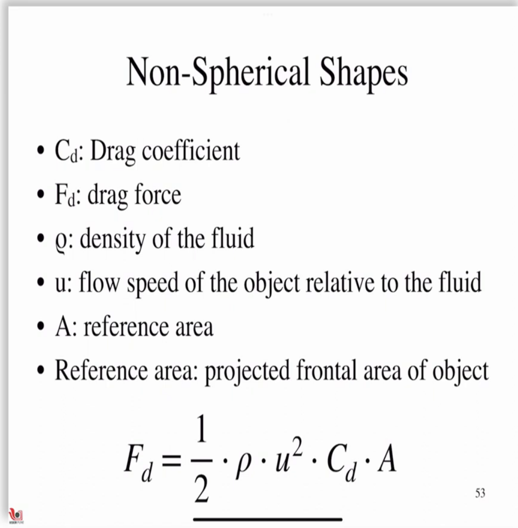

So for non-spherical shapes we need to consider something called the drag coefficient CD and the drag force calculation then becomes complex we have half into ρ into u square cd times a where a is the reference area, so v and u are used interchangeably I have just connected it.

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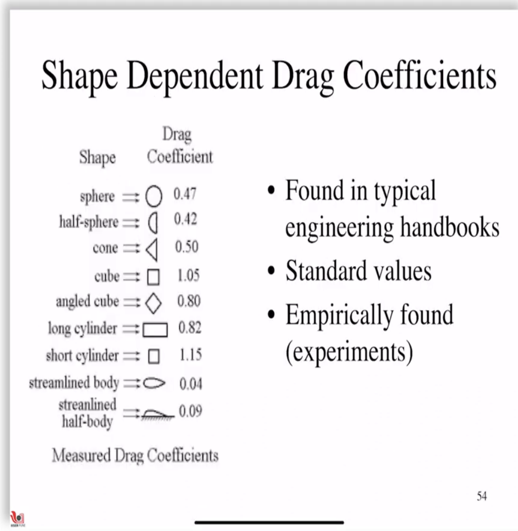

The shape-dependent drag questions are in terms of numbers that are dimensionless and you should always check that the right-hand side gives you force units and dimensions these values change depending on the cross-sectional area, where you find these numbers typically in engineering handbook standard values they are basically found namely by doing experiments.