Thermodynamics for Biological Systems:
Classical and Statistical Aspects

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Lecture – 50
Reaction in Liquid or Solid Phases

Welcome!

In the last class we looked at the temperature dependence of the equilibrium constant. We had looked at two equilibrium constants, K p and K f. All those were valid for gas phase reactions.

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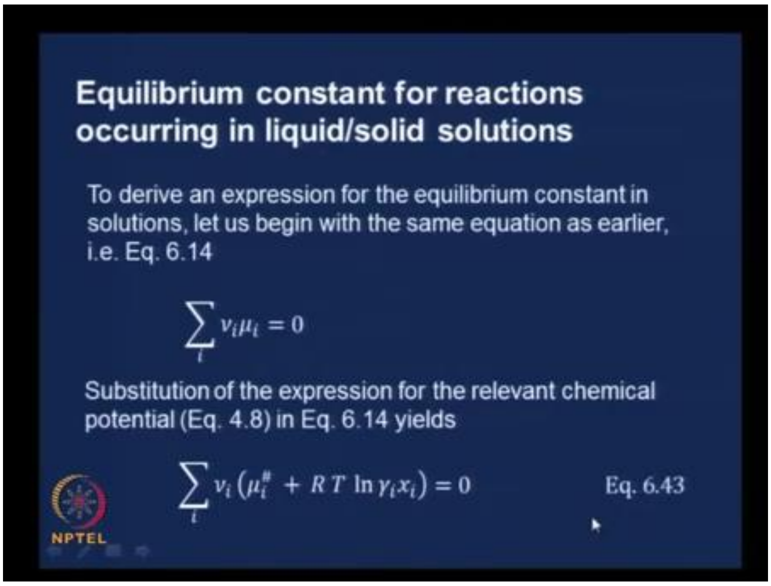

Equilibrium constant for reactions occurring in liquid/solid solutions

To derive an expression for the equilibrium constant in solutions, let us begin with the same equation as earlier, i.e. Eq. 6.14

$$\sum_i \nu_i \mu_i = 0$$

Substitution of the expression for the relevant chemical potential (Eq. 4.8) in Eq. 6.14 yields

$$\sum_i \nu_i \left(\mu_i^{\#} + R\,T \ln \gamma_i x_i\right) = 0 \qquad \text{Eq. 6.43}$$

What we are going to start doing in this class is to consider equilibrium constant for reactions that occur in liquid or solid solutions. To derive an expression for the equilibrium constant in solutions, let us begin with the same equation that is the equation 6.14, which is

$$\sum_i \nu_i \mu_i = 0$$

This is the criterion for reaction equilibrium; this is something that we said is very fundamental good to remember and so on. Simple – sum over all i nu i the stoichiometric coefficient times the chemical potential of the various species i mu i equals zero. If, we substitute the expression for the relevant chemical potential … this is for liquid or solid solution. Therefore, you know that it is going to be mu i equals mu i hash plus R T ln gamma i x i; that is how we characterize the liquid and solid solutions. If we do that sum over i nu i and we are going to replace mu i with mu i hash plus RT ln gamma i x i; this equals zero.

$$\sum_i \nu_i \left(\mu_i^\# + R\,T\,\ln \gamma_i x_i\right) = 0$$

Let us call this equation 6.43.

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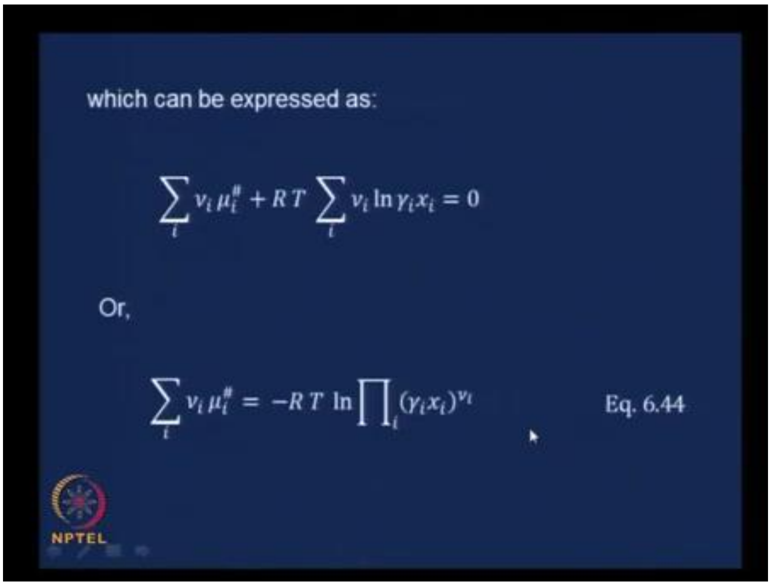

which can be expressed as:

$$\sum_i \nu_i \mu_i^\# + R\,T \sum_i \nu_i \ln \gamma_i x_i = 0$$

Or,

$$\sum_i \nu_i \mu_i^\# = -R\,T\,\ln \prod_i (\gamma_i x_i)^{\nu_i}$$

Eq. 6.44

Which can be expressed as … I am just going to expand the terms here …

$$\sum_i \nu_i \mu_i^\# + R\,T \sum_i \nu_i \, ln \gamma_i x_i = 0$$

RT is a constant. Therefore, I have taken it out here. Or you could write this as

$$\sum_i \nu_i \mu_i^\# = -R\,T\, ln \prod_i \left(\gamma_i x_i\right)^{\nu_i}$$

If I take the log out, the sum of the log is nothing but the log of the product of the various things and therefore, log of the product over all i gamma i x i the whole raised to nu i. We will call it equation 6.44.

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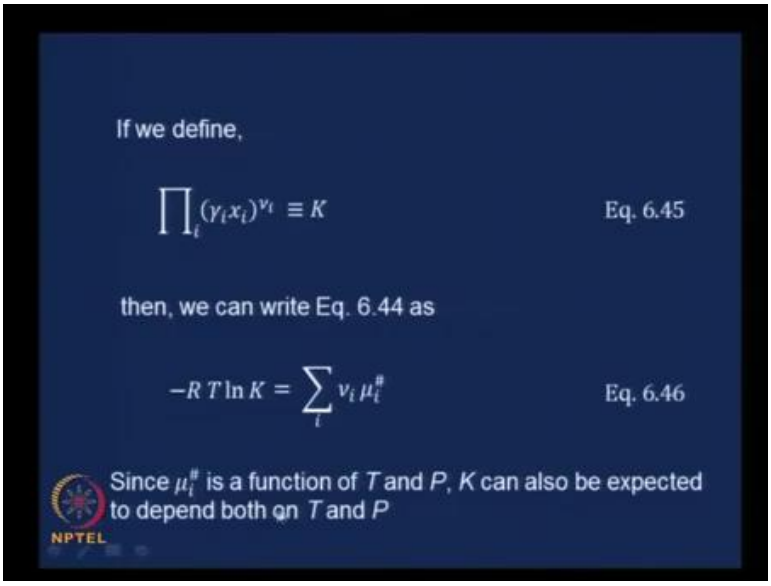

If we define,

$$\prod_i (\gamma_i x_i)^{\nu_i} \equiv K \qquad \text{Eq. 6.45}$$

then, we can write Eq. 6.44 as

$$-R\,T \ln K = \sum_i \nu_i \mu_i^{\#} \qquad \text{Eq. 6.46}$$

Since $\mu_i^{\#}$ is a function of T and P, K can also be expected to depend both on T and P

Now, if we define the same way that we did for Kp and so on, if we define

$$\prod_i \left(\gamma_i x_i\right)^{\nu_i} \equiv K$$

as a certain K, equation 6.45. This is the equilibrium constant for reactions occurring in solid or liquid phases. Then, we can write our previous equation as

$$-R\,T \ln K = \sum_i \nu_i \, \mu_i^{\#}$$

Now, are you able to see the relationship or the similarity between this expression 6.46 and minus R T ln K p equals sum over i nu i mu i naught? That was mu i naught, this is mu i hash.

There it was a function only of temperature mu i naught, but here since mu i hash is a function of both temperature and pressure, you expect the equilibrium constant, here also to be a function of both temperature and pressure – think, I say that here. Since, mu i hash is a function of temperature and pressure, K can also be expected to depend on both the temperature and pressure.

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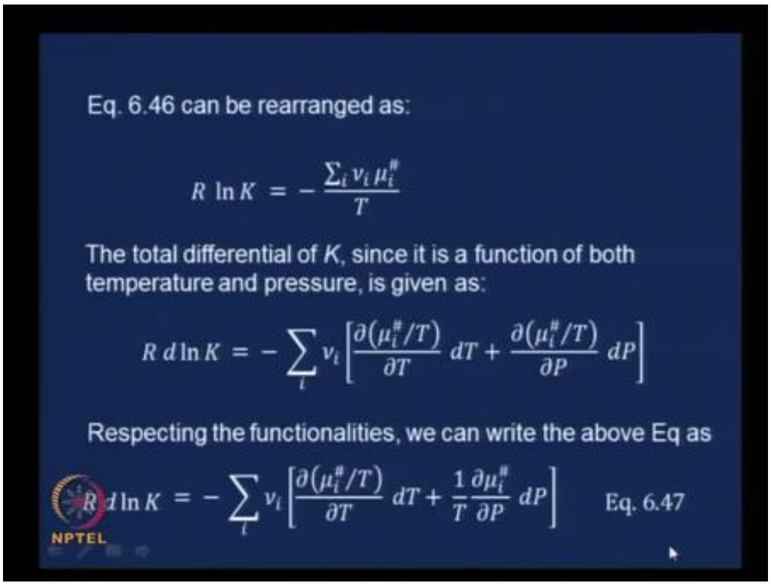

Eq. 6.46 can be rearranged as:

$$R \ln K = -\frac{\sum_i \nu_i \mu_i^\#}{T}$$

The total differential of K, since it is a function of both temperature and pressure, is given as:

$$R\, d\ln K = -\sum_i \nu_i \left[\frac{\partial(\mu_i^\#/T)}{\partial T}\, dT + \frac{\partial(\mu_i^\#/T)}{\partial P}\, dP\right]$$

Respecting the functionalities, we can write the above Eq as

$$R\, d\ln K = -\sum_i \nu_i \left[\frac{\partial(\mu_i^\#/T)}{\partial T}\, dT + \frac{1}{T}\frac{\partial \mu_i^\#}{\partial P}\, dP\right] \qquad \text{Eq. 6.47}$$

Now, to find the temperature pressure dependence, let us start by a rearranging equation 6.46 as

$$R \ln K = -\frac{\sum_i \nu_i \mu_i^\#}{T}$$

The total differential of K, since it is a function of both the temperature, and pressure is given as R d ln K. I am taking the total differential here. R is just a constant. R d ln K can be written as a function of temperature and a function of pressure. To do that, what I have done here is see here there is a sum … there is a minus here first, and then there is a sum here. … Inside the sum, each of these can be nu i which is not, of course, a function of temperature and pressure. Therefore, that is taken as a constant out here, whereas, mu i hash by T is … a function of both temperature and pressure.

So, that is written as the total differential here in terms of the partial derivatives

$$R\, d\ln K = -\sum_i \nu_i \left[\frac{\partial(\mu_i^\#/T)}{\partial T}\, dT + \frac{\partial(\mu_i^\#/T)}{\partial P}\, dP\right]$$

Therefore, R d ln K equals

$$R\,d\ln K = -\sum_i \nu_i\left[\frac{\partial(\mu_i^{\#}/T)}{\partial T}\,dT + \frac{1}{T}\frac{\partial\mu_i^{\#}}{\partial P}\,dP\right]$$

Again respecting the functionalities … in other words, it means just combining the various terms together, we can write this as R d ln K equals minus sum over i nu i dou mu i hash by T dou T dT. And, this is not going to vary with pressure. Therefore, you can take 1 by T out as a constant here. So, 1 by T dou dou P of mu i hash dP. Let us call this equation 6.47.

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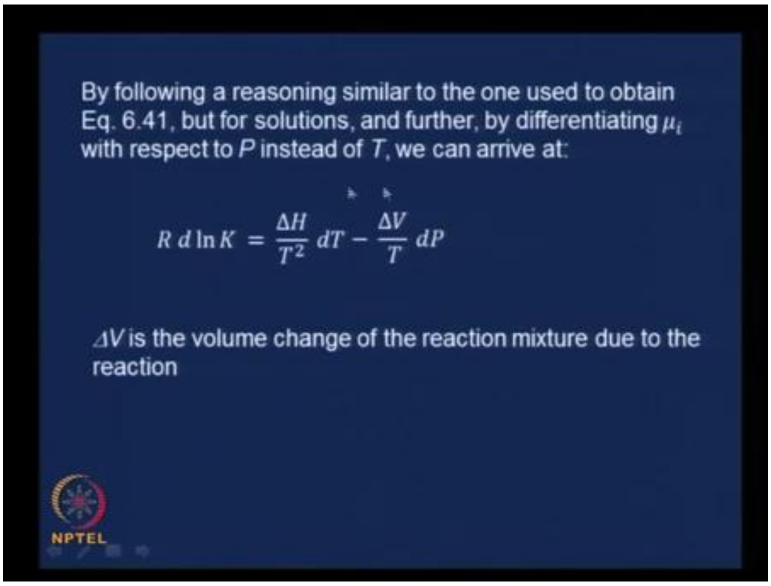

By following a reasoning similar to the one used to obtain Eq. 6.41, but for solutions, and further, by differentiating μ_i with respect to P instead of T, we can arrive at:

$$R\,d\ln K = \frac{\Delta H}{T^2}\,dT - \frac{\Delta V}{T}\,dP$$

ΔV is the volume change of the reaction mixture due to the reaction

By following a reasoning that is similar to the one used to obtain 6.41, but for solutions … in other words we had obtained 6.41 for the gas phase reactions. Here, we are going to follow the same reasoning, but for solutions, and further by differentiating mu i with respect to P instead of T. … This is what I would like you to do, and the hint here is that you will get it in terms of delta V, and so on. I will give you about … since you are already used to it may be about ten minutes. Just look at 6.41 … go back and look at 6.41. Go a few steps before that; see how we got that. Use the same reasoning to get this particular expression. Ten minutes.

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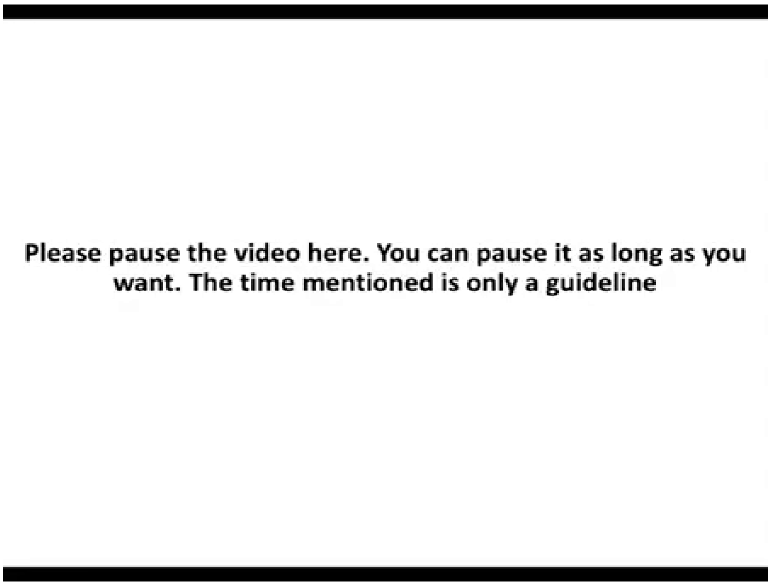

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You would have obtained

$$R\, d \ln K = \frac{\Delta H}{T^2}\, dT - \frac{\Delta V}{T}\, dP$$

Essentially, we have replaced the partial derivatives in terms of the expressions or thermodynamic variables and the relationships we had derived earlier. So, this first derivative gets replaced by delta H by T squared dT and the second derivate gets replaced by delta V by T dP. Therefore, R d ln K equals delta H by T squared dT minus delta V by T dP. Doing such exercises, small exercises, also breaks the monotony of just listening to this and getting actively involved in the process; thereby, the learning is that much better. We will call this equation 6.48 and of course, the delta V is the volume change of the reaction mixture due to the reaction.

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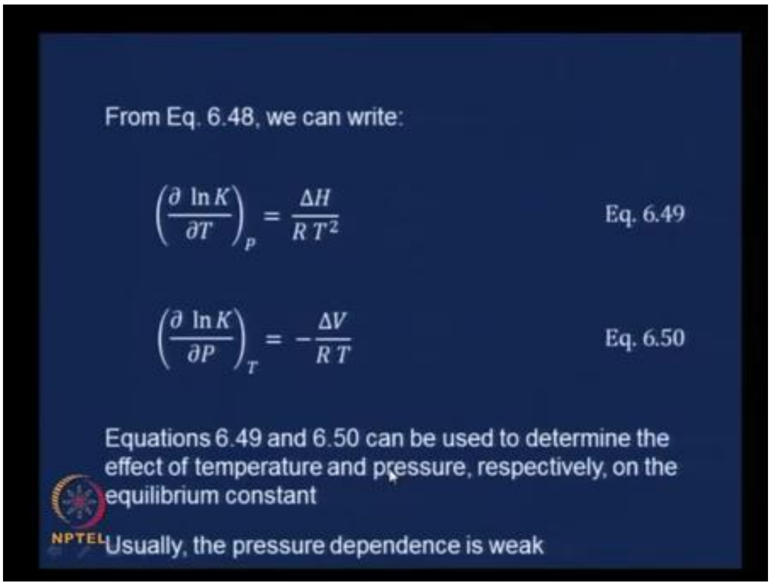

From Eq. 6.48, we can write:

$$\left(\frac{\partial \ln K}{\partial T}\right)_P = \frac{\Delta H}{R\,T^2} \qquad \text{Eq. 6.49}$$

$$\left(\frac{\partial \ln K}{\partial P}\right)_T = -\frac{\Delta V}{R\,T} \qquad \text{Eq. 6.50}$$

Equations 6.49 and 6.50 can be used to determine the effect of temperature and pressure, respectively, on the equilibrium constant

Usually, the pressure dependence is weak

From equation 6.48 which is this, we can write

$$\left(\frac{\partial \ \ln K}{\partial T}\right)_P = \ \frac{\Delta H}{R\,T^2}$$

– not quite easy to see that … we are writing total derivative in terms of the partial derivative. d ln K equals dou ln K dou T at constant P dT plus dou ln K dou P at constant T dP. So, just comparing the terms here we get dou ln K dou T at constant P equals delta H by RT squared, equation 6.49, and

$$\left(\frac{\partial \ \ln K}{\partial P}\right)_T = \ -\frac{\Delta V}{R\,T}$$

We will call this equation 6.50.

Equation 6.49 and 6.50 can be used to determine the effect of temperature and pressure on the equilibrium constant. So, that is what we set out to do, and we have done that here. Usually, the pressure dependence is rather weak; rather the pressure dependence of the equilibrium constant is rather weak. It is good to know that also.

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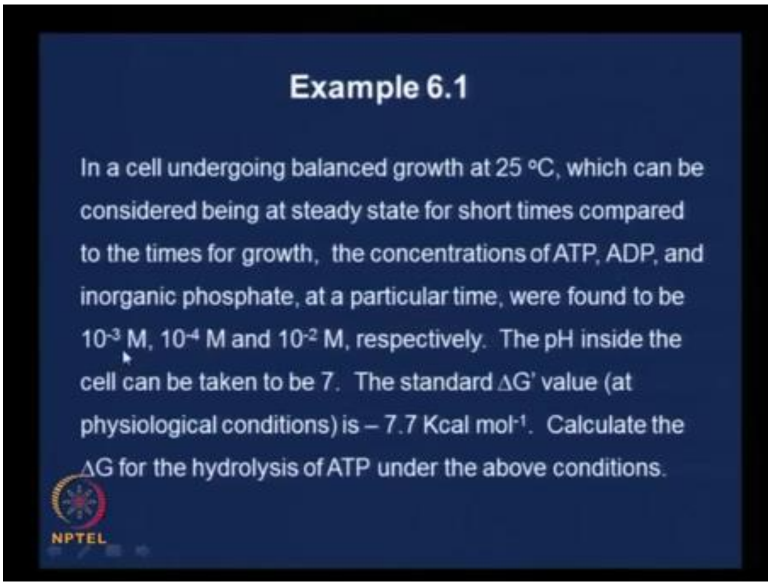

Example 6.1

In a cell undergoing balanced growth at 25 °C, which can be considered being at steady state for short times compared to the times for growth, the concentrations of ATP, ADP, and inorganic phosphate, at a particular time, were found to be 10^{-3} M, 10^{-4} M and 10^{-2} M, respectively. The pH inside the cell can be taken to be 7. The standard $\Delta G'$ value (at physiological conditions) is – 7.7 Kcal mol^{-1}. Calculate the ΔG for the hydrolysis of ATP under the above conditions.

Now, we have looked at equilibrium constants of various reactions, the effect of temperature on the gas phase equilibrium constants, the effect of temperature and pressure on the equilibrium constants for reactions that occur in the liquid or solid phases. Now, let us work out an example. There are three examples one after another, deliberately. The reason for having these examples is that these involve clarity in the concepts that you had learnt in the earlier classes also. So, this is some way of refreshing, because you need to know that as a part of thermodynamics of reactions. And therefore, we are going to kind of review that in the context of the problem.

We are going to see and understand better whatever is required to solve these problems. But the point that I am trying to make here is that you will need principles, clarity of principles that you had learnt much earlier in your twelfth standard and previous courses on chemical thermodynamics to be able to do these examples. Because of that I am going to give you significant time to do this before I present the solution. And example 6.1 the first one of the three, we will do one today and may be the other two in the next class. This is in the context of course, bio-systems. In a cell undergoing balanced growth at twenty five degrees C, which can be considered as being at steady state for short times compared to the times of growth. The concentrations of ATP … recall what ATP is? Adenosine Tri-Phosphate, the energy currency of the cell.

ATP, it is a nucleotide, if you recall the type of molecule – ADP, adenosine di-phosphate and inorganic phosphate. I am sure, now you recall the relationship between ATP, ADP and inorganic phosphate from your biochemistry course. The concentrations of ATP, ADP and inorganic phosphate, P i, at a particular time were found to be ten power minus three molar, ten power minus four molar and ten power minus two molar, respectively. The pH inside the cell can be taken to be 7, this is a reasonably good assumption.

For example, the pH inside a mammalian cell varies anywhere from about 7.2 to 7.3. Typically, the pH is very well regulated inside the cell, except if there is a drastic change in the function of the cell. For example, when the cell goes into hibernation, which is a very drastic change, then there is a change of about 0.8 to 1 unit in pH. For metabolic activities, there is a change of about 0.2 units, and so on and so forth, as the activity proceeds. And sometimes, it is as high as about 0.5 units. But, for the purposes of this particular problem we will consider the pH inside the cell as a constant.

The standard delta G dash value at physiological conditions, there is a whole meaning attached to this delta G dash value. It is not the same as delta G naught value. I hope you recall that at physiological conditions that it is delta G dash. What is physiological about it? You must be able to recall; it is minus 7.7 kilocalories per mole, and calculate the delta G, the Gibbs free energy change for the hydrolysis of ATP under the above conditions. What I would like you to do is take about twenty minutes to this. It is going to take your time, first to recall whatever is required. It is quite a simple calculation, if you know what is needed. But it is going to take your time to recall those – go back to your notes, check that of the previous courses and so on. So, take about twenty minutes or maybe even twenty, twenty five minutes and then come back with the solution. I will present the detailed solution after you come back. Go ahead, please.

You would have recalled quite a few principles from the earlier course. Let us see what these are or what is required to solve this particular example.

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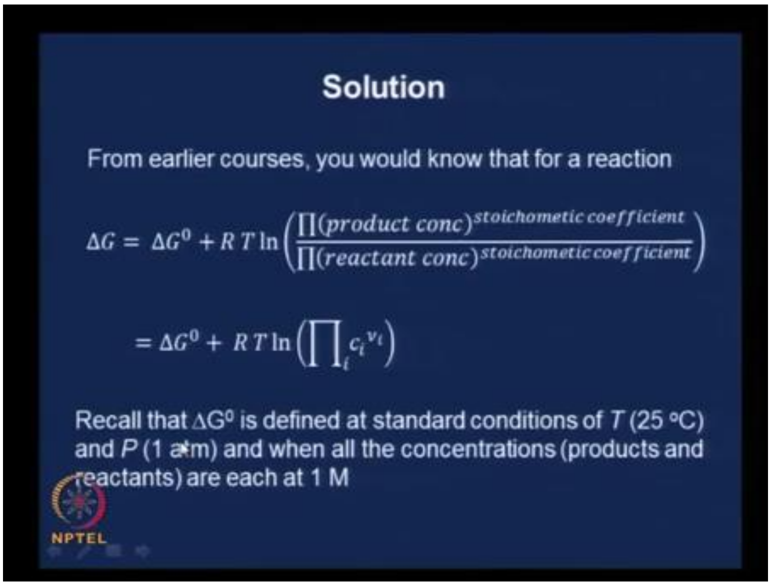

Solution

From earlier courses, you would know that for a reaction

$$\Delta G = \Delta G^0 + R\,T \ln\left(\frac{\prod(product\ conc)^{stoichometic\ coefficient}}{\prod(reactant\ conc)^{stoichometic\ coefficient}}\right)$$

$$= \Delta G^0 + R\,T \ln\left(\prod_i c_i^{\nu_i}\right)$$

Recall that ΔG^0 is defined at standard conditions of T (25 °C) and P (1 atm) and when all the concentrations (products and reactants) are each at 1 M

From earlier courses, we know that for a reaction,

$$\Delta G = \Delta G^0 + R\,T \ln\left(\frac{\prod(product\ conc)^{stoichometic\ coefficient}}{\prod(reactant\ conc)^{stoichometic\ coefficient}}\right)$$

In other words,

$$= \Delta G^0 + R\,T \ln\left(\prod_i c_i^{\nu_i}\right)$$

As you can recognize now, our convention of nu i being positive for products and negative for reactants automatically takes care of this particular formulation here.

Now, recall that delta G naught is defined at standard conditions of temperature, which is twenty five degree C and pressure, which is one atmosphere. And, more importantly in the context of whatever we are going to discuss now, when all the concentrations that is the concentrations of products as well as reactants are each at one molar concentration. hat is how delta G naught is defined. But what is the problem with this? It is perfectly fine for chemical reactions, but what is the problem with this for biological reactions? Would you recall?

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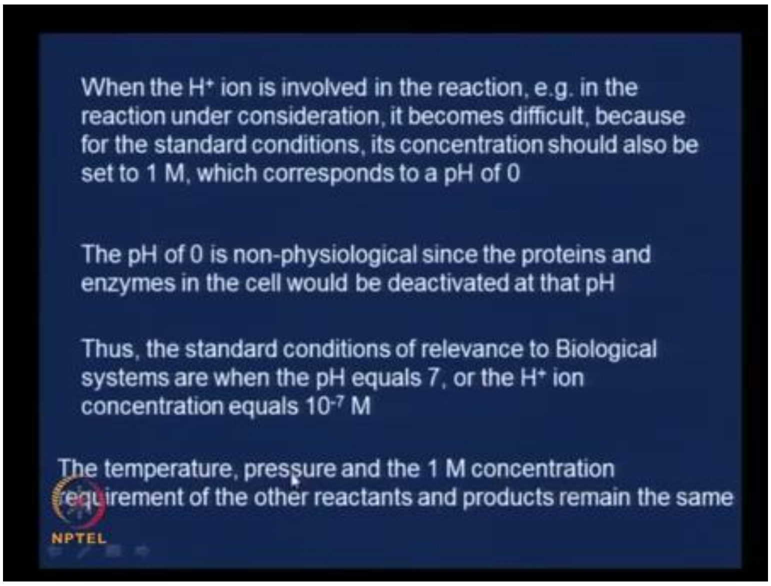

When the H^+ ion is involved in the reaction, e.g. in the reaction under consideration, it becomes difficult, because for the standard conditions, its concentration should also be set to 1 M, which corresponds to a pH of 0

The pH of 0 is non-physiological since the proteins and enzymes in the cell would be deactivated at that pH

Thus, the standard conditions of relevance to Biological systems are when the pH equals 7, or the H^+ ion concentration equals 10^{-7} M

The temperature, pressure and the 1 M concentration requirement of the other reactants and products remain the same

The problem comes about whenever hydrogen ion is involved in a reaction, which is pretty much most of the time, because most reactions that occur in a cell are ionic reactions. So, when the hydrogen ion is involved in the reaction it becomes difficult, because for the standard conditions it is concentration should also be set to one molar. And, what happens when you set the concentration of hydrogen ions to one molar, the pH which is nothing but … the negative log of the hydrogen ion concentration as a first approximation, turns out to be zero.

… The cell is completely gone at a pH of zero, because of the cell typically works at a pH of seven or around seven. And, pH of zero is non-physiological since the proteins and the enzymes in the cell would certainly be deactivated at that pH. And therefore, people came up with a different standard or a different set of standard conditions of relevance to biological systems. … That is, the pH is taken to be equal to seven or the hydrogen ion concentration is taken to be equal to ten power minus seven molar.

Whereas, the other conditions the other standard conditions, the temperature, twenty five degree C, pressure, one atmosphere, and the one molar concentration requirement for all the other reactants and products remain the same. I hope you recall this now. Therefore, you know, this and the hydrogen ion concentration being ten power minus seven molar is the set of standard conditions for biological reactions, which is different from the standard conditions for normal reactions.

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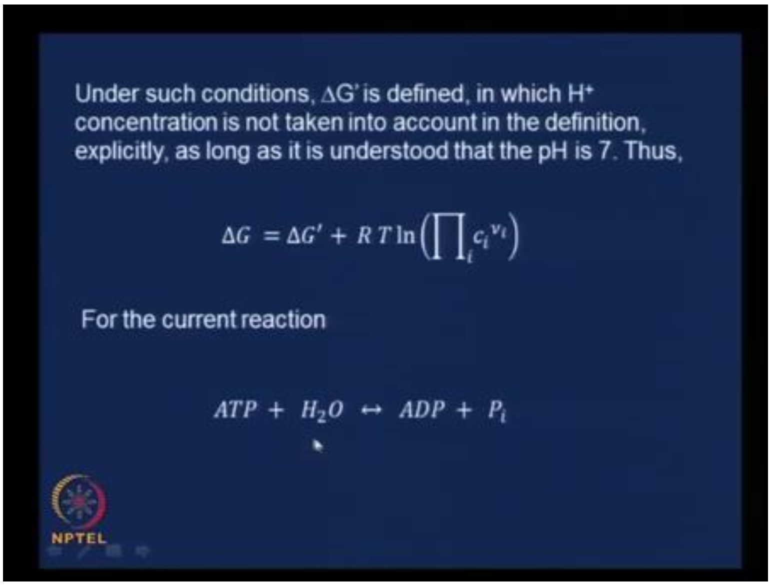

Under such conditions, $\Delta G'$ is defined, in which H^+ concentration is not taken into account in the definition, explicitly, as long as it is understood that the pH is 7. Thus,

$$\Delta G = \Delta G' + R\,T \ln\left(\prod_i c_i^{\nu_i}\right)$$

For the current reaction

$$ATP + H_2O \leftrightarrow ADP + P_i$$

Therefore, we use a different terminology also. Before that, I should say that under such conditions, delta G dash – this is the different terminology that I am talking about – delta G dash is defined instead of delta G naught, we use a delta G dash. This is defined in which the hydrogen ion concentration is not taken into account in the definition explicitly, as long as it is understood that the pH is seven. It is essentially saying that the hydrogen ion concentration is taken as ten power minus seven molar.

Therefore, for our biological reactions, the equation of relevance to use is

$$\Delta G = \Delta G' + R\,T \ln\left(\prod_i c_i^{\nu_i}\right)$$

For the current reaction, which is ATP plus water giving you ADP plus Pi – this is the reaction that we are considering here ATP, ADP and Pi, … the concentrations of which we knew earlier.

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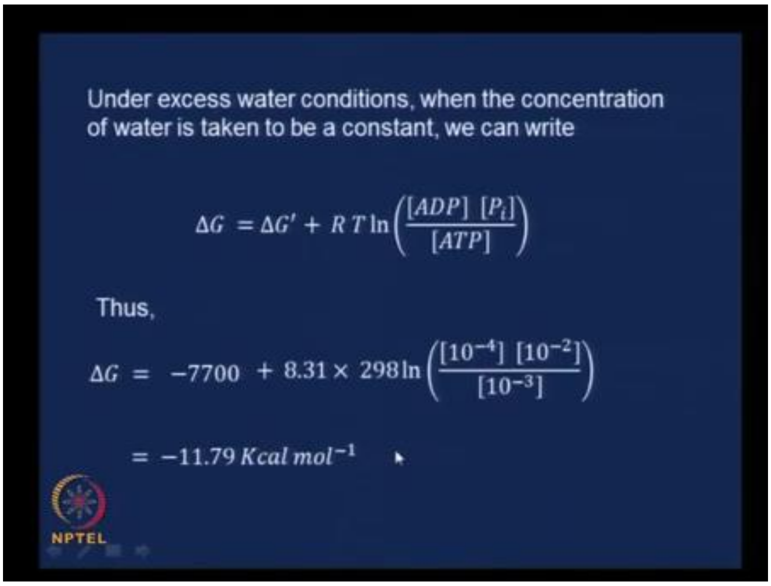

Under excess water conditions, when the concentration of water is taken to be a constant, we can write

$$\Delta G = \Delta G' + R\,T \ln\left(\frac{[ADP]\ [P_i]}{[ATP]}\right)$$

Thus,

$$\Delta G = -7700 + 8.31 \times 298 \ln\left(\frac{[10^{-4}]\ [10^{-2}]}{[10^{-3}]}\right)$$

$$= -11.79\,Kcal\,mol^{-1}$$

Under excess water conditions, when the concentration of water can be taken to be a constant, I hope you recall this trick that we use: we can say

$$\Delta G = \Delta G' + R\,T \ln\left(\frac{[ADP]\ [P_i]}{[ATP]}\right)$$

This is hydrolysis of ATP … so, concentration of the product, ADP, and concentration of the other product, P i, divided by the concentration of ATP. Under the snapshot conditions, that were given

$$\Delta G = -7700 + 1.983 \times 298\, ln\left(\frac{[10^{-4}]\ [10^{-2}]}{[10^{-3}]}\right) = -11.79\,Kcal\,mol^{-1}$$

R has to be in calories, it is not 8.31 it is 1.983. This will turn out to be minus 11.79 kilo calories per mole.

I think we are out of time. I think we need to redo this particular calculation, why do not you redo this particular calculation and actually tell me, because I have taken this to be in kilo calories I should also take the relevant set of units here; just check and tell me whether this value is correct. We will stop here for now. When we come back in the next class, we will work out the other two problems that I mentioned, which also will involve whatever we did in this particular module plus a lot of background information that you need to brush up.