Transport Phenomena in Biological Systems
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Module - 1
Lecture - 3
Mass Conservation for a Microscopic System

Welcome back to the next lecture in the course on transport processes in biological systems. Okay. In the previous class, we looked at the importance of rates; why we need to look at things in terms of rates; mass rate rather than mass; and rates of conserved quantities especially. Let us move forward. And before that, we said we would look at mass conservation itself for some time, to begin with in this course, because it is a very important and useful concept. Let us continue from there.

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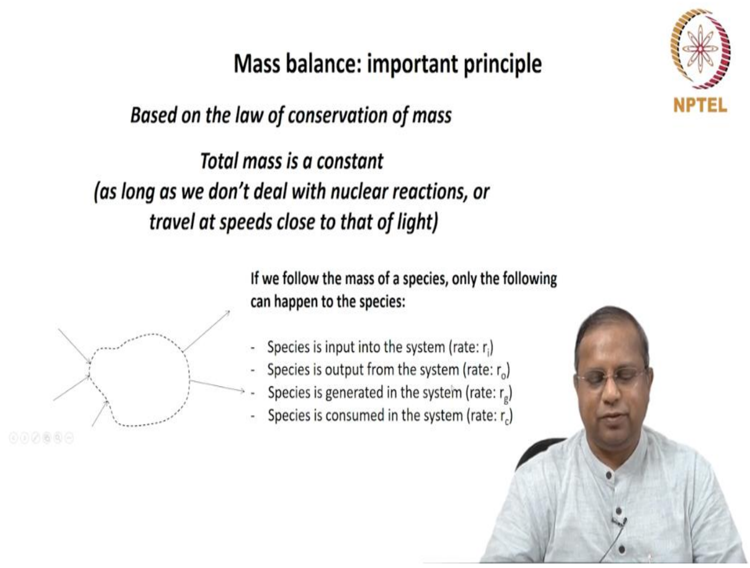

Mass balance is an important principle as has been to you, many times in this course already. Few more will come. This, of course is based on the law of conservation of mass which says that total mass is constant as long as we do not deal with nuclear reactions where mass to energy conversion is possible; or we do not travel at speeds close to that of light. To apply material balance, mass balance, we need to do it a certain way for it to be effective or easy to apply.

Always, we need to apply the mass balance over a system. A system is nothing but an aspect that we focus on. It could be as large as, say the entire country or the entire earth; or it could be the entire building; the entire university; or it could be as small as your pen, your body; or it could be even smaller as a cell and so on so forth. As long as it is a continuum system, you could choose that as your system.

That is what a system is; something that you focus your attention on. It could be whatever. The choice of the system depends on what you want to do. And the appropriate choice comes only with practice. You will need to get into it. You will need to practice a lot before you feel comfortable. So, for the initial stages, just look at what has been chosen as a system, as we go along.

And then, as you do more and more aspects, as more and more problems and so on so forth, you will get accustomed to choosing a system. In some cases, choice of a system can be very tricky to get the exact thing that we are looking for; that is; even for people who are very experienced. So, it is not a very easy thing to do. But, we will get there ultimately. So, for now, let us say that we are going to focus on a particular aspect and that aspect is shown by what is inside these dotted lines. These dotted lines give the periphery of the aspect that we need to focus on. So, that is our system. This dotted line represents the boundaries of a system. There could be various inputs to the system and there could be various outputs to the system, of mass because we are looking at the mass conservation that is the conserved quantity that we are looking at.

If we follow the mass of a species, only the following can happen to the species in this context of the system and inputs and outputs. The species comes into the system. It crosses the system boundaries and comes into the system at a rate of r_i. It is output from the system at a rate of r_o. Note that I am talking about rates. I am not talking about amounts. It could be generated in the system at the rate of r_g or it could be consumed in the system at a rate of r_c. These are the 4 things that will happen to a species, a mass of a species. You can think as much as you want, whatever happens can be categorized into one of these 4.

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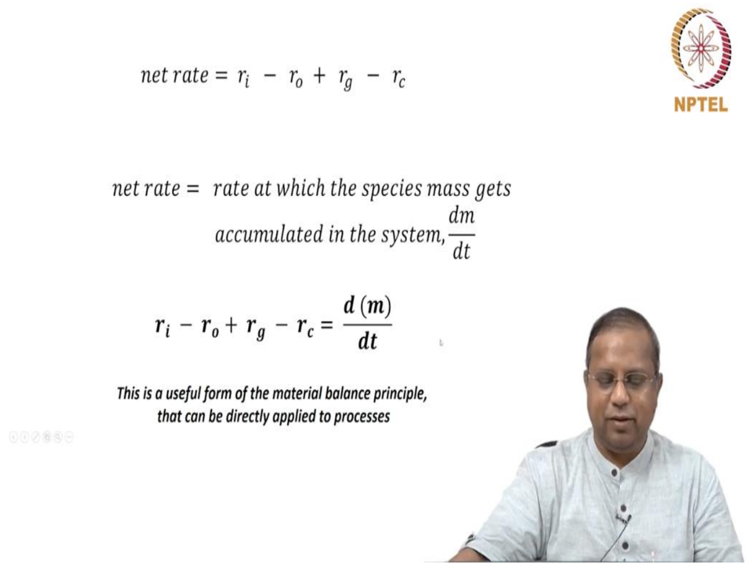


So, the net rate is nothing but; from the point of view of the system; is nothing but the rate of input of that mass into the system - the rate of output of that mass through the various streams from that system + the rate of generation of the same species - the rate of consumption of that species. That is the net rate of that species in that system, inside the system. So, the net rate is the rate at which the species mass gets accumulated in the system.

Net rate = $r_i - r_o + r_g - r_c$

The net rate is going to determine how the species mass is going to either increase or decrease with time in the system. In other words, $\frac{dm}{dt}$ the rate at which mass changes with time. This is the, you know, little bit of calculus. Therefore, you know that this is the standard way to represent the rate of change of a substance of a quantity with another quantity.

In this case, the rate of change of mass with time. So, $[r_i - r_o + r_g - r_c]$ is the rate of change of mass of that species in that system with time. This is the useful form of the material balance principles that can be directly applied to various processes. Just remember this. You go to a process; you blindly apply this; you will not go wrong.

Wherever you are, just make sure that not dealing with nuclear reactions; or you are not travelling at speeds close to that of light. You will be perfectly fine. This would always be valid. And that is very powerful. And that is what this course is giving you. It will give you a few such things. It is giving you a principle that can be applied anywhere, anytime to anything; and it will be valid. That is something very powerful. **(Refer Slide Time: 06:38)**

You do not have many such things. What we are going to do now is to apply this principle to a macroscopic system in this case. I will call this a reflection point or a practice point. I may term it as that, or I may implicitly think that it is that. You would be able to figure out very easily. So, we are at these points where you strengthen your understanding; you do a bit of calculation, maybe; or you think about something; essentially, you strengthen your understanding.

So, what we are going to do is, we are going to apply this principle, the useful form of the material balance. Material balance is nothing but, mass can neither be created nor destroyed; or mass is conserved. So, we use that over a system undergoing a process to get it off a form that can be useful to us. Right. And, that is what we are going to apply. Useful to us, just means that it has been presented in terms of quantities that can be measured.

That is essentially what it means. I will also mention this again. For this, let us consider this problem. A humidifier is fed with dry air. Dry air means, no water vapor is present. The water vapor is removed during the processing of air, to avoid contamination of the bioreactor. Because, there could be organisms present in the air, moist air especially. In the bioreactor, we want only the organism of interest of, for us to grow and nothing else.

And the way we make sure of that is: we first sterilize the bioreactor space, make sure that everything gets killed, all microorganisms are killed. And then, very carefully, we introduce the microorganism of interest to us, into the bioreactor space, in, maybe in a liquid medium and then, allow that to grow; all the time ensuring that, no other organism can get in.

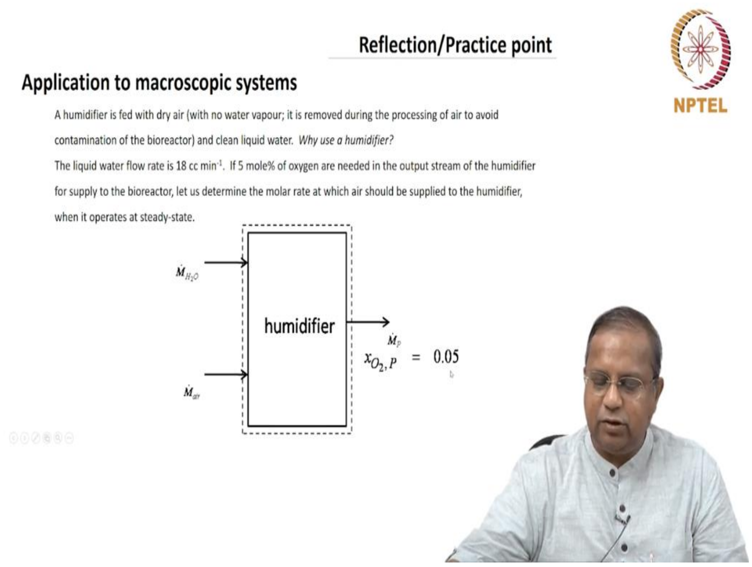
If you are supplying oxygen through air, and we have the organisms that are present in the air; that could very easily get in and contaminate our system. We do not want that to happen in a production system. So, that is what we do. So, the water vapor is removed to create this dry air during the processing, to avoid contamination in the bioreactor. And clean liquid water is also added to the humidifier.

Think about why we really need to use a humidifier. What is a humidifier? A humidifier is something that adds humidity, that adds moisture to something. You may be familiar with something, with things called room humidifiers. They are machines, you pour some water into them. They spray a fine mist of liquid water into the dry space and the humidity of the room increases. And we feel more comfortable in very dry climates. That is what a humidifier is. In this case, why do we need them? Think about it. Pause the video here and think about it. Right. Let me give you the answer. Hopefully you thought about it. Maybe this is what you thought about also. We removed the moisture from the air to prevent contamination; that is fine.

However, the air has a certain capacity to carry moisture. If it does not have that amount which is supposed to be naturally present, you can use thermodynamics to figure out how much should be actually, naturally present there. The vapor-liquid equilibrium concepts and so on so forth; you can go back and look at that. You can actually predict very accurately, how much there is going to be. That is what relative humidity is all about. We will get back. We have air from which the moisture has been stripped to avoid contamination. And then, you have cleaned the air. It has gone through various filters. Now, if we put it into the bioreactor broth, which is all liquid(water), it is going to take liquid from the bioreactor broth to satisfy its need to carry a certain amount of moisture under the given conditions of temperature and pressure.

Because, that is its source. Whenever it comes into contact with that source, they are going to start pulling out the water. If it is going to pull out the water, the water level in the bioreactor is going to reduce, because a good amount of evaporation is going to take place into the dry air. And once that starts happening, maybe the probes will get exposed. You know, the probes are no longer reading what they are supposed to do.

They are no longer submerged in the liquid; the water level keeps on going. Things get concentrated to levels that are far beyond acceptable. And probably, within a couple of days, you may not have any liquid left in a small bioreactor. That is the reason why we need to add moisture to probably equilibrium amounts back into the dry air, before we feed it into the bioreactor broth. Thereby, we have saturated the air with whatever moisture it is supposed to carry. It will not strip the bioreactor off the moisture, of the liquid. And the liquid level will remain pretty much constant. This is the reason why we use a humidifier.



Let us move further. The liquid water flow rate is 18 cc min^{-1}. If 5 mole% of oxygen are needed in the output stream of the humidifier, for supply to the bioreactor; let us determine the molar rate at which air should be supplied to the humidifier, when it operates at steady state.

You all know that steady state means that properties of interest at a particular point, do not change with time, in the system. That is essentially what steady state is. You must have picked this up earlier. If not, you can learn now. You take any property of interest to us; it does not change in that system, in that point in the system, with time. The unchanging nature with time is essentially what steady state is, but at a particular point. Remember this.

So, this is the problem. Think about this problem. Let me also present a figure to help you understand. So, this is the humidifier. Here, you have , dry air that is coming in; water that is probably sprayed into the humidifier, so that it saturates the air, hopefully. We are not really interested in that. And then, whatever is coming out here, the mole fraction of the product needs to be 0.05. This translates to 5 mole%. Therefore, the mole fraction of oxygen in the product stream needs to be 0.05 times the molar flow rate of the product stream, $\mathbf{\dot{M}_{O2,P} = 0.05\ \dot{M}_P}$.

The, this is the statement that is made. You go ahead and think about this. Try to do this problem. I will present the solution for this and then take a break. Yeah, that might be a little better. Pause the video here. Think about this. Read the problem a couple of times.

And then, let us see, whether you are able to do this. Do not take too much time, because I am going to give you some hints towards problem solving itself next. That I think should be done right away, so that it will help you the best way. In any case, pause the video for maybe a minute or two; read the problem; try to get a feel for the problem; and then, we will continue. Go ahead please. Hopefully, you got a feel for the problem. What I am going to do now, is tell you something about problem solving itself.

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We are going to solve a lot of closed-ended problems in this course. As a part of the course itself, to help you get better with the understanding and so on so forth. There would be assignments where you need to solve problems. And therefore, it is good to pick up some aspects of problem solving. Having said that, I should right away say that problem solving is a higher-level skill.

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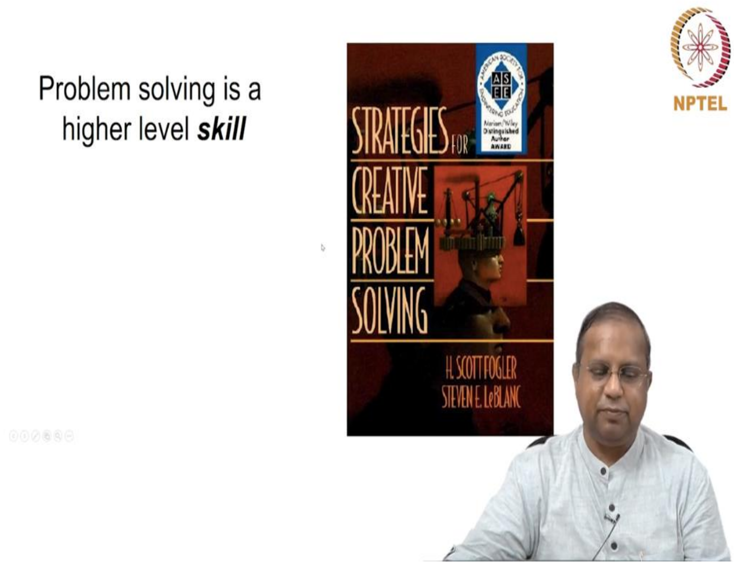

What do I mean by that? In learning, there are various levels at which one can learn. If one follows what is called the Bloom's taxonomy of learning objectives, things can be learned at a stage where you just recall them. You memorize, you recall them. So, that is the recall stage. Let me say recall is the most surface level stage. The next stage of depth is the understanding stage where we have understood the material.

The next level of depth is the apply stage, where you can apply the information. The next level of depth is the analyze level. The next level of depth is the evaluate level. The next level of depth is create level. So, any subject can be learned at any level of depth here. It is best that you learn it at the level of the deepest level of learning possible, which is the create level. But different courses are different. They will take you through different levels of learning. We try to get to at least the level of evaluation and creation. In my courses, in my direct contact courses, I go to level of creation all the way down, you know, the deepest level possible, through various exercises. In this format, that may not be so easy to achieve, but we will probably get to the level of analysis and evaluation, even in this kind of a format.

So, problem solving is a higher-level skill. Higher level skill means deeper level skill, which means, it needs the skill in analysis and needs a skill in evaluation, in creation and so on so forth. So, it is not easy to do. Okay. That is something that you need to understand. You need not be worried that you are unable to solve problems right away. You will not be able to.
You need to develop those skills, before you can solve the problem. That is what I mean. There are some books, nice books. This is a slightly old book. It is called Strategies for Creative Problem Solving. This is by Scott Fogler and Steven LeBlanc. It is a nice book. You can read

this book. If you can lay hands on that, check out your library and so on so forth. You could buy this book; this book is available. Or you could read other books too. This is something that I have read and found useful to recommend.

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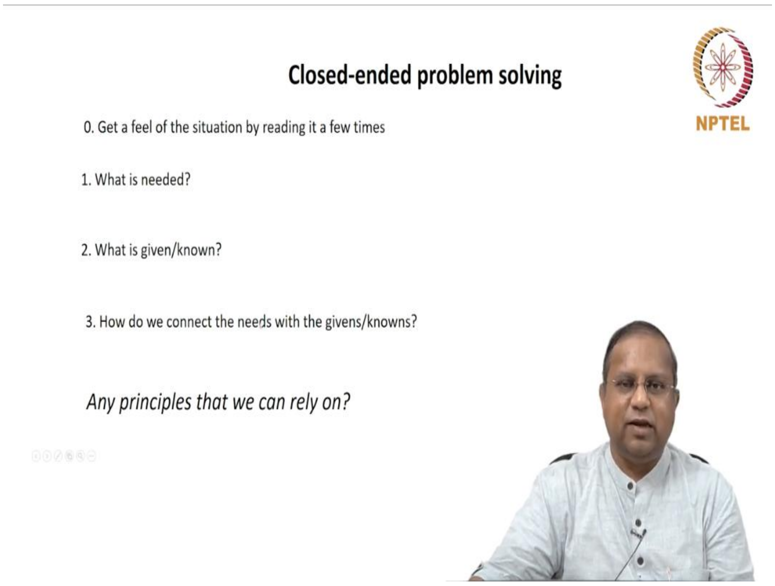

We will focus on closed-ended problem solving alone, which is a very small aspect of problem solving. Open-ended problem solving is much larger, much more difficult. Closed-ended problem solving. This is my recipe to you. Level 0: get a feel of the situation by reading the problem a few times. Then ask the question, what is needed? What is given or known?.

I am just presenting it as 1 and 2. Sometimes, you could ask this first; and then ask that next; and so on. Do not be very strict about following that. But, whatever, what is given, known, must be clear. What is needed must be very clear. It is good to start with what is needed. And then, ask the question, how do we connect the needs with the givens and knowns. This is what we need. This is what we know.

How do you connect the 2? That is in essence problem solving, right, closed-ended problem solving. In other words, are there any principles that we can rely on, to do this connection?

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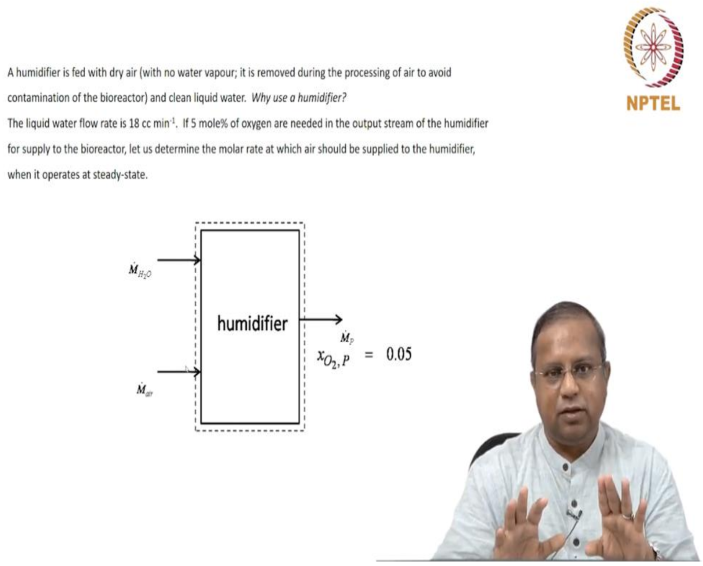

Now, let us do this. Let us read this a couple of times. Sorry, read this a couple of times. Be with that for some time, to get a feel of what it is. Okay. Let me read this at least for you. A humidifier is fed with dry air, which has no water vapor. The water vapor has been removed during the processing of air, to avoid contamination to the bioreactor.

So, the humidifier is fed with dry air and clean liquid water. The liquid water flow rate is 18 cc min^{-1}. If 5 mole% of oxygen are needed in the output stream of the humidifier, for supply to the bioreactor later, let us determine the molar flow rate at which air should be supplied to the humidifier, when the humidifier operates at steady state.

Steady state operation is very standard. That is the state at which many continuous things operate. You would like to be in that state. Otherwise, the control becomes difficult and so on. So, as it is operating at steady state and you want to find out the molar flow rate of air, given the molar flow rate of water and the composition of water in the stream. This is completely water. Humidifier is completely dry air. There is no moisture in this. This is the situation. So, you can look at it some more till you feel comfortable with it. And that, I think is essential. You need to feel comfortable with it. Now, that you are comfortable with it, let us ask this question.

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Get a feel of the situation by reading it a few times. We did that. What is needed? It is clear. Given the molar flow rate of air at the outlet of the humidifier, molar flow rate of air at the inlet of the humidifier is needed. What is given? Flow rates and compositions of some streams are given. How do we connect the needs with the givens and knowns? Or, are there any principles that we can rely on? Of course, mass balance principle is something that we can rely on. And that, we can use to connect the needs with the givens or the knowns. So, this is the material balance here. So, this is the essential thinking. Once you are clear about this, then the rest is processing. You will have to do somewhat mechanically that with this in the background.

Then, things will be fine. If you just start doing the process without the clarity in this, it is rather difficult to solve any problem, especially closed-ended problems. And that is what most of you face when you try to solve problems for the first time, without experience. So, it is recommended to do the above explicitly while solving closed-ended problems. We may or may not do explicitly this procedure in this course.

Do not worry about it. We would mostly do it implicitly in this course. But, keep this in mind. Always asked the question, I mean, do I have a picture of this? Then, what is needed; what is known; in whatever order. And then, how do I connect the needs to the unknowns or needs to the knowns, given the knowns, to arrive at the solutions needed to solve the problem. Let me present the solution; stop here for this lecture.

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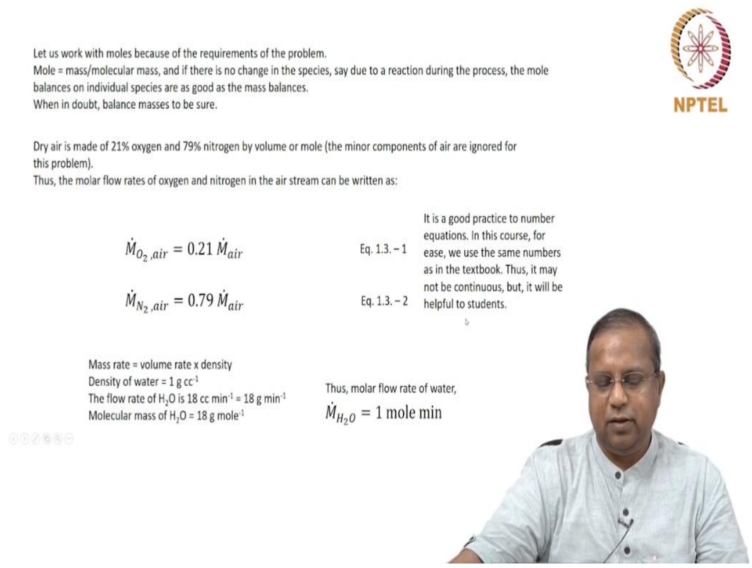


We are going to work with moles, because we need the molar flow rates. Typically, mass is the most comfortable to work in. Moles can change; moles of a species can change during a reaction. And therefore, moles are not as general; mole rates are not as general in applicability as mass rates. But here, that is not the case. Take it from me, that, I have ensured that that is not the case.

Therefore, we can work with moles itself. If you are not clear about that, it is better to work with mass; and then convert mass into moles and so on so forth. How do we convert from mass to moles? We know that by the definition of mole. It is nothing but the mass of a species divided by its molecular mass.

$$moles = \frac{mass}{molecular\ mass}\,.$$

If there is no change in the species, say due to a reaction during the process, the mole balances on individual species are as good as the mass balances. That I know in this case. If you are not sure, just use masses. Here, we will use moles. Dry air, we know is made up of 21% oxygen and 79% nitrogen, by volume or by mole. Okay, you know this. Of course, we are ignoring the minor components of air, which is carbon dioxide and the other minor gases. This is what it contains. This is dry air. Otherwise, it contains a good amount of moisture H_2O in it.

This dry air is 21% oxygen and 79% nitrogen by mole. The molar flow rates of oxygen and nitrogen in the air stream can be written in terms of molar flow rate of air. We have the air stream and water stream in the input and the product stream. So, the molar flow rate of oxygen in the inlet air stream is 0.21 times the molar flow rate of air, because the mole fraction is 0.21.

$M_{O2, AIR}$ = 0.21M_{AIR} **Equation 1.3-1**

Let us call this equation 1.3.-1.

This might look a little surprising to you. But there is a reason why I do this here. It is always good to number equations whenever we work out something; whenever we do a derivation, whenever we work out the solution to problems. It is always good to number equations. And in this course, that is being presented here, I am going to number the equations that are the same as in the textbook. Okay.

So, that is the reason why it looks a little odd. This has a certain order in the textbook. And that order brings out the number, Chapter 1, Section 3, the first equation of that. That is what this means. So, just to make sure it is easy for you to go and refer to the book, I am going to use the same equation number. Although it might look a little odd, it might not look continuous and so on so forth in this course.

Please ignore the continuity part of the equations. It is nice to number equations continuously. But for this bigger reason, I am going to sacrifice continuity in numbering, for ease of reference in the textbook. So, thus it may not be continuous, but it will be more helpful to the students. Coming back to this problem. The molar flow rate of nitrogen in the inlet air stream is 0.79 (1 - 0.21), that is 0.79 times the molar flow rate of air.

$M_{N2, AIR}$ = 0.79M_{AIR} **Equation 1.3-2.**

Mass rate is nothing, but the volume rate times density. We have already seen that.

The density of water is 1 g cc^{-1}. ρ=1g cc^{-1} or 1 g cm^{-3}.

The flow rate of water is 18 cc min^{-1}, which is the volumetric flow rate?

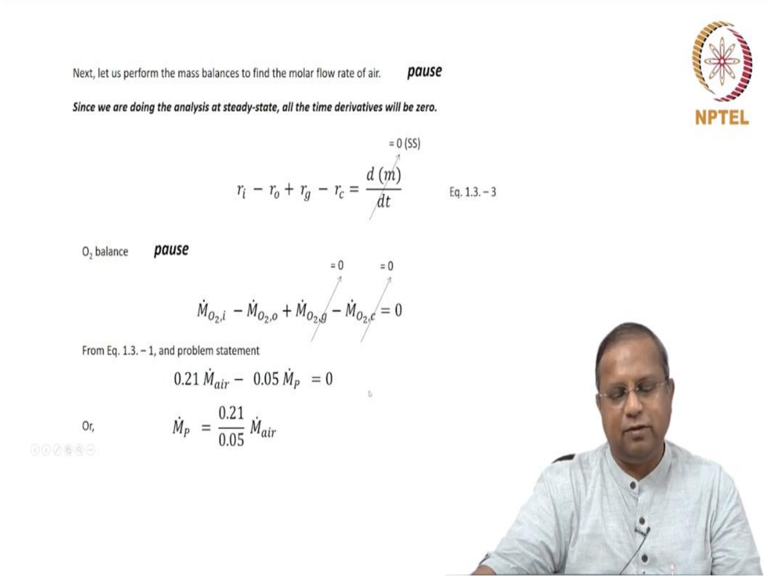
So, the mass flow rate of water = volumetric flow rate * density of water (Density is constant here).

Mass flow rate of water = 18 cc min^{-1}*1 g cc^{-1} = 18 g min^{-1}

The molecular mass of water is 18 $gmol^{-1}$. And therefore, the molar flow rate of water, from the data that has been given is 1mol min^{-1}.

We know that $mole = \frac{mass}{molecular\ mass}$, so $molar\ flow\ rate = \frac{mass\ flow\ rate}{molecular\ mass}$.

Thus, $molar\ flow\ rate\ of\ water = \frac{18}{18}$ = 1 mol min^{-1} **. $\dot{M}_{H20}$=1 mol min^{-1}**. **(Refer Slide Time: 26:36)**



Now, we will do mass balances to find the molar flow rate of air that is needed in the input. We are doing the analysis at steady state. And we said by the definition of steady state, there cannot be variation of the property of interest with time. And therefore, any time derivatives in the system need to be put to 0. That is the mathematical meaning of a steady state. Therefore, you would not have any derivatives in the equation.

You can put any time derivatives in the equation to 0. So, you have; this is the useful form of the mass balance equation.

Input rate – output rate + generation rate – consumption rate = the accumulation rate or the rate of change of mass in the system with time. Since we are looking at the steady state situation, the accumulation rate is 0. We will call this 1.3-3.

$r_i - r_o + r_g - r_c = 0$ Equation 1.3-3

Now, we do an oxygen balance. This is the mass balance that we are going to do; or we are going to apply this equation to the oxygen species. We blindly look at Equation 1.3-3. The molar flow rate of oxygen in the inlet - the molar flow rate of oxygen in the outlet + the molar flow rate of oxygen that is generated - the molar flow rate of oxygen consumed = 0.

$\dot{M}_{O2,i} - \dot{M}_{O2,o} + \dot{M}_{O2,g} - \dot{M}_{O2,c} = 0$

I thought, let me show this and then ask you to write something else, rather than ask you to write something right away. So, this is the application of the mass balance equation to oxygen.

We know that there is no oxygen that is generated in the system due to any reaction. There is no oxygen that is consumed in the system due to any reaction maybe. And therefore, the molar flow rate of oxygen in the inlet stream- the molar flow rate of oxygen in the product stream is 0 under steady state.

$$\dot{M}_{O2,i} - \dot{M}_{O2,o} = 0$$

From whatever we have already seen, we have already seen that the molar flow rate of oxygen in the air stream is 0.21 times the molar flow rate of air. The molar flow rate of oxygen in the outlet stream (product stream) is 0.05 times the molar flow rate of the product stream. We know the equations of molar flow rate of oxygen in the input and the product streams from our discussions so far, and can be written as

$$0.21\dot{M}_{AIR} - 0.05\dot{M}_{P} = 0$$

Input - output from the system, that is humidifier; is 0. You could write molar flow rate of the product stream is 0.21 divided by 0.05 or the molar flow rate of air. This is what we get from that balance.

$$\dot{M}_{P} = \frac{0.21}{0.05}\dot{M}_{AIR}$$

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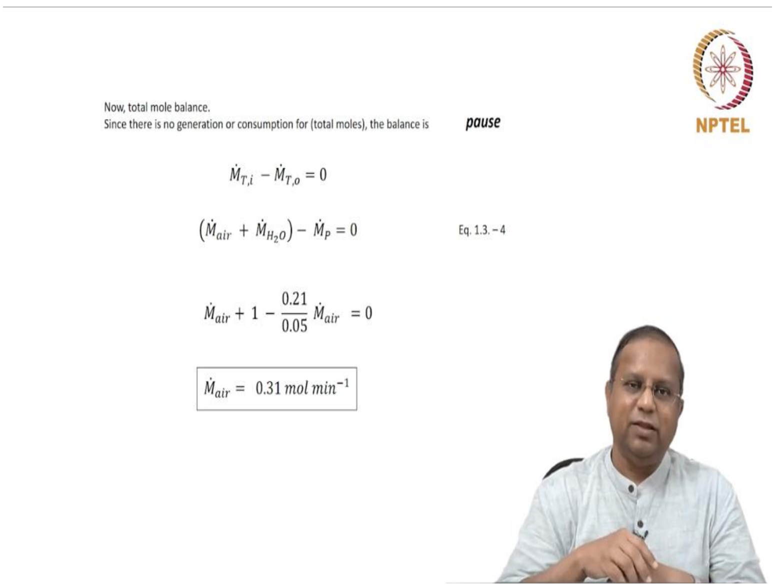

Now, I would like you to do a total mole balance. Take all the moles of the species; all the moles of the species; and do a balance. Let me give you some time. Do the balance. And then, let me show you the balance. Take maybe about 3-4 minutes. And then, do the balance. Let us see what you get. Go ahead please. Hopefully, you got something like this. When you consider the total number of moles, there is no generation or consumption.

Mass can neither be created nor destroyed. Therefore, the balance simply becomes the total molar rate in - the total molar rate out. This is again written on the system. Balance would make sense only in the system that it is written. The total molar rate is through the moles of air and the moles of water. Those are the 2 input streams. The total molar rate out is through the product stream. That is only output stream.

$(\dot{M}_{AIR} + \dot{M}_{H20}) - \dot{M}_{P} = 0$ **Equation 1.3-4.**

From whatever we know, the molar flow rate of water is 1; and the molar flow rate of the product stream is 0.21 divided by 0.05 times the molar flow rate of air. This is what we got in this step. And all this, we are substituting here, to get the molar flow rate of air is, if you do this calculation. You have only molar flow rate of air is the variable here.

$$(\dot{M}_{AIR} + \dot{M}_{H20}) - \frac{0.21}{0.05}\dot{M}_{AIR} = 0$$

$$\dot{M}_{AIR}\left(1 - \frac{0.21}{0.05}\right) + 1 = 0$$

$$-3.2\ \dot{M}_{AIR} = -1$$

$$\dot{M}_{AIR} = 0.3125 \text{ mol min}^{-1}$$

Here, you work things out. Then you will get molar flow rate of air to be 0.31 mol min^{-1}. So, that is the needed answer. This is a typical design situation. You want to know what is the molar flow rate of air to be fed for the humidifier to work appropriately? This is a design operation question, an input quantity that we need. And that, we could easily get by using the material balance principle or doing mass balances.

That is how powerful it is. So, you need to know this. What I will do is, I think I will stop here. Let me stop here for this lecture. When we come back, let me show you the application of the material balance principle to a microscopic system such as a cell, which is of relevance to biological systems. Hopefully, you have been having fun in the course so far.

It would not be mathematical, there is a certain need for mathematics to come in. Because it makes it very gentle. We saw that, you know, when we talk in terms of conserved quantities, it makes, it gives you tools that are so general. Mathematics is something that is very general.

You put it in a mathematical context, then it becomes applicable to pretty much a large variety of situations.

And that is why we take the mathematical route. There is a typical question asked by many students: why so much mathematics? This is the reason why we would like to look at it from a mathematical point of view. You put it in a mathematical plane, then it becomes more generally applicable. It has a very nice set of rules, consistent set of rules and so on so forth. So, we are very clear as to what we are doing in the mathematical plane.

And whatever we can get out of that is the clarity that we need for design and operation. If we can do that, it is much better. Okay. That is the reason why we take the mathematical route. Fine. Go ahead, take a break. Then when we come back to the next lecture, we will go forward with the application to microscopic systems. See you then.