Material and Energy Balances
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Lecture - 68
Unsteady State Material Balances

Hello everybody. Welcome to the last topic as part of this NPTEL course on material and energy balances. Today we will be talking about unsteady state material and energy balances. Until now, all the processes that we have looked at are at steady state. So we have always assumed accumulation to go to 0.

However, under certain special conditions this does not have to be true. So in some cases like batch systems or semi batch systems or continuous systems during startup and shutdown, the processes would be unsteady. So we have to account for the accumulation term as well when we perform material or energy balances.

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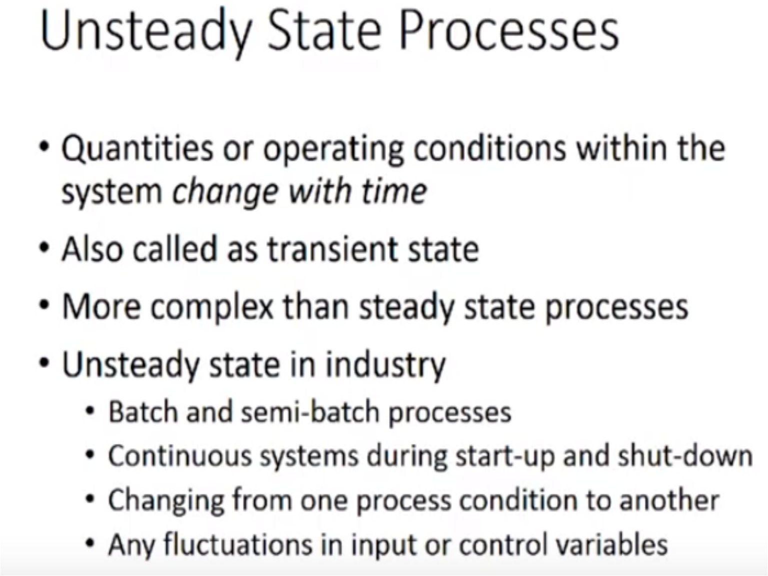

Unsteady State Processes

- Quantities or operating conditions within the system *change with time*
- Also called as transient state
- More complex than steady state processes
- Unsteady state in industry
  - Batch and semi-batch processes
  - Continuous systems during start-up and shut-down
  - Changing from one process condition to another
  - Any fluctuations in input or control variables

Today, we will see how to perform such unsteady state balances. So here what happens is the quantities or the operating conditions within the system will be changing with respect to time. Because these parameters are dependent on time, they are unsteady state. They are also called as transient processes. So these complex systems are actually more tedious to solve compared to steady state process.

Unsteady state processes in the industry could be seen in batch or semi batch processes or continuous systems during the startup and shutdown. So when the system is fully operational, they usually operate at steady state. However, every year or every month depending on the size of the company and the process, the processes might be shut down and started up again. So during this initial phase of startup and final phase of shutdown you would be experiencing unsteady state processes.

There can also be unsteady state when one of the process condition is being changed to another. Also if there are fluctuations in the input or control variables you might experience unsteady state process.

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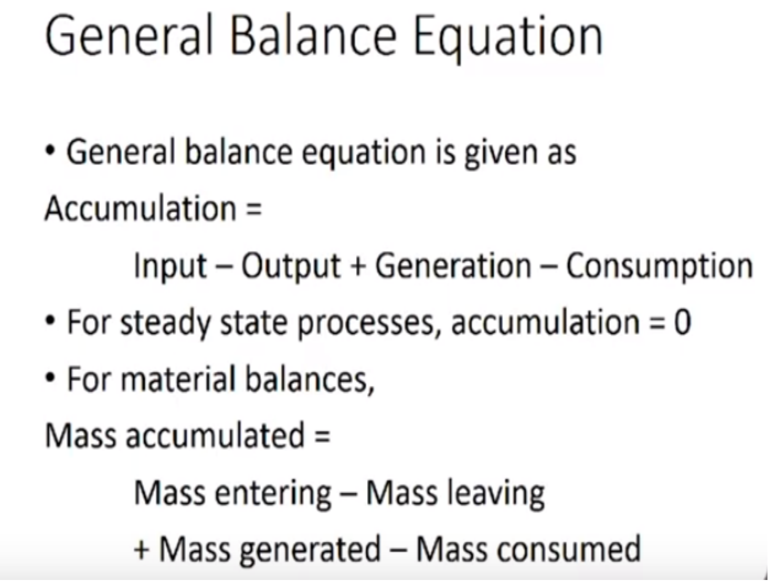

General Balance Equation

- General balance equation is given as

Accumulation =

Input – Output + Generation – Consumption

- For steady state processes, accumulation = 0
- For material balances,

Mass accumulated =

Mass entering – Mass leaving

+ Mass generated – Mass consumed

So before we start solving unsteady state problems let us visit the general balance equation. So the general balance equation which we have been using throughout this course is given here. Accumulation = input – output + generation – consumption. So in this equation we have always assumed accumulation to be 0 when the process is steady state. Until now, we have always ignored that term.

We have only worried about input, output, generation, and consumption. Generation and consumption terms had to be calculated only for reactor processes and if we had total mass balances then generation and consumption went to 0 and also for nonreactive components. Input and output are the terms which we almost always calculated. In some cases, those were also going to 0 if you are talking about close systems, okay?

So now, let us look at how this equation would be for a mass balance. So for material balances, mass accumulated = mass entering – mass leaving + mass generated – mass consumed.

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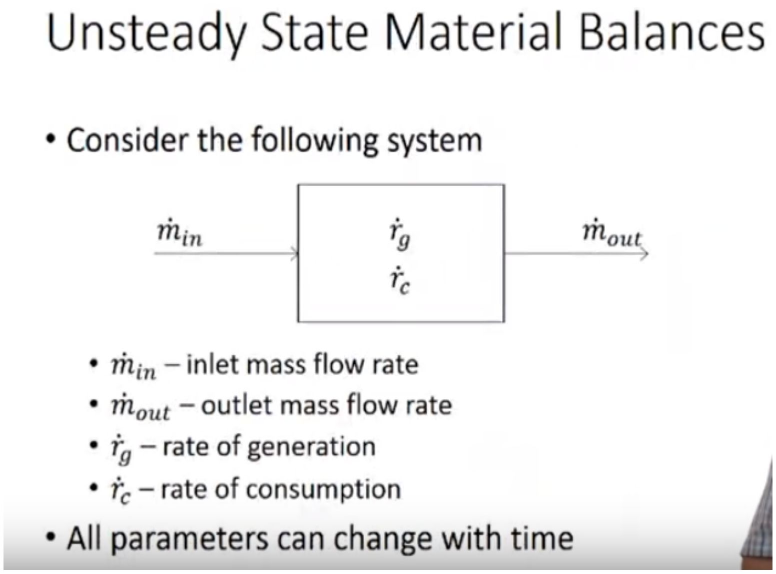

Unsteady State Material Balances

- Consider the following system

  - $\dot{m}_{in}$ – inlet mass flow rate
  - $\dot{m}_{out}$ – outlet mass flow rate
  - $\dot{r}_g$ – rate of generation
  - $\dot{r}_c$ – rate of consumption
- All parameters can change with time

So if we were to take the following system which is a continuous system with m dot in as the inlet mass flow rate m dot out as the outlet mass flow rate, r g is the rate of generation of a component and r c is the rate of consumption of a component, then we can actually write general material balance equation. So in these parameters, until now we have assumed that none of the parameters will change with respect to time.

However, in an unsteady state material balance these parameters can actually change with respect to time.

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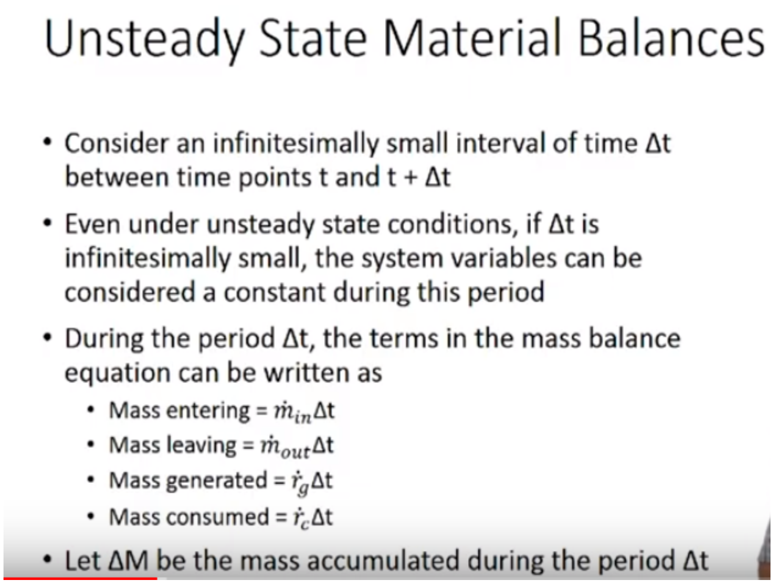

Unsteady State Material Balances

- Consider an infinitesimally small interval of time Δt between time points t and t + Δt
- Even under unsteady state conditions, if Δt is infinitesimally small, the system variables can be considered a constant during this period
- During the period Δt, the terms in the mass balance equation can be written as
  - Mass entering = $\dot{m}_{in}\Delta t$
  - Mass leaving = $\dot{m}_{out}\Delta t$
  - Mass generated = $\dot{r}_g\Delta t$
  - Mass consumed = $\dot{r}_c\Delta t$
- Let ΔM be the mass accumulated during the period Δt

So to account for that our balance equation would have to be considered for an infinitesimally small time interval. So if you take delta t as an infinitesimally small time period between times points t and t + delta t then you can assume that the system variables are a constant for this particular small period. So even for an unsteady state process, we can assume constant system variables for very small time periods.

So using that we would actually be able to calculate the mass entering, mass leaving, mass generated, mass consumed within this time period as the rates time the time. So if we were to calculate mass entering it would be m dot in which is the rate of the mass flow rate for the inlet stream times delta t which is the time period which we are looking for.

So it will be flow rate times time. So that will give you the mass which is entering into the system during the time period delta t. So that is m dot in delta t. Similarly, mass leaving during the period delta t is given as m dot out delta t. Mass generated is given as r dot g delta t and mass consumed is given as r dot c delta t. so now that we have these terms we can plug them into the general material balance equation we have.

However, we still have to account for accumulation. So accumulation is nothing but change in mass over the period of time. So we can assume delta t is the change in mass inside the system during this period delta t. So this delta m is the accumulation.

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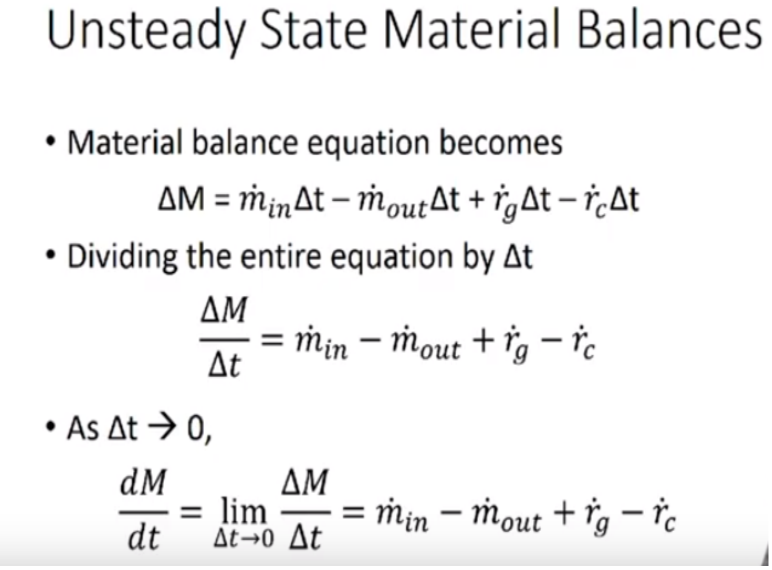

Unsteady State Material Balances

- Material balance equation becomes

$$\Delta M = \dot{m}_{in}\Delta t - \dot{m}_{out}\Delta t + \dot{r}_g\Delta t - \dot{r}_c\Delta t$$

- Dividing the entire equation by Δt

$$\frac{\Delta M}{\Delta t} = \dot{m}_{in} - \dot{m}_{out} + \dot{r}_g - \dot{r}_c$$

- As $\Delta t \to 0$,

$$\frac{dM}{dt} = \lim_{\Delta t \to 0} \frac{\Delta M}{\Delta t} = \dot{m}_{in} - \dot{m}_{out} + \dot{r}_g - \dot{r}_c$$

So substituting all these in the general material balance equation we had, the equation becomes delta M which is accumulation = m dot in delta t the inlet stream – m dot out delta t the mass of the outlet stream + r dot g delta t which is the generation term – r dot c delta t which is the consumption terms. If you were to divide the entire equation by delta t, then you would get delta M/delta t = m dot in – m dot out + r dot g – r dot c.

As delta t tends to 0, basically as delta t becomes smaller and smaller and becomes completely infinitesimally small then this equation can be further modified to dM/dt = limit delta t tends to 0 delta M/ delta t = m dot in – m dot out + r dot g – r dot c.

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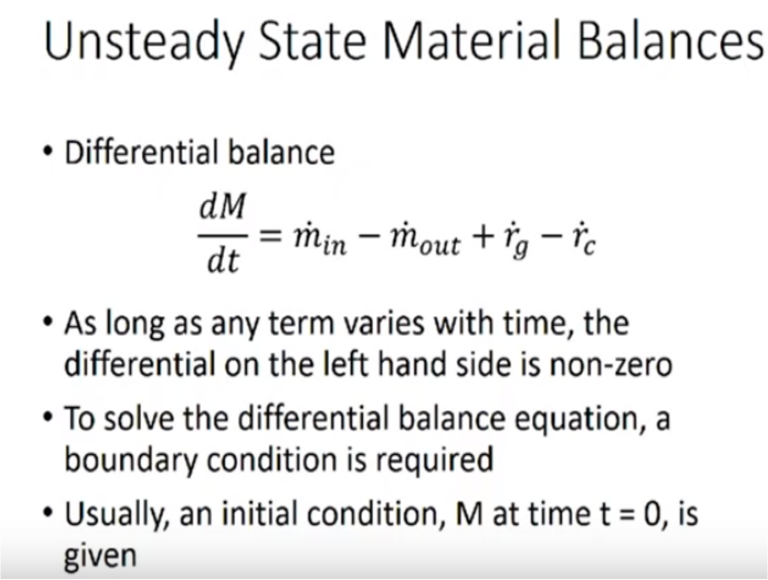

Unsteady State Material Balances

- Differential balance

$$\frac{dM}{dt} = \dot{m}_{in} - \dot{m}_{out} + \dot{r}_g - \dot{r}_c$$

- As long as any term varies with time, the differential on the left hand side is non-zero
- To solve the differential balance equation, a boundary condition is required
- Usually, an initial condition, M at time t = 0, is given

So the differential balance equation for an unsteady state material balance would be dM/dt = m dot in – m dot out + r dot g – r dot c. as long as any of the term varies with time, the differential on the left had side which is dM/dt is non-zero. So if all of them are independent of time, then your accumulation will go to 0 and it becomes a steady state process. For unsteady state processes one of these 4 components which you see on the right hand side input, output, generation or consumption is actually dependent on time.

There could be more which are dependent on time as well. But at least one has to be dependent on time. If that happens then the process is unsteady state and dM/dt cannot go to 0. To solve the differential balance equation, you need a boundary condition because it is a first order differential equation and to solve this you need at least one boundary condition usually an initial condition where M at the time t = 0 is given. So once you have this information you can actually solve the differential equation which is given here.

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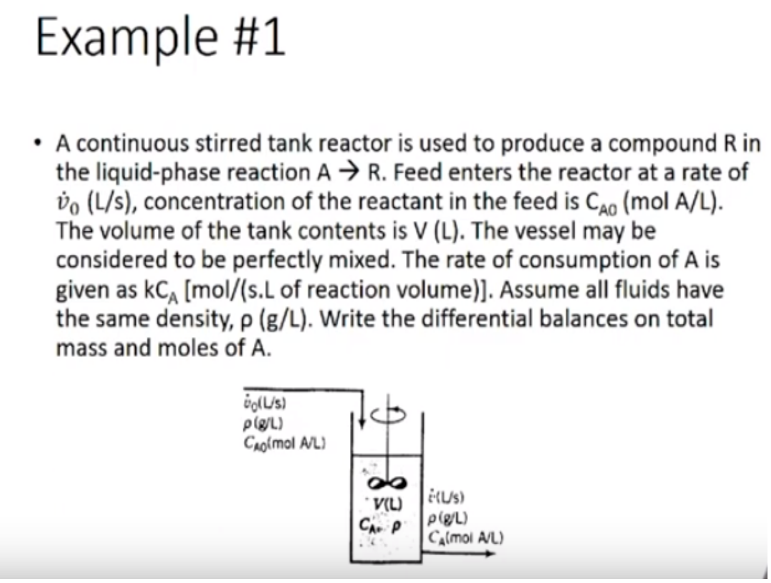

Example #1

- A continuous stirred tank reactor is used to produce a compound R in the liquid-phase reaction A → R. Feed enters the reactor at a rate of $\dot{v}_0$ (L/s), concentration of the reactant in the feed is C_{A0} (mol A/L). The volume of the tank contents is V (L). The vessel may be considered to be perfectly mixed. The rate of consumption of A is given as kC_A [mol/(s.L of reaction volume)]. Assume all fluids have the same density, ρ (g/L). Write the differential balances on total mass and moles of A.

So here is an example problem which will help us construct such differential balances for the total mass and for a component A. So the problem statement says a continuous stirred tank reactor is used to produce a compound R in the liquid phase reaction A gives R. feed enters the reactor at a rate of v naught dot L/s, concentration of the reactant in the feed is C A naught moles of A/L. The volume of the tank contents is V in liters.

The vessel may be considered to be perfectly mixed. So the rate of consumption of A is given as kC A mol/s L where liter is the reaction volume. Assume all fluids have the same density rho. So basically density is not changing. You are asked to write the differential balances on the total mass and moles of A. So let us try to write this differential balance.

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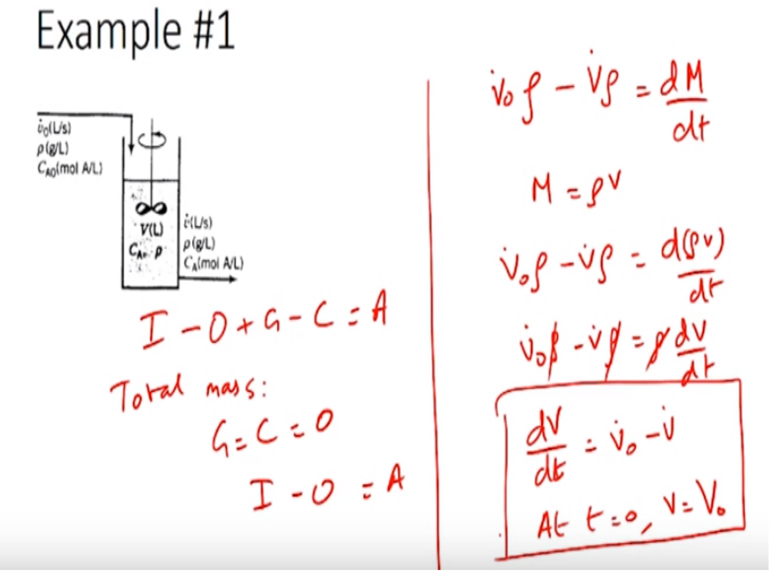

Example #1

$$I - O + G - C = A$$

Total mass:

$$G = C = 0$$

$$I - O = A$$

$$\dot{v}_0 \rho - \dot{v}\rho = \frac{dM}{dt}$$

$$M = \rho V$$

$$\dot{v}_0 \rho - \dot{v}\rho = \frac{d(\rho V)}{dt}$$

$$\dot{v}_0 \rho - \dot{v}\rho = \rho \frac{dV}{dt}$$

$$\boxed{\begin{array}{l} \frac{dV}{dt} = \dot{v}_0 - \dot{v} \\ \text{At } t = 0,\ V = V_0 \end{array}}$$

As we have always done the general balance equation you would have would be input – output + generation – consumption = accumulation. So if we look at this equation we have been asked to write the differential balance for total mass and moles of A. So let us first start with total mass. So for total mass you would have generation and consumption will go to 0 as mass can neither be created nor be destroyed.

So this would mean you are left with input – output = accumulation. So once you have this equation, we now have to account for the mass which is entering into the system and the mass which is leaving the system. So the total mass which is entering the system can be obtained by the volumetric flow rate multiplied with density. So that would be V naught dot times rho giving liters per second times grams per liter which would give a mass flow rate of grams per second.

And the output stream would be V dot times rho. So here V dot is the volumetric flow rate in liters per second and rho is density in grams per liter giving you grams per second, again a mass flow rate. So now the accumulation term has to be accounted for. The accumulation term would be dM/dt. So this M is the total mass of the components inside the reactor. So how do we account for this M?

So that would be equal to rho times V where rho is the density and V is the volume in liters of the reaction mixture inside the reactor. So now we have this. So the equation then becomes V naught dot rho – V dot rho = d rho V/dt. So where rho is actually a constant so it can be taken out of the differential. So it will become rho dV/dt would be = V naught rho – V dot rho. So the rho can be cancelled.

So the equation becomes dV/dt which is the change in volume with respect to time = V naught dot – V dot. So this is the differential equation. From here we have to get, we can solve the equation only if we have the initial condition. So what would be the initial condition? At time t = 0, v should be equation to V naught which is the volume at time t = 0. So this would be the differential equation with the initial condition.

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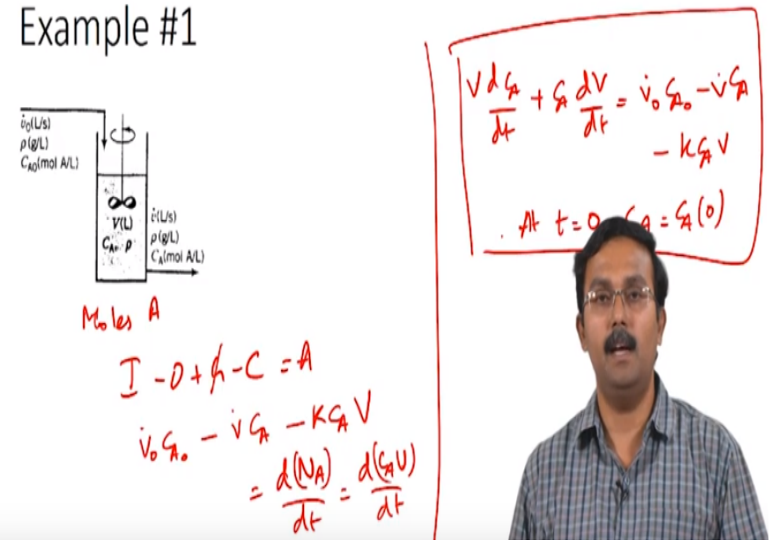

Example #1

Moles A

$$I - O + G - C = A$$

$$\dot{v}_0 C_{A0} - \dot{v} C_A - k C_A V = \frac{d(N_A)}{dt} = \frac{d(C_A V)}{dt}$$

$$\boxed{V\frac{dC_A}{dt} + C_A\frac{dV}{dt} = \dot{v}_0 C_{A0} - \dot{v} C_A - k C_A V}$$

$$At\ t = 0\ C_A = C_A(0)$$

The next aspect of the question is to solve for the differential balance for number of moles of A. So let us try and do that. So we are trying to write a balance for moles of A. So this would have input – output + generation – consumption = accumulation. So here only generation term will go to 0 because A is a reactant and not a product. You have A entering the reactor, so you have an input term. A is leaving the reactor, so you have an output term.

A is being consumed by the reaction, so there would be a consumption term and accumulation would be happening because it is an unsteady state process. So in the input term, we would have to calculate the number of moles of A which is entering the reactor. So earlier we multiplied density with the volumetric flow rate to get the total mass entering the reactor. Here we are interested in knowing the number of moles of A which is entering the reactor.

So which would mean the rate at which the volume is entering, the volumetric flow rate times the concentration can give you the molar flow rate of A into the reactor. So you would have V naught dot times C A naught will give you moles of A per second. Similarly, for the output term, it would be V dot times C A. So the next term would be consumption. For identifying what the consumption would be you would have to go back and see what the rate equation given was.

So the rate equation was given as KC A. So your consumption term becomes K times C A and if you were to look at the units for the consumption term, it was moles of A per second liter. So to convert this into the number of moles of A consumed per second, you would have to multiply this term with the volume of the reactor which is V. So with this you get the consumption term. So the last term you have to look for is the accumulation term.

So the accumulation term would be dN A d t. I am using N instead of M because I am using number of moles. So this number of moles of A can be calculated by multiplying the concentration of A inside the reactor times the volume. So this would be d C A times V dt. So what you would notice is both for the consumption term and for the accumulation term I have used C A which is the concentration of A which is leaving the reactor.

So this is because we have been told that the system is perfectly mixed. That means that the concentration of the outlet stream would be equal to the concentrations in the system. So that is why we are using C A which is the concentration of the exit stream for A as the concentration of A inside the reactor as well. So now that we have this, the equation can further be simplified to get V d C A/dt + C A dV/dt = V naught dot C A naught – V dot C A – K C A V.

So this would be the differential equation. Now we also need to write what would be the initial condition. So at time = 0, we do not know what the concentration of A would have been. So we can just say C = C A of time 0. So this would be the initial condition. With this we have the differential balance equation for number of moles of A and the initial condition.

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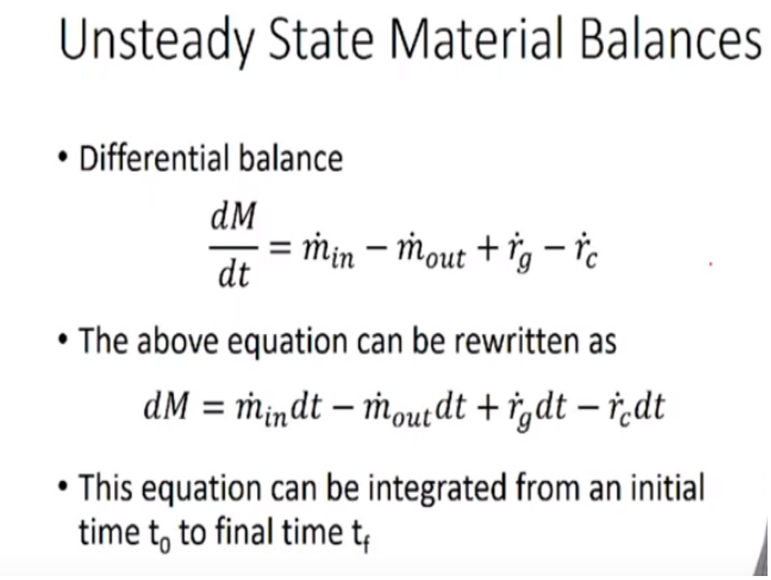

Unsteady State Material Balances

- Differential balance

$$\frac{dM}{dt} = \dot{m}_{in} - \dot{m}_{out} + \dot{r}_g - \dot{r}_c$$

- The above equation can be rewritten as

$$dM = \dot{m}_{in}dt - \dot{m}_{out}dt + \dot{r}_g dt - \dot{r}_c dt$$

- This equation can be integrated from an initial time t_0 to final time t_f

So in the previous example, we looked at how to write the differential balance for an unsteady state process. So this was the differential balance which we wrote. So to solve this differential balance we would have to rearrange this equation and integrate it. So the above equation can actually be rewritten as dM = m dot in dt – m dot out dt + r dot g dt – r dot c dt. So this can then be integrated between the limits of initial times t naught and final time t f and when you do this integration, you will be able to solve the equation.

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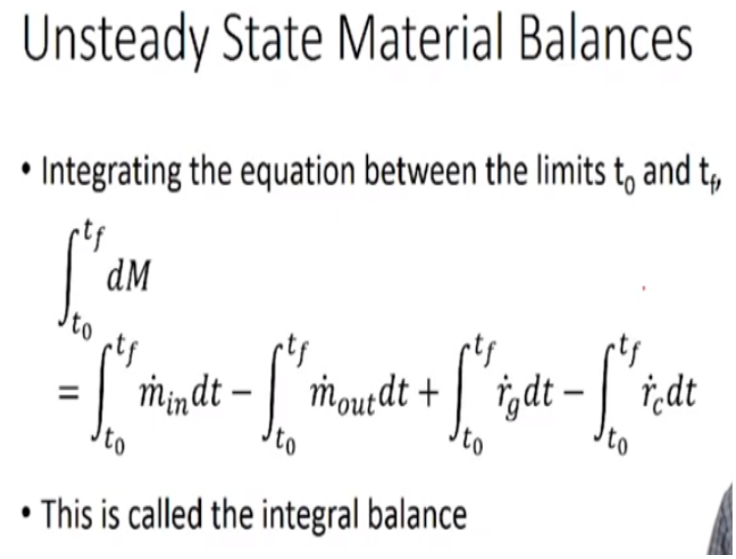

Unsteady State Material Balances

- Integrating the equation between the limits t_0 and t_f,

$$\int_{t_0}^{t_f} dM$$

$$= \int_{t_0}^{t_f} \dot{m}_{in}dt - \int_{t_0}^{t_f} \dot{m}_{out}dt + \int_{t_0}^{t_f} \dot{r}_g dt - \int_{t_0}^{t_f} \dot{r}_c dt$$

- This is called the integral balance

So while you are integrating the equation between the limits t naught to t f you will get this equation which can then be simply called as the integral balance. So once you know how these terms are actually changing with respect to time, you can plug in these functions into this equation and solve for the unknown. So this is called as an integral balance. Let us try an example problem which will help us practice how to get integral balances and solve for the desired values.

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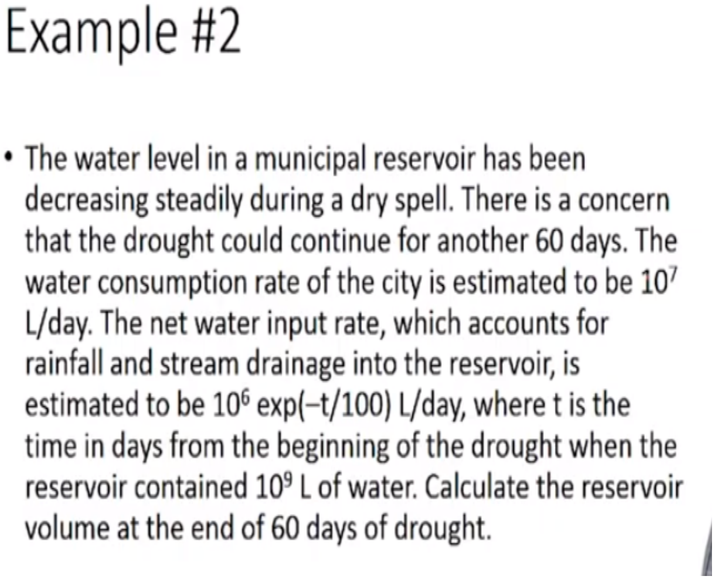

Example #2

- The water level in a municipal reservoir has been decreasing steadily during a dry spell. There is a concern that the drought could continue for another 60 days. The water consumption rate of the city is estimated to be 10^7 L/day. The net water input rate, which accounts for rainfall and stream drainage into the reservoir, is estimated to be $10^6 \exp(-t/100)$ L/day, where t is the time in days from the beginning of the drought when the reservoir contained 10^9 L of water. Calculate the reservoir volume at the end of 60 days of drought.

So here is an example. The water level in a municipal reservoir has been decreasing steadily during a dry spell. There is a concern that the drought could continue for another 60 days. The water consumption rate of the city is estimated to be 10 power 7 L/day. The net water input rate which accounts for rainfall and stream drainage into the reservoir is estimated to be 10 power 6 times exponential of –t/100 L/day where t is the time in days from the beginning of the drought when the reservoir contained 10 power 9 L of water.

Calculate the reservoir volume at the end of 60 days of drought. So how would we go about solving this problem?

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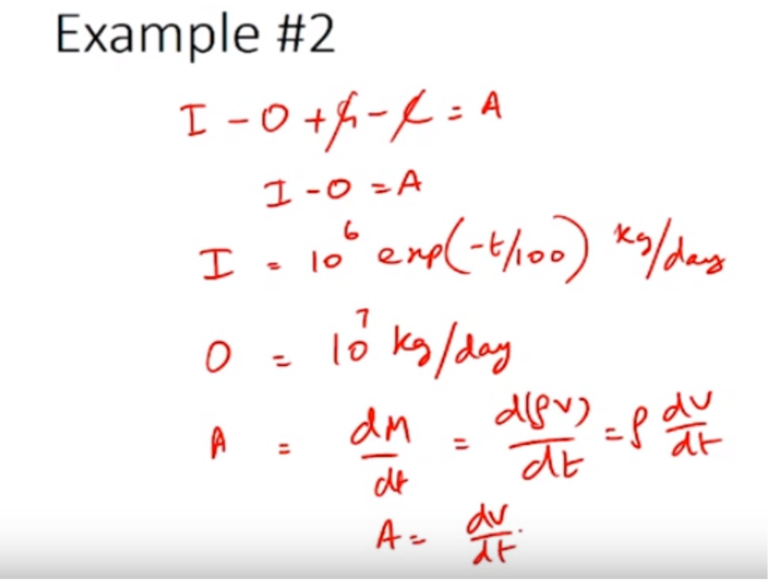

Example #2

$$I - O + \cancel{G} - \cancel{C} = A$$

$$I - O = A$$

$$I = 10^{6} \exp(-t/100) \text{ kg/day}$$

$$O = 10^{7} \text{ kg/day}$$

$$A = \frac{dM}{dt} = \frac{d(\rho v)}{dt} = \rho \frac{dv}{dt}$$

$$A = \frac{dv}{dt}$$

So first we would have to write the general balance equation. We will have to write the differential balance and then rearrange it, integrate it to get the final values which is the final volume of the reservoir after 60 days of drought. So let us try and do this. So as usual, we would start with the general balance equation, general material balance equation which is input – output + generation – consumption = accumulation.

So generation and consumptions terms will go to 0 because there is no reaction happening. So you would end up with input – output = accumulation. So the input term has been given as 10 power 6 times exponential of –t/100 and this is given in terms of liters per day. To convert it into mass you would have to multiply it with density because this is water we know the density which is 1 kg/L.

So multiplying this with 1 kg/L as the density you would get this input in terms of kg/day. Similarly, output stream can be calculated. So it is 10 power 7 L/day converting into mass you would get 10 power 7 kg/day. So the accumulation term has to be accounted for. The accumulation term is dM/dt. So this dM/dt can be written as d rho v/dt where rho is the density and v is the volume.

So this will become rho dv/dt and because rho = 1 kg/L the equation term for accumulation would be dv/dt.

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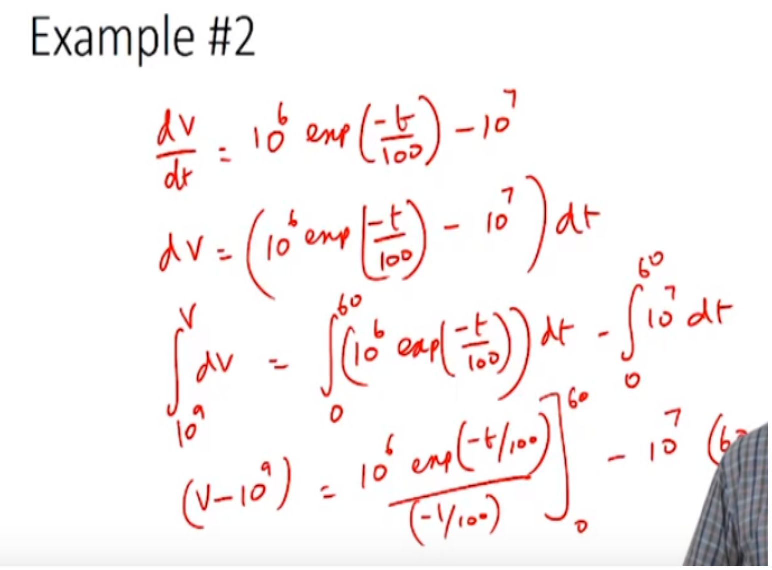

Example #2

$$\frac{dV}{dt} = 10^6 \exp\left(\frac{-t}{100}\right) - 10^7$$

$$dV = \left(10^6 \exp\left(\frac{-t}{100}\right) - 10^7\right) dt$$

$$\int_{10^9}^{V} dV = \int_0^{60} \left(10^6 \exp\left(\frac{-t}{100}\right)\right) dt - \int_0^{60} 10^7 \, dt$$

$$(V - 10^9) = \left. 10^6 \frac{\exp(-t/100)}{(-1/100)} \right]_0^{60} - 10^7 \, (6$$

Plugging these values back into the equation you would get dV/dt = 10 power 6 exponential of – t/100 – 10 power 7. So this equation has to be solved. So rearranging this equation, you would get dV = 10 power 6 exponential of –t/100 – 10 power 7 times dt. So integrating this equation you would get integral dV = integral 10 power 6 exponential of –t/100 dt – integral 10 power 7 dt. So now, what would be the limits for this differential?

So we know the initial volume of the reservoir before the drought started was 10 power 9. We have to find the final volume. Let us call that V. So we know that the drought lasted for 60 days and which means that the day in which the drought started can be considered to be time = 0 and the last day of drought would be 60. So the time frame is from 0 to 60. So this would be the limit with which we would have to integrate this.

So this would mean the equation becomes V – 10 power 9 = 10 power 6 exponential of – t/100/-1/100 with the limits of 0 to 60 – 10 power 7 60 – 0. So solving this equation so we can actually substitute the values of t from the limits.

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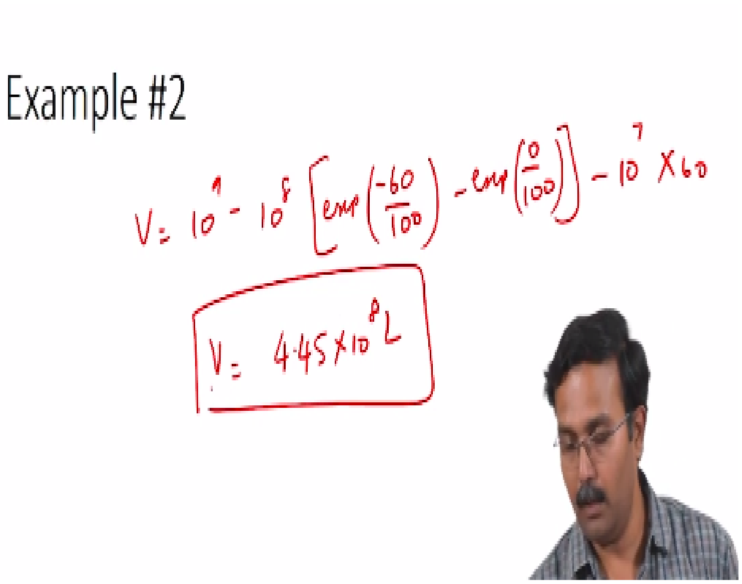

Example #2

$$V = 10^9 - 10^8 \left[\exp\left(\frac{-60}{100}\right) - \exp\left(\frac{0}{100}\right) \right] - 10^7 \times 60$$

$$\boxed{V = 4.45 \times 10^8 L}$$

By applying the limits and solving the given equation we can get V = 10 power 9 minus 10 power 8 times exponential of – 60/100 – exponential of 0/100 – 10 power 7 times 60. So you can solve this equation and you will be able to get the value for V as 4.45 times 10 power 8 L. So this would be the final volume of the reservoir after 60 days of drought.

As you can see from these calculations when we did the differential balance we were able to get an equation which can actually explain the correlation between the terms which are changing with respect to time and accumulation. When we actually use the integral balance we can integrate and get numerical values based on the limits that we apply and the initial conditions that are provided. So we will move on to the next concept.

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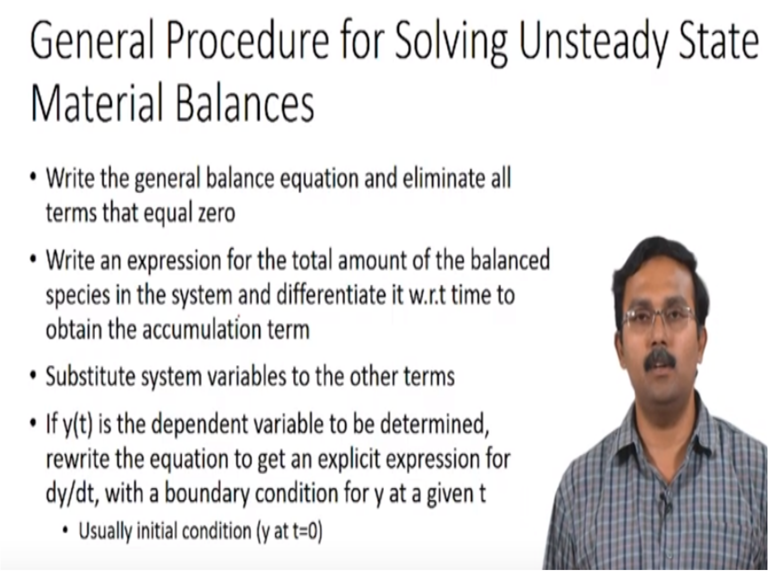

General Procedure for Solving Unsteady State Material Balances

- Write the general balance equation and eliminate all terms that equal zero
- Write an expression for the total amount of the balanced species in the system and differentiate it w.r.t time to obtain the accumulation term
- Substitute system variables to the other terms
- If y(t) is the dependent variable to be determined, rewrite the equation to get an explicit expression for dy/dt, with a boundary condition for y at a given t
    - Usually initial condition (y at t=0)

So when we are solving unsteady standard material balances there is a general procedure which we should follow. So we will go through that. The first step is like for any other material balance problem you would write the general balance equation and eliminate terms that are equal to 0. The next thing is you would write an expression for the total amount of the balance species in the system and differentiate it with respect to time to get the accumulation term.

So for example when we first did the total mass we did dM/dt whereas in the second part of the first example we were looking to write the balance equation for number of moles of A so we wrote d N A/dt. So basically what you have done is you have written the total amount of the balance species in the system and you have differentiated it with respect to time and this becomes your accumulation term.

You are asked to substitute the system variables to the other terms basically the input, output generation and consumption terms need to be substituted based on information which is available. If there is a function of time, if something is actually y of t is a dependent variable to be determined then you rewrite the equation to get an explicit expression for dy/dt with a boundary condition for y at given t. So usually initial condition which is y equals something at t = 0 is provided to u.

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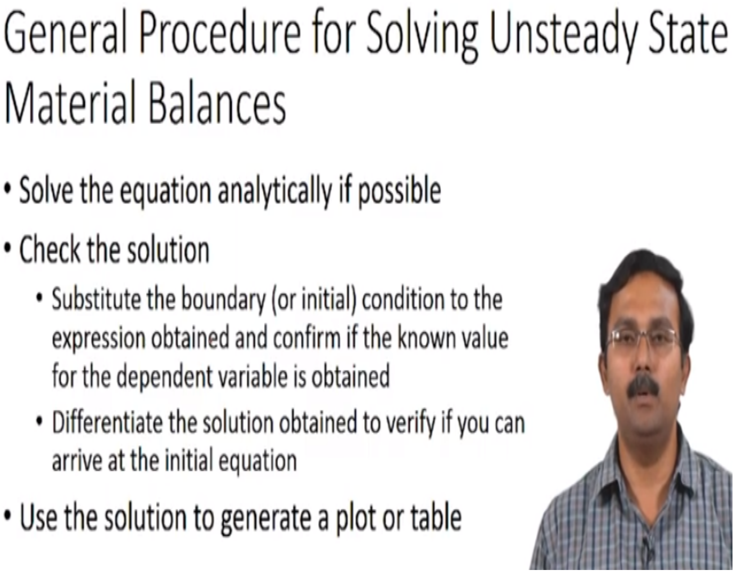

General Procedure for Solving Unsteady State Material Balances

- Solve the equation analytically if possible
- Check the solution
  - Substitute the boundary (or initial) condition to the expression obtained and confirm if the known value for the dependent variable is obtained
  - Differentiate the solution obtained to verify if you can arrive at the initial equation
- Use the solution to generate a plot or table

So then the last step is to solve the equation analytically if possible. So if not you would use numerical methods to solve it. So you can also check the solution. So here if you were to check the solution what you would do is you can substitute the boundary or the initial condition to the expression obtained to confirm if the known value of the dependent variable is obtained when you actually do this.

Then another method can also be employed to check the solution which is you can differentiate the solution obtained to verify if you can arrive at the initial equation. By doing these kinds of verification you can confirm if the solution you arrived at is accurate. Last step if required might be to plot the solution using a graph or to build a table which will give you the values.

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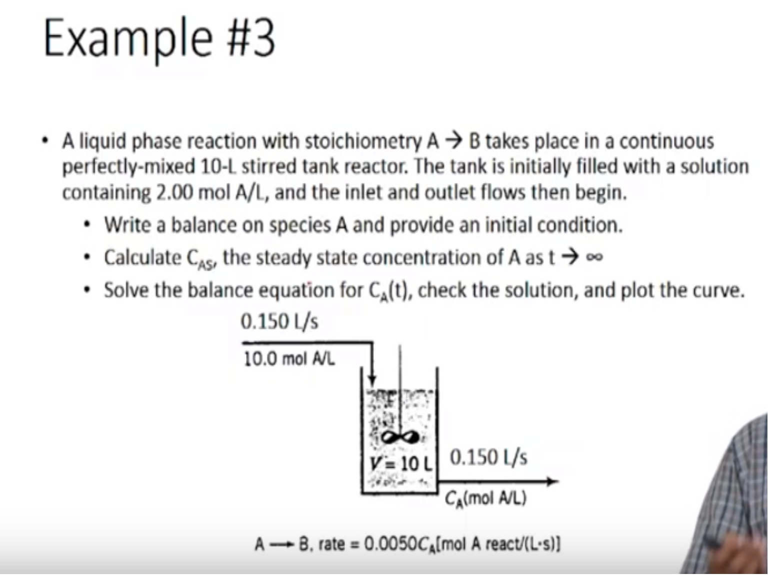

Example #3

- A liquid phase reaction with stoichiometry A → B takes place in a continuous perfectly-mixed 10-L stirred tank reactor. The tank is initially filled with a solution containing 2.00 mol A/L, and the inlet and outlet flows then begin.
  - Write a balance on species A and provide an initial condition.
  - Calculate C_{AS}, the steady state concentration of A as t → ∞
  - Solve the balance equation for $C_A(t)$, check the solution, and plot the curve.

A ⟶ B, rate = $0.0050C_A$[mol A react/(L·s)]

Here is one last example for unsteady state material balances. We will try to solve this to get clarity on the procedure which we just described. A liquid phase reaction with stoichiometry A gives B takes place in a continuous perfectly mixed 10 L stirred tank reactor. The tank is initially filled with a solution containing 2 moles of A/L and the inlet and the outlet flows then begin. Write a balance on species A and provide an initial condition.

Calculate C AS the steady state concentration as t tends to infinity. Solve the balance equation for C of A and check the solution and plot the curve. So here we are expected to first write the differential balance and then apply the limit of t tends to infinity to get what would be the concentration of the substance A which would be the steady state concentration and then the last objective is to develop an equation which would represent concentration of A as a function of time and then we have to plot a curve. So let us see how to go about solving this problem.

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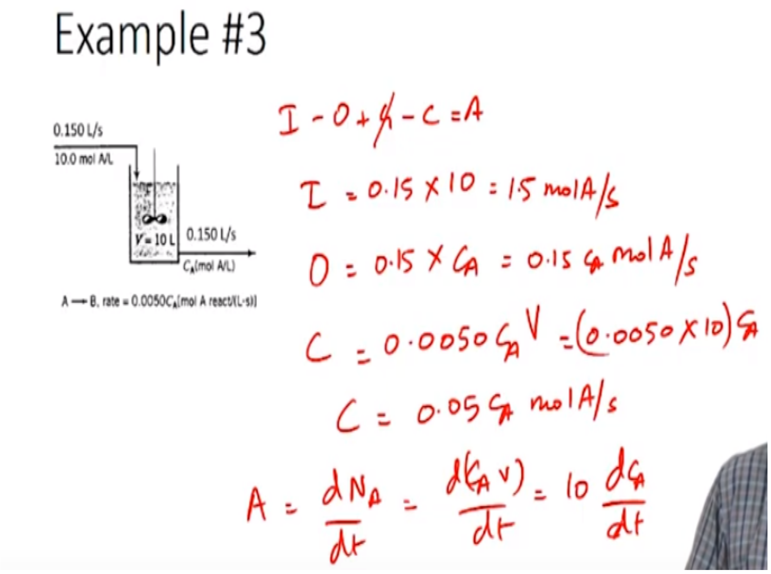

Example #3

$I - O + \cancel{G} - C = A$

$I = 0.15 \times 10 = 1.5 \text{ mol A/s}$

$O = 0.15 \times C_A = 0.15\, C_A \text{ mol A/s}$

$C = 0.0050\, C_A V = (0.0050 \times 10)\, C_A$

$C = 0.05\, C_A \text{ mol A/s}$

$$A = \frac{dN_A}{dt} = \frac{d(C_A V)}{dt} = 10\frac{dC_A}{dt}$$

As usual we start with the general balance equation which is input – output + generation – consumption = accumulation. So here you do not have a generation term for component A. However, you would have a consumption term. You do have input and output and accumulation terms. So the input term would be equal to 0.15 times 10 because 1.15 is the volumetric flow rate, 10 is the concentration of A in terms of moles per liter.

So doing this multiplication you would get moles of A per second which is the molar flow rate of A entering into the system. So this would become 1.5 moles of A/s. so output would be 0.15 times C A. We do not know the outlet concentration. So we will just call it C A as given in the problem. So it will be 0.15 C A would be the moles of A in the outlet stream per second. So consumption term has been given as 0.0050 C A which is the rate equation.

And as we had discussed already this has to be multiplied with the volume of the reactor to get the moles of A which is consumed within the reactor per unit time. So this would give 0.0050 times 10 C A. So your consumption term is actually equal to 0.05 C A. It should again be in terms of moles of A/s. accumulation term as we had already described would basically be d times number of moles of A dt.

So it is the differential with respect to time and this N which is the number of moles of A can be written as the number of moles of A which is present inside the volume would be volume of the reactor would be d times C A times V dt. C A is the concentration of A inside the reactor and V is the volume. Again we are using C A in both consumption and in accumulation because the system is perfectly mixed.

As the inlet and the outlet flow rates are equal, the volume of the reactor will not be changing and the volume of the reactor has been given as 10 L. So this d C A V/dt can be written as 10 times d C A/dt. So this would be the equation for accumulation.

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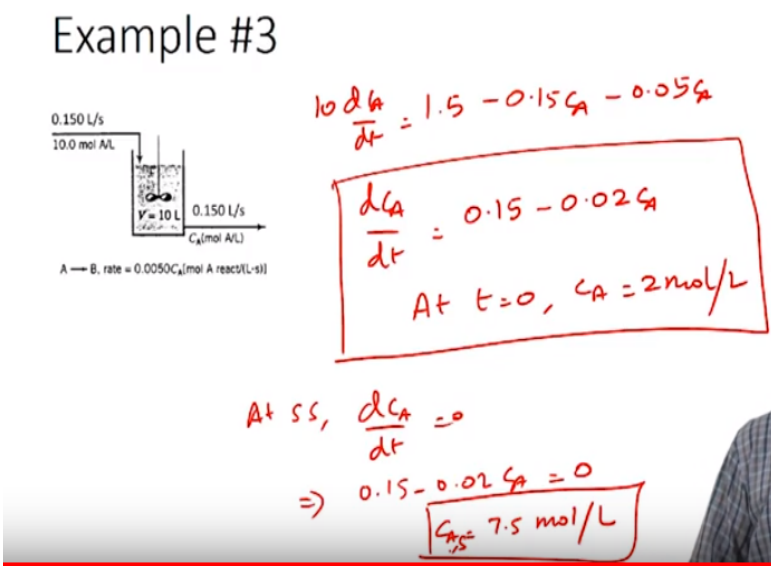

Example #3

A ⟶ B, rate = $0.0050C_A$[mol A react/(L-s)]

$$10\frac{dC_A}{dt} = 1.5 - 0.15C_A - 0.05C_A$$

$$\boxed{\frac{dC_A}{dt} = 0.15 - 0.02C_A \quad \text{At } t=0,\ C_A = 2\ \text{mol/L}}$$

$$\text{At SS, } \frac{dC_A}{dt} = 0$$

$$\Rightarrow 0.15 - 0.02C_A = 0$$

$$\boxed{C_{A,S} = 7.5\ \text{mol/L}}$$

Now substituting all these back into the general material balance equation we had we would get 10 d C A/dt = 1.5 – 0.15 C A – 0.05 C A where 10 times d C A/dt is the accumulation term, 1.5 is your inlet term and 0.15 C A is your outlet term and 0.05 C A is the consumption term. So this equation becomes d C A/dt = 0.15 – 0.02 C A. So we also need to identify what would be the initial condition.

So it has been given that at time t = 0 the reactor is charged with a concentration of 2 mol/L. So the concentration of A in the reactor at time t = 0 is given as 2 mol/L. So this would be the differential balance equation and the initial condition which can be used to solve this problem. So the next part of the problem asks us to calculate the value of C A,s which would be the steady state concentration of A. So at steady state what would happen? d C A/dt will be = 0.

So substituting this back into this equation you would have 0.15 – 0.02 C A = 0. So that would give C A = 7.5 mol/L. So that would be the steady state concentration of A when time tends to infinity.

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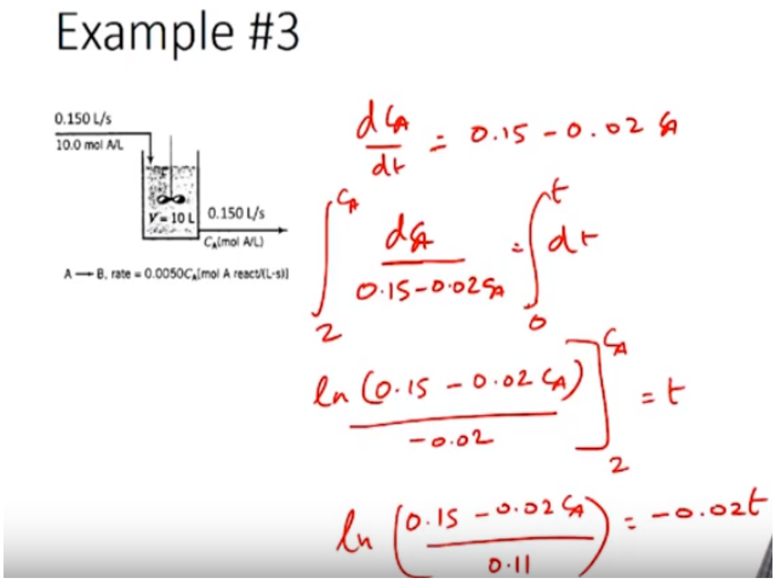

Example #3

$$\frac{dC_A}{dt} = 0.15 - 0.02\,C_A$$

$$\int_{2}^{C_A} \frac{dC_A}{0.15-0.02C_A} = \int_{0}^{t} dt$$

$$\left.\frac{\ln(0.15-0.02\,C_A)}{-0.02}\right]_{2}^{C_A} = t$$

$$\ln\left(\frac{0.15-0.02\,C_A}{0.11}\right) = -0.02t$$

For the last part of the problem, we have been asked to calculate, we have been asked to write C A as a function of time. So that can be obtained by integrating the differential equation. So we had d C A/dt = 0.15 – 0.02 C A. So this would be d C A/0.15 – 0.02 C A = dt. So integrating this equation, we would be able to solve for the required correlation. So what would be the initial and final condition? What would be the limits?

So C A is basically from 2 moles which would be the initial condition. And final concentration is C A which is the outlet stream concentration or the concentration inside the reactor at time t and similarly time would be from 0 to t. So because we want to start from time 0 where concentration of A is 2 mol/L and go to time t where the concentration of A is C A. So using this equation we can actually integrate and get this as the solution. You would have ln(0.15 – 0.02 C A)/ - 0.02 with limits of 2 to C A = t. So this can then be further solved to get ln (0.15 – 0.02 C A)/0.11 = -0.02 t.

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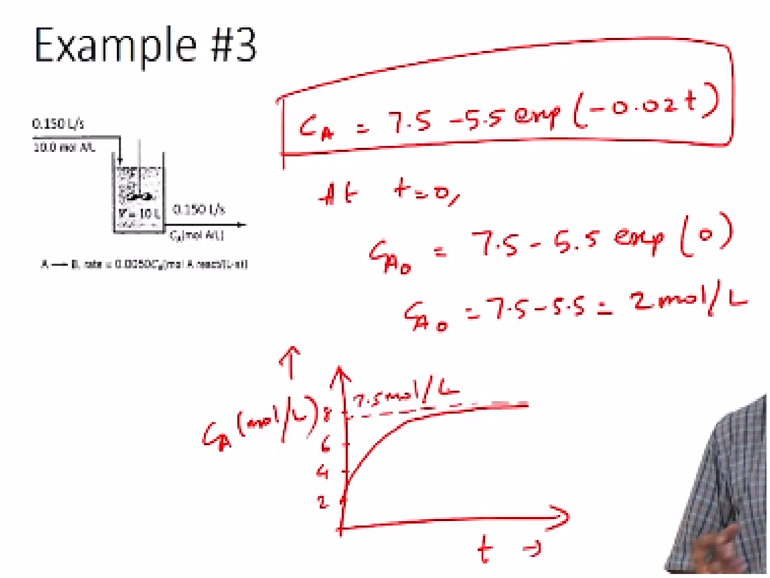

Example #3

$$\boxed{C_A = 7.5 - 5.5\,\mathrm{exp}(-0.02t)}$$

At $t = 0$,

$$C_{A_0} = 7.5 - 5.5\,\mathrm{exp}(0)$$

$$C_{A_0} = 7.5 - 5.5 = 2\ \mathrm{mol/L}$$

So from the previous equation we can get C A as C A = 7.5 – 5.5 times exponential of -0.02 t. So we can verify this as well. All we need to do is differentiate this equation or substitute the initial condition. So let us try substituting the initial condition. The initial condition was at time t = 0 C A was 2. So let us try to do that. At t = 0 what would happen? You would have C A naught would be = 7.5 – 5.5 times exponential of 0.

So this would make the equation to be C A naught = 7.5 – 5.5 which is = 2 mol/L. So we are able to verify and get the correct answer so that it means that the equation we have which is the correlation for C A as a function of time is accurate. So the last part of this problem is to plot the curve. So let us try to plot the curve. So this would be C A in terms of mol/L and this would be time as time goes here. So let us mark the points. 2, 4, 6, and 8. So at time t = 0, C A = 2.

So from here if you were to plot the curve, it would look something like this where it plateaus off at 7.5 mol/L. So that is the maximum concentration you would be able to get because that is the steady state concentration which we calculated at time t equals infinity. So with this we have solved an unsteady state material balance problem. So we will continue in the next section where we talk about unsteady state energy balances.