

**Cellular Biophysics**  
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**Reynolds Number**

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Hi. So, we are going to talk today about Reynolds numbers and centrifugation in a series of lectures, know something that all of you are familiar with are Newton's laws and Newton's laws for motion of solids. As you can see, if I take this pencil here and throw it on my table, it

continues to, you could say that is because it is rolling. So, if I throw it like this, it moves a little bit, I release it, it moves a bit.

And depending on the geometry and orientation, I will get different kinds of momentum. Now, the funny part is that for cellular life, things do not exactly work like that. And we will see a little bit why, we will understand the physics of it. And will also try to see how that affects cell behavior, basically, physiology.

And, to me, this is very important, because biophysics as such is not interesting and less, it in the sense of quantitative biology teaches are something new about the big questions of biology, why is something how it is? How did it evolve to be this? So, let us get to it.

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Previously, we had gone over a series of topics with regard to fluids, fluid flows, concepts of viscosity, Newton's law of viscoelasticity, molecular nature of water hydrophobic effect. And today, we are talk about Reynolds number.

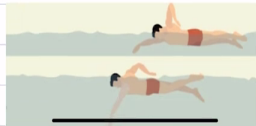
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Reynold's number and cellular life

- 1) Motion in water at macroscopic scale
- 2) Stokes drag force
- 3) Effect of Reynolds number on flows
- 4) Life at low R numbers - Ed Purcell's Essay

SUMMARY

1) Motion in water at a macroscopic scale

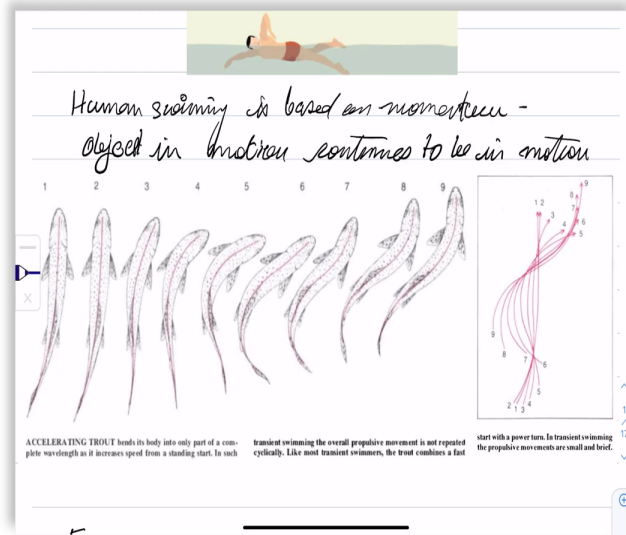


1) Motion in water at a macroscopic scale



Human swimming is based on momentum -  
object in motion continues to be in motion





So, what happens when there is motion in water at a microscopic scale? What is the Stokes drag force? What is the effect of Reynolds number on flows? And how about life at low Reynolds numbers. So, those of you who have been fortunate to swim in the ocean or in the river or in a lake or in a well, you know that when you perform a stroke, like this young man is doing, you reach ahead, pull.

And in the meantime, as your second arm is coming down again, the momentum of motion continues you for a certain amount of distance. So, in that sense, humans swimming is based on momentum plus propulsion, an object in motion continues to be in motion with his third law. This is also true fish but a little more complex.

As you can see this fish as a trout has a slender body and it turns one way, then turns the other way. And if you plot the long axis of the body, in a plot as in on the right hand side, then you will see that there is increasing speed, and acceleration, deceleration and a cyclical transition. And at the end, it moves ahead. And it is beautiful. I always enjoy watching fish swim, because I think we are very elegant in it, something fascinating about how things move in fluids.

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ACCELERATING TROUT bends its body into only part of a complete wavelength as it increases speed from a standing start. In such transient swimming the overall propulsive movement is not repeated cyclically. Like most transient swimmers, the trout combines a fast start with a power turn. In transient swimming the propulsive movements are small and brief.

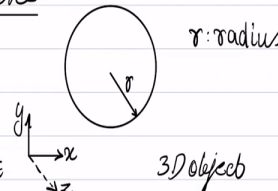
$F = ma$

Momentum - a body in motion continues to be in motion  
Opposing force: friction, drag

2) Stokes drag force

Simplest object for calculating drag force: SPHERE

(i) Object in motion (ii) Fluid in motion



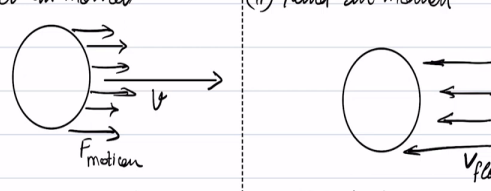
The diagram shows a circle representing a sphere with a radius vector labeled 'r'. To its left is a 3D Cartesian coordinate system with x, y, and z axes. The text '3D object' is written to the right of the sphere.

2) Stokes drag force

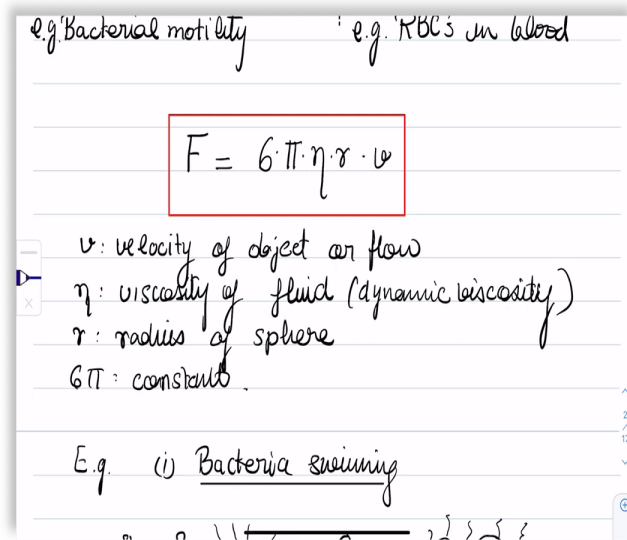
Simplest object for calculating drag force: SPHERE

(i) Object in motion (ii) Fluid in motion

e.g. Bacterial motility e.g. RBC's in blood



The diagram illustrates two scenarios. On the left, a sphere is shown with several horizontal arrows pointing to the right, labeled 'F\_motion'. On the right, a sphere is shown with several horizontal arrows pointing to the left, labeled 'v\_flow'. A vertical dashed line separates the two scenarios. Above the spheres, a circle with radius 'r' and a 3D coordinate system are shown, identical to the first slide.



As you remember, F is equal to ma motion, which is basically mass x dv/dt change in velocity needs to force but is also an opposing force of friction or drag. And that is why the things come to a stop eventually. So, in fluids, we are most familiar with Stokes drag force and many of you may have come across this already.

For simplest objects for calculating drag force we assume a sphere, why do we assume a sphere? This is a good question, and I want you to contemplate it. The short answer is simple geometry. So, it has one dimension radius. That becomes what we call the length scale, the characteristic length that defines the object and its motion.

In the case of an actively propelled motion is driven by some force that drives motility. There is an opposite situation where the object could be static, but there is flow coming at it, the fluid is in motion. And then it experiences drag again, the first case motion of the object itself by active motility like bacteria or Leishmania or any motile self-propelled cell and the passive motion meaning to say the object is suspended in a motion of a fluid.

In a fluid which is an itself undergoing motion is like red blood cells in blood, where they then follow the flow. So, the Stokes drag equation is this

$$F=6\pi\eta rv$$


what do these symbols mean F is the force, that is the drag force, v is the velocity of the object or the flow,  $\eta$  is the viscosity of the fluid dynamic viscosity remember we talked about kinematic

and dynamic this dynamic viscosity,  $r$  is the radius of sphere this  $\pi$  is a proportionality constant you can actually derive this we are not going to derive it today. Let us plug in some numbers since we mentioned bacteria swimming, what are the typical drag forces experienced?

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$\eta$ : viscosity of fluid (dynamic viscosity)  
 $r$ : radius of sphere  
 $6\pi$ : constant

E.g. (i) Bacteria swimming

Shape? 

Diversity  
 ↓  
 BIOPHYSICS  
 ↓  
Simplify (assumption)

Simplify (assumption)

↓  
 SPHERE 
 ↓  
 Model organism  
 E. coli

$v \approx 30 \mu\text{m/s}$   
 diameter  $\sim 1 \mu\text{m}$   
 $r = 0.5 \mu\text{m}$   
 $\eta = 10^{-3} \text{Pa}\cdot\text{s} = 10^{-3} \frac{\text{pN}\cdot\text{s}}{\mu\text{m}^2}$

$1 \text{Pa}\cdot\text{s} = \frac{1 \text{N}\cdot\text{s}}{\text{m}^2}$
$= \frac{10^{-18} \text{pN}\cdot\text{s}}{(10^9)^2 \mu\text{m}^2}$

$\therefore F_{\text{drag}} = 6 \cdot \pi \cdot \frac{10^{-3} \text{pN}\cdot\text{s}}{\mu\text{m}^2} \cdot 0.5 \mu\text{m} \cdot \frac{30 \mu\text{m}}{\text{s}} = 1 \frac{\text{pN}\cdot\text{s}}{\mu\text{m}^2}$

Model organism  
*E. coli*

$$v \approx 30 \mu\text{m/s}$$

$$\text{diameter} \sim 1 \mu\text{m}$$

$$r = 0.5 \mu\text{m}$$

$$\eta = 10^{-3} \text{Pa}\cdot\text{s} = 10^{-3} \frac{\text{pN}\cdot\text{s}}{\mu\text{m}^2}$$

$$\therefore F_{\text{drag}} = 6 \cdot \pi \cdot 10^{-3} \frac{\text{pN}\cdot\text{s}}{\mu\text{m}^2} \cdot 0.5 \mu\text{m} \cdot \frac{30 \mu\text{m}}{\text{s}} = 1 \frac{\text{pN}\cdot\text{s}}{\mu\text{m}^2}$$

$$= 0.287 \text{ pN}$$

(ii) Red Blood Cell in bloodstream

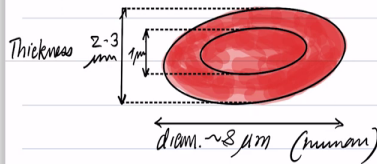
Blood is an aqueous suspension of various constituents.

$$= 0.287 \text{ pN}$$

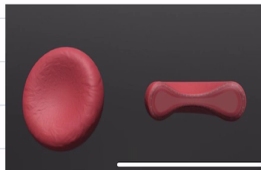
(ii) Red Blood Cell in bloodstream

Blood is an aqueous suspension of various constituents.

RBC's most numerous cell types.

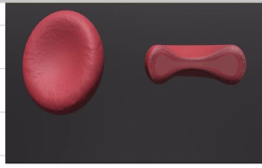


Capillaries form the smallest, Terminal blood vessels.



Capillary diameter  $\sim 2-5 \mu\text{m}$   
Flow velocity  $\sim 0.8 \text{ mm/s}$



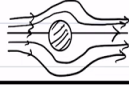


Capillary diameter  
 $\sim 2-5 \mu\text{m}$   
Flow velocity  $\sim 0.8 \text{ mm/s}$

$$F_{\text{drag}} = 6\pi \cdot 10^{-3} \frac{\text{pN} \cdot \text{s}}{\mu\text{m}^2} \cdot 2.5 \mu\text{m} \cdot \frac{800 \mu\text{m}}{\text{s}}$$
$$= 37.7 \text{ pN}$$

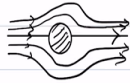
### 5) Effects of Reynolds Numbers on flows

$$R \approx 0.01$$



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$R \approx 0.01$



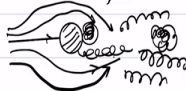
$R \approx 20$



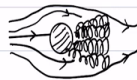
$R \approx 10^2$



$R \approx 10^4$



$R \approx 10^6$



$v \sim 1-10 \text{ cm/s} = 10^{-2} \text{ m/s}$

$\eta = 10^{-3} \text{ Pa}\cdot\text{s}$

$L \sim 30-50 \text{ cm}$   
 $\sim 10^{-1} \text{ m}$

$L$ : length scale

$u$ : velocity

$\rho$ : density of fluid

$\eta$ : viscosity

$R = \frac{Lu}{\eta} = \frac{L\rho u}{\eta}$

$\nu = \eta / \rho$

kinematic viscosity

$\rho_{\text{H}_2\text{O}} \approx 10^3 \frac{\text{kg}}{\text{m}^3}$

$R_{\text{fish}} = 0.1 \text{ m} \times 10^{-2} \text{ m/s} \times 10^3 \text{ kg/m}^3 = 10^3$

Kinematic viscosity  $\nu_{H_2O} \approx 10^{-6} \frac{kg}{m^3}$  fluid 'U'  
 $\eta$ : viscosity

$$R_{pin} = \frac{0.1 m \times 10^{-2} m/s \times 10^3 kg/m^3}{10^{-3} Pa \cdot s} = 10^3$$

NOTE: Units of Reynolds number?

$$R = \frac{L u e}{\eta} \Rightarrow \frac{m \times m/s \times kg/m^3}{(kg \cdot m/s^2) \times s/m^2}$$

$$\text{UNITS: } \frac{m^2 \times m^2 \times kg \times s^2}{kg \cdot m \cdot s^2 \times s \cdot m^2}$$

NOTE: Units of Reynolds number?

$$R = \frac{L u \rho}{\eta} \Rightarrow \frac{m \times m/s \times kg/m^3}{(kg \cdot m/s^2) \times s/m^2}$$

UNITS:  $\frac{m^2 \times m^2 \times kg \times s^2}{m^3 \times m \times s \times s \times kg}$

UNITS CANCEL OUT

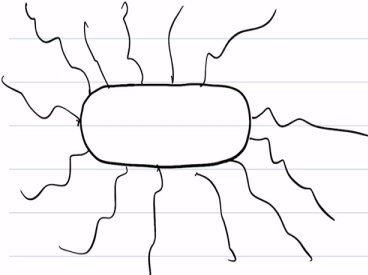
Dimensionless  $[M^0 L^0 T^0]$

Unitless

Contrast to cellular Movement

Unitless

Contrast to cellular Movement



Model bacterium  
Gut bacteria

Well studied  
Genome size  
 $4.6 \times 10^6$  bps

E. coli

Hydrodynamics: Cell size

So, for bacteria swimming, they could take in take many shapes as you know they can be rod shaped, they can be spherical, they can be ellipsoid, they can be curved, they can be spiral. So, this diversity, we have to deal with it somehow and in biophysics we always try to take the path of the simplest situation, not because we are not aware of other situations being in existence, but because otherwise you will never be able to draw general laws, and that is what you want to do.

So, one of the simplifying assumptions is a spherical assumption. The other almost sort of you could argue biological simplifying assumption is take a model organism. And one of the things we do for bacterial motility is to take *Escherichia coli* as a model organism. You remember in chemistry, we always talked about hydrogen atoms, what is so special about hydrogen?

Well, many things are specially with hydrogen, but most importantly, it is simple, if we at least understand the simple things and we can go and try and understand the more complex things. So, velocity of bacteria swimming like *E. coli* is 30 micrometers per second between 10 to 30. So, let us take 30 as an upper limit, the diameter is approximately 1 micron, so the radius is 0.5 microns, this is assuming a sphere and the viscosity is  $10^3$  1000 Pascal second which in the units comparable to the micron is pico Newton second per micrometer square.

How did we get this conversion is written over here 1 Pascal second is 1 Newton per meter square into second. That is, when you go to pico Newton, pico remember is  $10^{-12}$ , meter is minus 6, minus 6 squared is minus 12. So, minus 4 minus 4 cancels out, 1 Pascal second is equal to 1 pico Newton second per micro meter squared that is how I got this answer.

So, coming back to the equation itself  $F_{\text{drag}}$  the cost of drag is equal to  $6\pi\eta rv$ . So,  $r$  is  $10^{-3}$  pico Newton second micrometers square, that is  $\eta$  is  $10^{-3}$  pico Newton second per micrometer square,  $r$  is 0.5 micron,  $v$  is 30 micrometer per second, the answer is point 0.287 pico Newton, that is the force experienced by yourself. Now, if you do not know anything else, it is unnerving to think what does this mean?

What is 0.287 pico Newton mean or let us say rounding 0.3 pico Newton we will see in a minute, because everything will depend on the motile force, which to which against which this thing is acting, the drag force in active motility, in the case of red blood cells, as you recall, they are the most numerous cells that is why our blood is red, human blood. By the way, there are animals with blue blood and plants do not have blood.

So, blood is not ubiquitous, but at least in vertebrates, we see, in animals, large animals with circulatory systems we see it, small insects have lymphoid systems and diffusion is used to transfer oxygen. Blood is an increased suspension, it has many constituents. RBCs are a major cellular constituent. In humans, red blood cells, mature red blood cells do not have nuclei they are enucleated when they are born, however, they have nuclei. So, the chromosomal DNA.

Capillaries that form the smallest terminal blood vessels have a diameter of around 2 to 5 micrometers. Whereas the RBCs have a diameter of approximately 8 microns 7 point something you will find in the textbooks. The flow velocity is 0.8 millimeters per second.

So, knowing all this 0.8 millimeters per second is 800 micrometers per second, radius is 2.5 microns. We took an average of the thickness and the diameter and  $6\pi\eta r v$ , so we get 37.7 pico Newton, that is a high amount of drag. And this probably comes from the very high velocity as compared to the bacteria, as well as the larger size, so these two multiplying effects of, think about it like this  $6\pi\eta r v$  tells us that the drag force increases proportionately with the size of the object and the velocity of the flow or the object that is in motion itself.

If you increase the viscosity you also get higher drag, if you decrease the viscosity you reduce the drag. Very often, we may not have a choice about the fluid we are in like a submarine is moving in water in the ocean that does not go in a soup of honey.

So, in some senses, some things we can treat as constants, some things we need to take care of the variables. So, what is the effect of this viscosity on motion and on closed turns out that Osborne Reynolds came up with such a number called Reynolds number, where he showed that these qualitatively different scenarios of a static object sphere enough flow will experience different kinds of streamlines of the flow.

So, you remember laminar flow in your high school textbooks. So, it turns out at very low Reynolds numbers, you will get laminar flow, but anything higher than 1 around 10 to the power 1, 20 over here, you will see little Eddie's, 100 you will see large vortices and  $10^4$  and  $10^6$ , the motion is chaotic of the fluid flow lines. So, what kind of Reynolds numbers can we expect?

So, at a microscopic level fish like labeo rohu fish has an approximate velocity of 1 to 10 centimeters per second, which is  $10^{-2}$  meters per second, viscosity remains  $10^{-3}$  Pascal second. So, substituting the values we of Reynolds number and the equation is given here, which is

$$R = Lu/\nu$$

$\nu$  is nothing but kinematic viscosity, but kinematic viscosity itself can be written in terms of the terms  $\eta/\rho$ . So, when we write it out, we get something that then looks like this  $L\rho v/\eta$ ,  $\rho$  being the density of the fluid,  $\eta$  being the dynamic viscosity that we have been familiar with. Now, something to bear in mind. If we substitute these numbers we get a value we get  $10^3$  this is for fish Reynolds number for fish labeo 1000. And if you recall, this  $10^3$  somewhere between these turbulent phase but what are the units, what are the units of Reynolds numbers?

So, let us try to calculate this we substitute units for  $L$  which is meter, for velocity which is meters per second all in SI units, density is kg per meter cube,  $\eta$  is viscosity, which is Pascal second which is Newton per meter square x second, which Newton itself is mass x acceleration. So, it is kg meters per second square.

So, all of that put together becomes kg meters per second square in the numerator multiplied by second divided by meter square. All this when we write it all out comes to meter square into meter square into kg into second square upon meter cubed into meter into second into second into kg. And all units cancel out. What does this mean?

This means that Reynolds number is dimensionless and therefore, also unitless. These kinds of numbers dimensionless numbers unitless numbers are quite interesting because they suggest that this one number as we saw here, by just knowing this, irrespective of the physical nature of the object that we are studying, can give us an idea of what its properties might be and this is exciting.

I hope that you realize that the very elementary concepts in fluid mechanics apply very nicely to biology and it seems right away because compared to our fish, when we take a *E. coli* model bacterium, gut bacteria were studied genome size  $4.6 \times 10^6$  base pairs, that is to say is a 4 million base pairs.

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Well studied  
Genome size  
 $4.6 \times 10^6$  bps

E. coli

Hydrodynamics: Cell size  
 $L \sim 2 \mu\text{m}$   
 $w \sim 1 \mu\text{m}$   
Avg diameter  $\sim 1.5 \mu\text{m}$   
 $r \sim 0.75 \mu\text{m}$

Swimming speed  $\sim 10-30 \mu\text{m/s}$

$R = \frac{L u \rho}{\eta}$

$R = \frac{L u \rho}{\eta}$

Using approximations  $L \approx 1 \mu\text{m} = 10^{-6} \text{m}$   
 $u \approx 30 \mu\text{m/s} = 3 \times 10^{-5} \frac{\text{m}}{\text{s}}$

Substituting  
 $R = \frac{10^{-6} \times 3 \times 10^{-5} \times 10^3}{10^{-3}} = 3 \times 10^{-5} \approx 10^{-5}$

Very LOW REYNOLDS NUMBER!

4) Life at low Reynolds Numbers

It is hydrodynamics are dependent on the cell size, with a length of 2 micron with a 1 micron average diameter 1.5 micron radius of 0.75 micron, and swimming speed of 10 to 30 micrometers per second. So, let us substitute the numbers using approximations L is approximately 1 micron  $10^{-6}$  meters 30 microns per second can be approximated at  $3 \times 10^{-5}$  meters per second.



Velocity can be taken as 30 microns per second, which is upper limit, we can make it even smaller, substituting the values you get  $10^{-5}$ . Now please note, this is minus 5, for fish it was plus 3, it is 1000. And this is hundred thousand one by hundred thousand.

This is a very low number, and this is therefore defined as low Reynolds number. So, when we return, I am going to talk about what happens to living systems at a cellular scale. Remember the cellular biophysics when the Reynolds numbers are this low.

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$10^{-5}$

Very LOW REYNOLDS NUMBER!

4) Life at low Reynolds Numbers

Ref. article from a lecture by Ed. Purcell (1976)  
CLASSIC PAPER

(i) Newton's law  
Objects in motion continue to be in motion

At low Reynolds numbers this does not

Objects in motion continue to be in motion

At low Reynolds numbers this does not hold true - ~~of~~ inertia (massif)

$$Re = \frac{\rho U L}{\eta} = \frac{\rho U L}{\eta} \approx \frac{\text{Inertial Forces}}{\text{Viscous Forces}}$$

Osbourne Reynolds

low  $Re$  (or  $Re$ ) indicates a dominance of viscous over inertial forces. Things move if the action to move continues.

$$\eta^2/\epsilon = \frac{\text{Pa} \cdot \text{s}^2}{\text{kg}/\text{m}^3} = \frac{\text{N}^2 \cdot \text{s}^2 \cdot \text{m}^3}{\text{m}^4 \cdot \text{kg}}$$

$$= \frac{\text{kg}^2 \cdot \text{m}^2}{\text{s}^4 \cdot \text{m}^4 \cdot \text{kg}} \cdot \text{m}^3$$

$$= \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = \text{N} \rightarrow \text{Force}$$

(ii) Let us substitute some numbers to know the magnitude of the viscous force  $\sim \eta^2/\epsilon$

SI  $\sim \frac{(10^{-3} \text{ Pa} \cdot \text{s})^2}{10^{+3} \text{ kg}/\text{m}^3} = 10^{-6-3} \text{ N} = 10^{-9} \text{ N} = 10^{-4} \text{ dyn}$

CGS  $\epsilon = 1 \text{ gm}/\text{cm}^3$

So, he has spoken about Reynolds numbers, and how we can obtain very low Reynolds numbers in the case of cells. It took the work of Ed Purcell in 1976. In a classic paper, which is in fact, the text of speech he gave to highlight the special properties of such low Reynolds numbers. So, we are familiar and I mentioned it earlier to, that by Newton's law of motion.

Those objects that are in motion will continue to be in motion this is inertia this is your standard experience also in our scale of the world as human beings. That is to say if you are swimming you make 1 stroke and you hold on you wait for a bit you will continue gliding in water. I am not all of us are in water all the time we are not fish.

With fish we see this, but with single cells at low Reynolds numbers, this does not hold true and this is the really interesting part because there is almost no inertia or it is insignificant. It is not that it does not exist. And we will see exactly what we mean by that more precisely in a bit, part of this is driven can be seen from the equations themselves.

So, if

$$R = Lu/v = \rho L u^2 / \eta = \text{Inertial forces} / \text{Viscous forces}$$

when  $v$  as the kinematic viscosity then rewriting it becomes  $L u$  times  $\rho$  upon  $\eta$ ,  $\rho$  being density and  $\eta$  being kinematic viscosity, I am sorry dynamic viscosity. This is in terms of the ratio of inertial to viscous forces. And of course, the  $R$  itself is attributable to Osborne Reynolds, he used it for flows to decide whether they are turbulent or not.

But Purcell found that low Reynolds numbers indicate the dominance of viscous however inertial forces. And he then went on to discuss what that means for single celled organisms. For example, he back calculated what happens when you take the ratio of  $\eta^2$  by  $\rho$ .  $\eta$  is in SI units Pascal second which is Pascal squared second square,  $\rho$  is  $\text{kg/m}^3$  that is density.

We write Pascal in terms of  $\text{N/m}^2$  that is  $\text{N}^2/\text{m}^4$ . And we are still left with  $\text{kg}$  in the denominator. So, Newton's comes from force which is mass into acceleration mass is  $\text{kg}$ , acceleration is  $\text{m/s}^2$ . Substituting that we get something that has  $\text{kg}^2 \text{meter}^2/\text{s}^4$ .

We already had  $\text{s}^2$ , we have  $\text{m}^3/\text{kg}$  and we had  $\text{N/m}^2$  which became power 4 and this resolves to  $\text{kg m/s}^2$ . Which is nothing but Newton and this is force and this is thought to be at least in concept to be the quantification in some senses of the viscous force.

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$\text{N} \cdot \text{s}^2$

(ii) Let us substitute some numbers to know the magnitude of the viscous force  $\sim \eta^2/\rho$

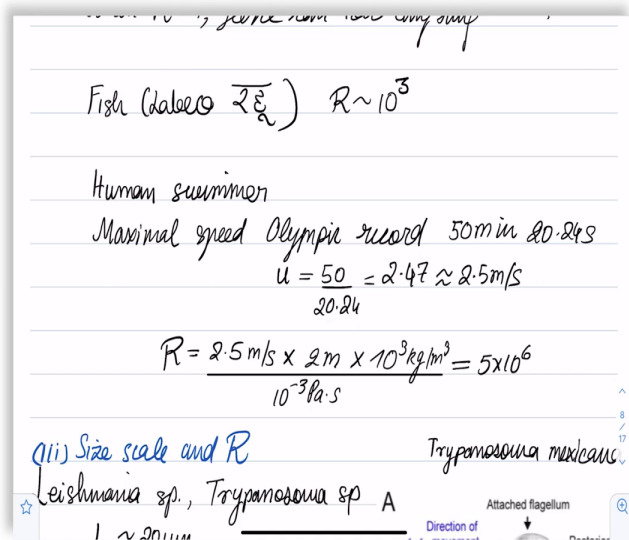
SI  $\sim \frac{(10^{-3} \text{ Pa} \cdot \text{s})^2}{10^{+3} \text{ kg/m}^3} = 10^{-6-3} \text{ N}$   
 $= 10^{-9} \text{ N} = 10^{-4} \text{ dyn}$

CGS  $\rho = 1 \text{ gm/cm}^3$

$\eta = 10^{-2} \frac{\text{dyn} \cdot \text{s}}{\text{cm}^2}$   $10^5 \text{ dyn} = 1 \text{ N}$

$\therefore \eta^2/\rho \sim 10^{-4} \text{ dyn}$

When  $R=1$ , force can flow any thing.



In order to know the magnitude of this viscous force, we can substitute numbers and for water, we know the values of both  $\eta$  and  $\rho$ . So, substituting them we end up with  $10^{-9}$  Newton or 1 nano newton, which also  $10^{-4}$  dyn. So, when  $r$  is equal to 1, the force can, a force can tow almost anything because the opposing force is quite smaller as we said nano newton.

So, for fish, labeo  $R$  is  $10^3$  that is Reynolds number 1000, for humans swimmers, we have taken not a typical human but the best human Olympic swimming record 50 meters freestyle 20.24 seconds works out to  $10^6$ ,  $5 \times 10^6$  that is 5 million is the Reynolds number. So, clearly at a microscopic scale, we are way above  $R$  is equal to 1 and the force that matters most is the inertial force, because it is nano newton though viscous force.

(Refer Slide Time: 21:18)

(iii) Size scale and  $R$

*Leishmania* sp., *Trypanosoma* sp

$L \sim 20 \mu\text{m}$

$v \sim 20 \mu\text{m/s}$

$R = \frac{(20 \mu\text{m}) \times (20 \mu\text{m/s})}{10^{-6}}$

$= 4 \times 10^{-4}$

*Trypanosoma mexicana*

Anterior pole, Attached flagellum, Direction of movement, Posterior pole, Cell body, Leading free flagellar tip, Base of flagellum in flagellar pocket, 20  $\mu\text{m}$

Hoddergott et al (2012)  
PLoS Pathogens

(iv) Coasting distance and time at low  $R$

Some theory

Horizontal damped motion.

(iv) Coasting distance and time at low  $R$

Some theory

Horizontal damped motion

! How far (dist)  
How long (time)  
STOP FLAGELLA

At stop,  $F_{\text{drag}} = -F_{\text{motion}}$

$C_d \cdot v = -m \frac{dv}{dt}$

drag constant, velocity, acceleration

drag constant velocity  $\frac{dv}{dt}$  acceleration

Rearranging

$$\frac{dv}{dt} = -\frac{c_1 \cdot v}{m} \quad \text{--- (1)}$$

By definition  $\frac{de^x}{dx} = e^x$ ,

So,

$$v(t) = v_0 e^{-(c_1 \cdot t/m)} \quad \text{--- (2)}$$

At  $t=0$ ,  $v(t=0) = v_0$  When motion just stopped!

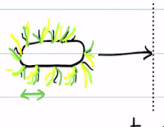
$\frac{dx}{dt}$

So,

$$v(t) = v_0 e^{-(c_1 \cdot t/m)} \quad \text{--- (2)}$$

At  $t=0$ ,  $v(t=0) = v_0$  When motion just stopped!

Initial velocity



$t=0$

Further differentiation of (2) by  $dt$  results in

Further differentiation of (2) by chain rule

$$\begin{aligned} \frac{d}{dt} (v_0 \cdot e^{-c_1 t/m}) &= v_0 \frac{d}{dt} (e^{-c_1 t/m}) \\ &= v_0 \frac{d}{dt} \left( \frac{-c_1 t}{m} \right) \cdot \frac{d e^x}{d x} \end{aligned}$$

where

$$x = \frac{-c_1 t}{m} \quad (\text{by definition})$$

$$\text{RHS} = v_0 \left( \frac{-c_1 t}{m} \right) e^x$$

$$x = -\frac{c_1 t}{m} \quad (\text{by definition})$$

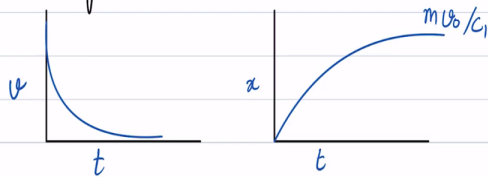
$$\begin{aligned} \text{RHS} &= v_0 \left( \frac{-c_1 t}{m} \right) e^x \\ &= -\frac{c_1}{m} v_0 e^{-(c_1 t/m)} \end{aligned}$$

$$= -\frac{c_1}{m} v(t) \quad (\text{from (2)})$$

Graphically



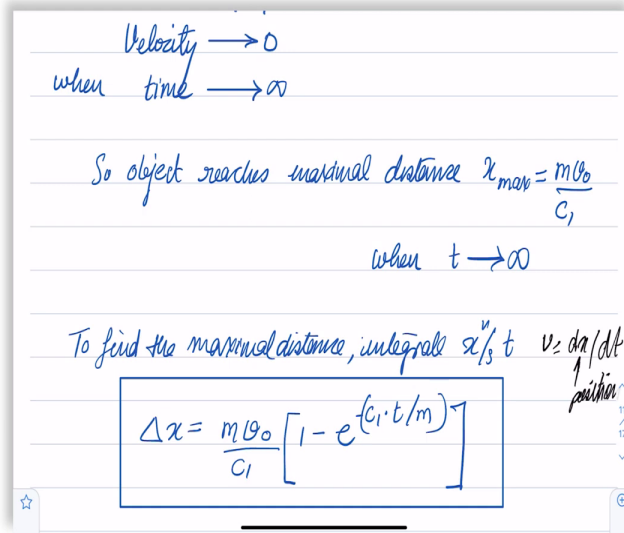
Graphically



Time scale of velocity decay defined by

$$\begin{aligned} & \frac{m}{c_1} \\ \text{Velocity} & \rightarrow 0 \\ \text{when time} & \rightarrow \infty \end{aligned}$$





But at small size scales, length is 20 microns, velocity is 20 microns per second, like for example, the Parasite Protozoan *Trypanosoma mexicana*, which has a length of 20 microns, which includes its flagellar tip, the leading edge hazard Reynolds number  $10^{-4}$ . So, what about the coasting distance and time at low Reynolds number.

So, in order to get the coasting distance, so, remember coasting distance is nothing but the distance for which once the actual motion stops, how long does the object or the cell continue to propagate? That is what it is coasting distance? For horizontal damped motion, we can then keeping this thought in mind that we are interested in the time after flagella stop beating hypothetically we can equate the drag force.

$$F_{\text{drag}} = F_{\text{motion}}$$

And the force due to motion, force due to motion can be written as,

$$F_{\text{motion}} = ma = m \frac{dv}{dt}$$

and drag force is written as,

$$F_{\text{drag}} = c_1 v$$

$c_1$  times  $v$ ,  $c_1$  is in turn the drag constant and  $v$  is the velocity,  $m$  the mass.

Rearranging we get,

$$\frac{dv}{dt} = \frac{-c_1 \cdot v}{m} (1)$$

by definition,

$$\frac{de^x}{dx} = e^x$$

we can use this to solve the integrating in some senses the dv/dt equation 1 to arrive at

v(t) is equal to,

$$v(t) = v_o e^{-\left(\frac{c_1 t}{m}\right)} (2)$$

So, this exponent is in terms of time, the drag coefficient and the mass. At t is equal to 0, the velocity is some initial velocity  $v_o$  that is what we will call  $v_o$ . Further differentiation of equation 2 by dt results in,

$$\frac{d(v_o) \cdot e^{-\frac{c_1 t}{m}}}{dt} = v_o \left(\frac{-c_1}{m}\right) \cdot e^{-\frac{c_1 t}{m}} = v(t) \left(\frac{-c_1}{m}\right)$$

and by doing some arithmetic and we ended up with this expression.

So, graphically this implies that the timescale of velocity is set by m by  $c_1$  velocity goes to 0, when time goes to infinity and displacement when time goes to infinity goes to m by  $V_o/c_1$ , what to call x max the maximal displacement. To find the maximal distance traveled, then we only need to integrate x versus t.

$$\Delta x = \frac{mv_o}{c_1} \left[ 1 - e^{-\left(\frac{c_1 t}{m}\right)} \right]$$

And we have this slightly more clunky looking expression which is in terms of the displacement after stoppage of active motion or for example, beating of flagella. So, in order to find the

coasting distance in time, we need to just substitute values and we will get the number so you should try it yourself.

(Refer Slide Time: 25:07)

In order to find coasting distance & time we need to substitute values and

To FIND:  $m$  (mass) &  $C_1$  (drag coeff.)

For  $E. coli$   $m = \rho \times Vol$

Approximate  $E. coli$  density as that of water & spherical in shape

$$m \sim 4.2 \times 10^{-15} \text{ kg}$$

$C_1$ : drag coefficient

$$C_1 = 6\pi\eta r = 1.9 \times 10^{-8} \text{ kg/s}$$

TRY IT YOURSELF

$$C_1 = 6\pi\eta r = 1.9 \times 10^{-8} \frac{\text{kg}}{\text{s}}$$

A. Coasting distance

$$x = \frac{m v_0}{C_1} = 4.4 \times 10^{-12} \text{ m}$$

$$\sim 4 \times 10^{-2} \text{ \AA}$$

(compare to cell size  $\sim 1 \mu\text{m}$ )

B. Coasting Time

$$t = \frac{m}{C_1} = 2.21 \times 10^{-7} \text{ s}$$

$$= 0.2 \mu\text{s}$$

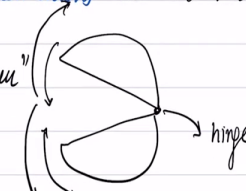
For  $E. coli$   $m$  is equal to  $\rho$  times the volume, but it is approximately  $10^{-15}$  kg  $C_1$  is about  $1.9 \times 10^{-8}$  kg per second, Coasting distance comes to  $4/100$  angstroms, that is  $0.04$  angstroms. Whereas coasting time comes  $2.2$  microseconds, where microseconds is  $10^{-6}$  seconds. This is only simply telling us there is no coasting. And it is going to a very short time if there is any.

(Refer Slide Time: 25:41)

(v) Swimming at low  $R$  and reversibility

Cyclical deformations - no net movement

"Scallop theorem"



In macroscopic world scallop can move by adjusting rates of open  $\mathcal{J}$  and close  $\mathcal{\Theta}$

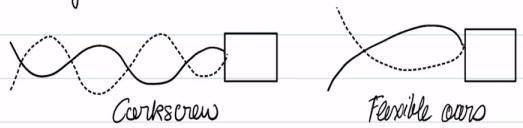
At low Reynolds numbers remains stationary.

world scallop

can move by adjusting rates of open  $\mathcal{J}$  and close  $\mathcal{\Theta}$

At low Reynolds numbers remains stationary.

Biological Solutions:



(vi) Efficiency

Another theorem, which is not very rigorously proven one in the case of Purcell, but later on was addressed a bit more carefully, is what he refers to as the Scallop theorem. Scallops are nothing but single hinged, microscopic shelled organisms, they have a muscle that contracts and close contracts and opens the mouth, I am sorry the two shells.

Creating a sort of clapping motion, sort of what we would call clapping with one hand at low Reynolds numbers, this motion would remain stationary because of what it is called reciprocity. As you go from open to closed and close to open stages, it does not matter whether you do well faster or the other or slower, you will, because of the lack of inertia, be at the same position.

And biological solutions to that is to avoid reciprocity of motion, meaning to say that the motion is one way and the motion the other way are not the same. They do not retrace the path, for example, a corkscrew or a flexible for some of these are seen in sperms and flagella. And therefore, we could argue that this may be a solution that biology has already arrived at.

(Refer Slide Time: 26:55)

(vi) Efficiency

$$\text{Bacterial flagellar energy efficiency} = \frac{\text{work to move object}}{\text{metabolic energy spent}}$$

$\sim 1\%$

looks low, but if  $v = 30 \mu\text{m/s}$  (*E. coli*)  
@ 1% efficiency  $2 \times 10^8$  ergs/s

Given mass of *E. coli*  $\sim 0.5$  W/kg

Contrast : Engineered atoms/molecules use

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Given mass of *E. coli*  $\sim 0.5$  W/kg

Contrast : Engineered atoms/molecules use  
 $\sim 15$  W/kg.

What about Efficiency? How efficient is the bacterial flagella motor work by work of moving the object versus metabolic energy spent is what we would call this efficiency, multiply 100 we get a percentage it is about 1 percent. That does not sound like lot 1 percent a very small minority most of the energy spent dissipated.


So, but interestingly, the small number is actually very important, because if we now consider velocity of *E. coli* of 30 microns per second, 1 percent efficiency is  $2 \times 10^8$  ergs per second. And given the mass of *E. coli* about 1 picogram, it is about 0.5 watts per kilogram. What about engineered systems what humans have been, engineers of human nature have been able to do all

the engineering graduates should very excited for our engines are more efficient than package legends. Well, turns out they are about 15 watts per kg, which is almost 30 times less efficient than bacteria.

(Refer Slide Time: 27:58)

$\sim 15 \text{ W/Kg.}$

(vii) Why bacteria swim and diffusion



time for stirring transport  $\sim \frac{l}{v}$

time for transport by diffusion  $\sim \frac{l^2}{D}$

$D$ : Diffusion Coefficient

$$\frac{\text{Diffusion time}}{\text{Stirring time}} = \frac{l^2}{D} \cdot \frac{v}{l} = \frac{vl}{D} \equiv \text{Stirring Number (So)}$$

### D: Diffusion Coefficient

$$\frac{\text{Diffusion time}}{\text{Stirring time}} = \frac{l^2}{D} \cdot \frac{v}{l} = \frac{vl}{D} \equiv \text{Stirring Number } (S)$$

$$D_{\text{macromol}} \sim 10^{-5} \text{ cm}^2/\text{s}$$

$$l \sim 1 \mu\text{m}$$

$$S = \frac{30 \mu\text{m}/\text{s} \times 1 \mu\text{m}}{10^3 \mu\text{m}^2/\text{s}} \approx 10^{-2} \text{ [small value]}$$

$S \ll 1$ , stirring accesses nothing

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$$S = \frac{30 \mu\text{m}/\text{s} \times 1 \mu\text{m}}{10^3 \mu\text{m}^2/\text{s}} \approx 10^{-2} \text{ [small value]}$$

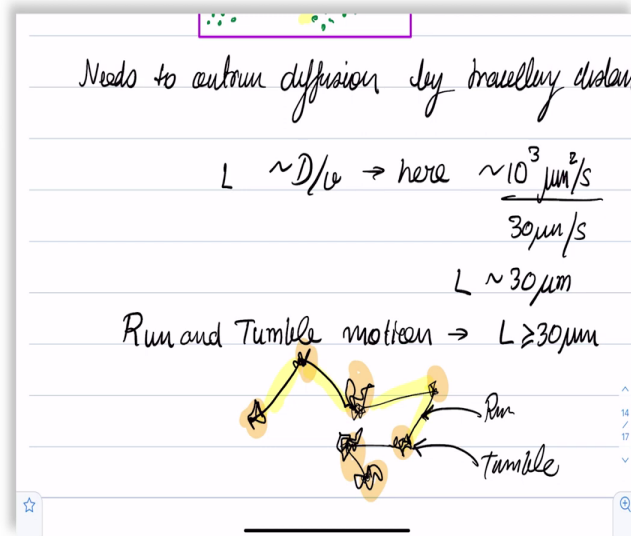
$S \ll 1$ , stirring accesses nothing



Needs to overcome diffusion by travelling distance

$$L \sim D/v \Rightarrow \text{here } \frac{\sim 10^3 \mu\text{m}^2/\text{s}}{30 \mu\text{m}/\text{s}}$$





So, lastly, why bacteria swim versus the effect of diffusion can be studied in terms of something called a time for stirring transport, this is a number that Ed Purcell came up with, with the idea that if you divide  $l$  by velocity, you end up with a time characteristic, which is a characteristic time which can tell us about the transport by velocity by active motion and  $l^2$  square by  $d$ , which will tell us what the typical timescale for transport by diffusion and if these the diffusion coefficient.

Then the diffusion time by the stirring time tells you which dominates. So, if the value is greater than 1, then diffusion time is greater and stirring time is very small, or the other way round, then if the ratio stirring number is very small, then the stirring time dominates. We need some numbers diffusion macromol diffusion coefficient of macromolecules.

We are interested in how cells may move based on the mobility of the molecules they want to eat. So, we take molecule like sugar molecule, of  $10^{-5}$  centimeter squared per second diffusion coefficient, length scale of the bacterium of about 1 micron. And velocity we know is about 30 microns per second of *E. coli*.

So, we now substitute all these numbers, and we end up with  $10^{-2}$  it is a small value like I said earlier, if the value is small, then it is probably is not diffusion limited or diffusion is a very small timescale. Studying is much longer timescales. In other words, studying does not achieve much. So, this is sort of meant to be used to help us think of the following scenario that there are

molecules of nutrient type in various parts the environment of the bacterium and it has to decide what to do does it, should it just wait in one place?

Should it be like a sort of baby and very, very helpless student who is waiting to be spoon fed? In this case spoon fed not by its mother but by diffusion. Or should it move around try to move its hands and legs and get somewhere and see what happens. So, it turns out that bacteria need to outrun diffusion by traveling a certain distance.

So, what is the minimal distance is what Ed Purcell asked? And that distance can be found by simply thinking in terms of the ratio of the diffusion coefficient of the molecules to the velocity of the bacterial motion and that gives us something in the range of 30 microns. So, 30 microns is what they need to swim before they can outrun diffusion and get somewhere and the running tumble motion that is actually observed in *E. coli* in bacteria of this particular type at least.

This is about a little equal to 1 more than 30 microns, the regions where back where *E. coli* undergoes directed motion, after that, it diffuses around, which are called the tumbles. So, in a sense, at Ed Purcell's point is that bacteria have discovered the physical laws that govern their behavior, and have tuned their behavior at a single cell, very simple organism level with no brain no neurons to do something very, very intelligent.

(Refer Slide Time: 31:41)

## Summary

- 1) Fluids and flows in biology
- 2) Concept of viscosity
- 3) Newton's law of viscosity
- 4) Classic bulk measurement of viscosity
- 5) Viscoelastic material

- 3) Newton's law of viscosity
- 4) Classic bulk measurement of viscosity
- 5) Viscoelastic material
- 6) Molecular nature of water
- 7) Statistical redundancy of water conformations
- 8) Hydrophobicity & Entropy
- 9) Reynolds number
- 10) Motion at cellular scale - of and inside

## 9) Reynolds number

### 10) Motion at cellular scale - of and inside c

11) Motion in water at a macroscopic scale

12) Effect of Reynolds number on flows

13) Life at low Reynolds numbers

- Newton's law and momentum

- Some typical values of  $R$

- Size scale and  $R$

- Coasting distance and cellular motility

- Reversibility at low  $R$  - Scallop theorem

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- Reversibility at low  $R$  - Scallop theorem

- Efficiency of motion

- Stirring vs diffusion

So, with that, I end this segment and you have gone through a lot of these topics fluid flows, concepts of viscosity, Newton's law visco elasticity, Reynolds number, life at low Reynolds number, and we will now start a new topic.