Material and Energy Balances
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Lecture - 65
Energy Balances: A Review - Part 2

Welcome back to the review of energy balances. We looked at all the fundamentals. We looked at the terminologies which are used in energy balances. We also looked at the procedures, objectives and all the fundamental concepts like how to account for different things like phase change. We also looked at how solutions and reactions are uniquely treated. We looked at all the basics which are required for performing energy balances.

So this review gave a short and a brief explanation on how energy balance problems can be done. So now we will try to apply these principles and perform a few example problems. So these problems have been chosen so that they are reasonably complex. So this gives a perspective of how energy balances can be done for complicated systems and we will be looking at these problems which are probably a lot more intense than the example problems we have chosen.

So this session is for us to get a feel for how energy balance problems can actually be approached when you have reasonably complicated systems. So we will talk about 3 different example problems. So the first example problem which we are going to start with is here.

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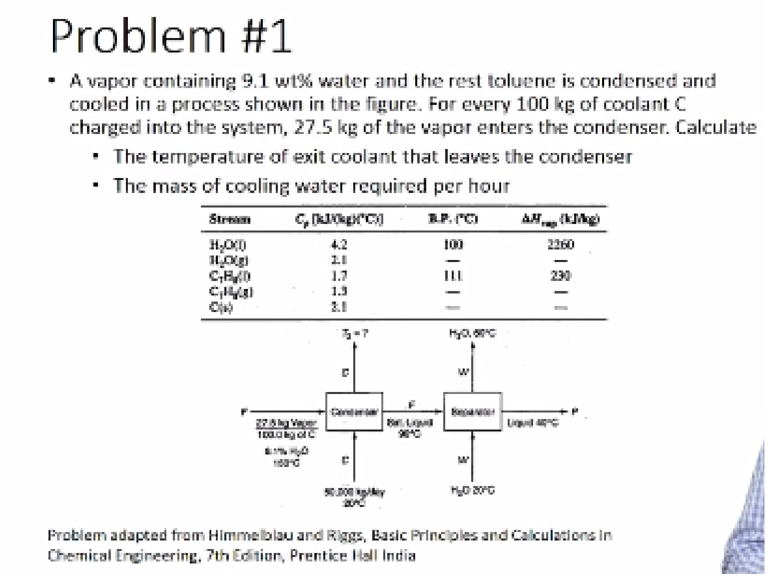

Problem #1

- A vapor containing 9.1 wt% water and the rest toluene is condensed and cooled in a process shown in the figure. For every 100 kg of coolant C charged into the system, 27.5 kg of the vapor enters the condenser. Calculate
  - The temperature of exit coolant that leaves the condenser
  - The mass of cooling water required per hour

Stream	C_p [kJ/(kg)(°C)]	B.P. (°C)	ΔH_{vap} (kJ/kg)
$H_2O(l)$	4.2	100	2260
$H_2O(g)$	2.1	—	—
$C_7H_8(l)$	1.7	111	230
$C_7H_8(g)$	1.3	—	—
C(s)	2.1	—	—

Problem adapted from Himmelblau and Riggs, Basic Principles and Calculations in Chemical Engineering, 7th Edition, Prentice Hall India

A vapor containing 9.1 weight percent water and the rest toluene is condensed and cooled in a process shown in the figure. For every 100 kg of coolant C charged into the system, 27.5 kg of vapor enters the condenser. You are asked to calculate the temperature of the exit coolant that leaves the condenser, the mass of the cooling water required per hour. So these two things have to be calculated.

So the problem also gives us the data for C p boiling point and the heat of vaporization for different components. So you have water liquid with a C p of 4.2 at a boiling point of 100 degree Celsius and heat of vaporization as 2260 kJ/kg. So you have water vapor where the C p is given as 2.1 kJ/kg degree Celsius and C7H8 is toluene. For toluene liquid and gas you have the C p values given and for the so toluene liquid you also have the boiling point and the heat of vaporization which has been given.

So C is the coolant which is being used in the first unit which is the condenser. So the C p for that is also given. So now look at the process. You actually have 2 units in this process. So until now we have always been using a single unit system when we performed energy balances. Primarily we have focused energy balances done only on one unit even when there are multiple units in some examples.

We had energy balances in one and the other one was only material balance and so on. So here these are 2 systems where both are involved in energy balance operations. So we will look at this process and what you see here is a feed which contains 9.1% water. It is a vapor feed containing 9.1% water which is entering at 150 degree Celsius. And you have a coolant which is coming in at 50,000 kg per day and 20 degree Celsius and entering the condenser.

So the vapor gets cooled because of the action of the coolant and you have a saturated liquid containing both water and toluene which is coming into the stream F which is leaving the condenser. The coolant also leaves the condenser at a temperature of T and the saturated liquid then enters into a separator where it is actually coming in contact with the liquid water at 20 degree Celsius and water leaves the system as liquid water at 60 degree Celsius.

The saturated liquid is further cooled to get a liquid which is at 40 degree Celsius. So this would be the liquid which contains both water and toluene. So as you can see the boiling point of water is 100 degree Celsius and boiling point of toluene is 111 degree Celsius. So in both cases, it is actually going to be only liquid and we are not worried about the pressure here.

For vapor phase we will assume ideal gas behavior and for liquid phase we will ignore the effect of change in pressure because the change in pressure for such process here would be very small and the effect on enthalpy for such small changes in pressure can be ignored. So taking that into account this is the flow chart we have and one thing we need to be clear about is what is the heat which is being transferred between the system we are interested in and the surroundings?

So we can have different systems which we can choose here. So we could either choose the condenser as a system or the separator as the system or the overall system. These are the 3 possible systems that can be chosen and for each of these we would have to understand that there is no heat which is going to be lost between the system and the surroundings. Because here you have not been given any heat which is being transferred to or from.

So whatever heat is being transferred for example in the condenser would be only the heat transferred between the coolant and the vapor. So heat is lost by the vapor which is coming in and gained by the coolant which is coming in. Similarly, in the separator as well you would have heat lost by the saturated liquid. Therefore it is leaving at a lower temperature and heat is gained by water and it is leaving at a higher temperature.

So this is an assumption we can make based on the system we have. So each of these do not have any heat lost to the surrounding which is outside the system. Heat transfer is happening only between the coolant and the vapor or coolant and the liquid. So in the second case, the coolant is the water. So with this information let us now look at solving this problem.

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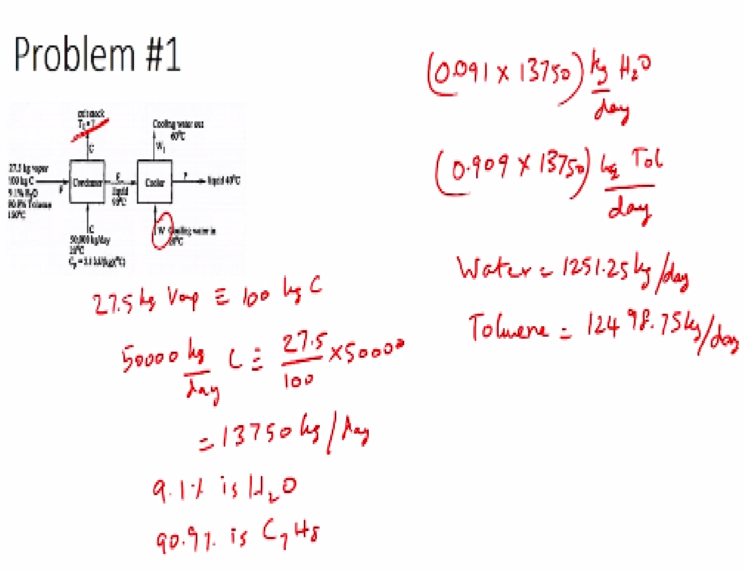

Problem #1

27.5 kg Vap ≡ 100 kg C

$50000 \frac{kg}{day}$ C ≡ $\frac{27.5}{100} \times 50000$

= 13750 kg/day

9.1% is H_2O

90.9% is C_7H_8

$(0.091 \times 13750) \frac{kg\ H_2O}{day}$

$(0.909 \times 13750) \frac{kg\ Tol}{day}$

Water = 1251.25 kg/day

Toluene = 12498.75 kg/day

So here is the flowchart. We have the information that we have entered. So you have 50000 kg per day of coolant coming in and we know that for every 100 kg of coolant you have 27.5 kg of vapor which is being fed. So let us first perform the basic material balance which is required. So as I always say, material balance is the first step for any energy balance problem. So let us perform the material balance to get the feed input.

So we have 27.5 kg of vapor fed per 100 kg of coolant. So we actually have 50000 kg per day of coolant coming in which means the amount of vapor which is, the mass of vapor which is fed would be 27.5/100 times 50000. So this is equal to 13750 kg/day. So this is the mass of vapor which is being fed to the system. So we know that out of this 9.1% is water and the rest is toluene which would be 90.9 % is toluene C7H8.

So now we need to calculate the mass of water and toluene which is coming in so that we will know the mass of water and toluene which is leaving through the F stream here. So mass of water would be 0.091 times 13750 kg/day. This is the mass of water entering the system and mass of toluene entering the system would be 0.909 times 13750 kg of toluene per day. So these can be calculated as water = 1251.25 kg/day and toluene = 12498.75 kg/day.

So now that we have the mass of toluene and water entering, we can start looking at the energy balance aspect. So what do we do for the energy balance? So we have to write energy balances to identify two things. First one is to identify the temperature T 2 and then the second one is to identify W which is the mass of water which is being fed in the second system. So for finding out temperature T 2 we can choose a system which would be only the condenser.

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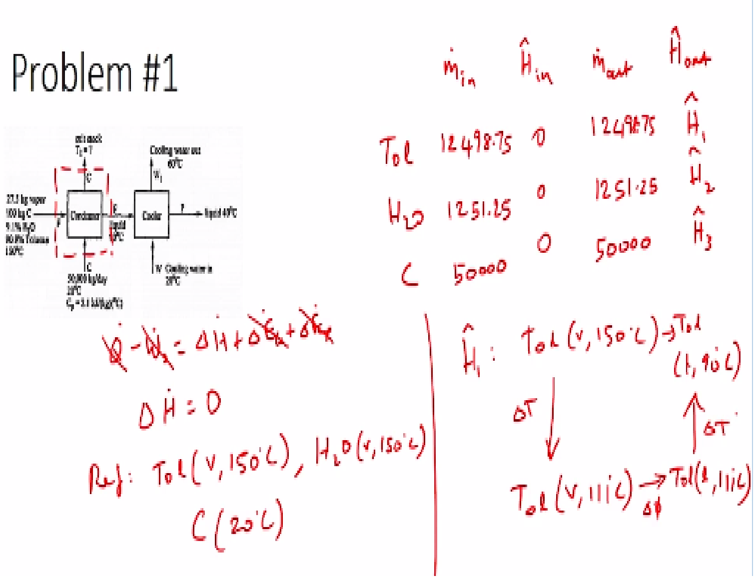

Problem #1

	$\dot{m}_{in}$	$\hat{H}_{in}$	$\dot{m}_{out}$	$\hat{H}_{out}$
Tol	12498.75	0	12498.75	$\hat{H}_1$
H_2O	1251.25	0	1251.25	$\hat{H}_2$
C	50000	0	50000	$\hat{H}_3$

$$\cancel{\dot{Q}} - \cancel{\dot{W}_s} = \Delta\dot{H} + \cancel{\Delta\dot{E}_k} + \cancel{\Delta\dot{E}_p}$$

$$\Delta\dot{H} = 0$$

Ref: Tol(v, 150°C), H_2O(v, 150°C)

C(20°C)

$\hat{H}_1$:

$$\begin{tikzcd} \text{Tol}(v, 150^\circ C) \arrow[r] \arrow[d, "\Delta T"'] & \text{Tol}(l, 90^\circ C) \\ \text{Tol}(v, 111^\circ C) \arrow[r, "\Delta\hat{H}"'] & \text{Tol}(l, 111^\circ C) \arrow[u, "\Delta T"'] \end{tikzcd}$$

So let us choose this as the system of interest. So for the condenser let us start writing the energy balance equation. So the condenser is an open system because you have material crossing the system boundary. So the equation would then become Q dot – W s dot = delta H dot + delta E k dot + delta E p dot. As there is no change in kinetic and potential energy or moving parts, these terms go to 0.

As I had already mentioned the process does not have any heat being lost between the system and the surrounding, so Q dot will also be = 0 giving you delta H dot = 0. So now that you have delta H dot = 0 we can write the expansion for H dot and identify the value for the temperature. So here you also have only constants given as C p values which means you would not have a very complex polynomial which we used to end up with when we calculate temperature.

So usually we would have a third order or a fourth order polynomial which would be difficult to solve. Here it would be a simple linear equation which can be solved. So let us try to do that. So for doing that we first need to identify the reference states so that we can build the enthalpy table. So let us look at what could be the reference states. So here we are not going to be using any enthalpy table as such like steam tables or anything.

So here we are going to use the C p values which have been given for calculating enthalpy which means we can actually use either the inlet or outlet conditions. So I have chosen to use the inlet conditions for the vapor and the coolant as the reference states. So which means the reference state for toluene would be vapor at 150 degree Celsius, water would also be vapor at 150 degree Celsius and you have the coolant which is coming in at 20 degree Celsius.

So these are the information we have and now let us build the enthalpy table. So the enthalpy table would have toluene, water, and coolant and we will have m in dot, H cap in, m out dot, and H cap out. And toluene coming in we had calculated it to be 12498.75 and water coming in we have calculated to be 1251.75. Coolant, it has been given as 50000 and so whatever is coming in is also leaving the system.

There is no change when it comes to the chemical composition of the system. It is only the phase which is being changed. So we will have 12498.75 kg of toluene leaving per day. 1251.75 sorry it should be 1251.25 kg of water leaving per day and C is 50000 kg of coolant leaving per day. So now which of the enthalpies do we have to calculate. We had identified the inlet condition as the reference state.

Which means these enthalpies which are present for the inlet streams would all be 0. We would only have to worry about the enthalpies for the exit streams which would be H 1, H 2, H 3. So we need to now calculate H 1, H 2 and H 3. Let us first look at H 1 cap. So what is H 1 cap? So H 1 cap is nothing but toluene vapor at 150 degree Celsius which is the reference state forming the exit condition which is toluene liquid at 90 degree Celsius.

So the process is toluene which is from vapor 150 degree Celsius goes to toluene which is a liquid at 90 degree Celsius. So here you have both temperature and phase changing which means you have to build a hypothetical path. So the path can be toluene vapor at 150 degree Celsius is cooled to toluene vapor at 111 degree Celsius. So I choose 111 degree Celsius because that is the boiling point of toluene and we have the information about the heat of vaporization at 111 degree Celsius which has been given to us in the problem.

So if you look at the problem statement you can see that the toluene boiling point is 111 degree Celsius and the heat of vaporization for toluene at that temperature is also known. So based on this I have chosen 111 degree Celsius for the change in phase. And from here it goes to toluene liquid at 111 degree Celsius which would be the saturated liquid at 111 degree Celsius. From here it is cooled to 90 degree Celsius.

In the first step you have delta T and in the second step you have delta phi which is the change in phase and the last step you again have delta T which is change in temperature.

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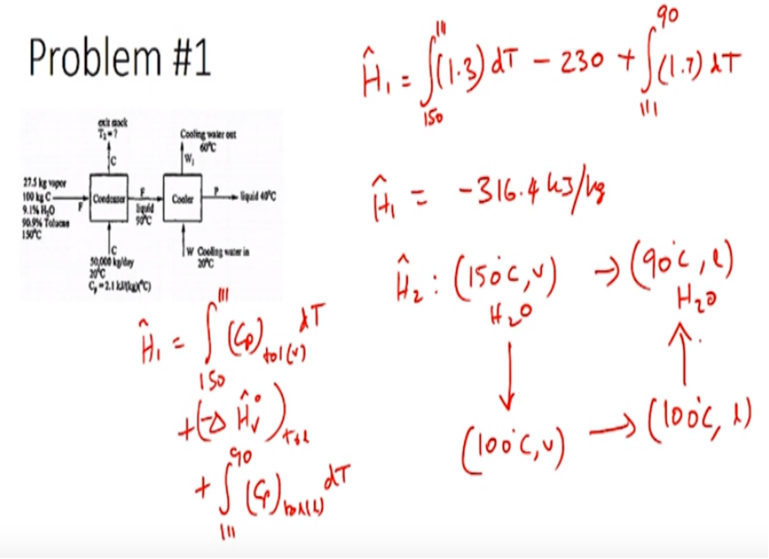

Problem #1

$$\hat{H}_1 = \int_{150}^{111} (C_p)_{tol(v)}\,dT + (-\Delta \hat{H}_v^\circ)_{tol} + \int_{111}^{90} (C_p)_{tol(l)}\,dT$$

$$\hat{H}_1 = \int_{150}^{111} (1.3)\,dT - 230 + \int_{111}^{90} (1.7)\,dT$$

$$\hat{H}_1 = -316.4 \text{ kJ/kg}$$

$$\hat{H}_2 : \begin{tikzcd} \underset{H_2O}{(150^\circ C, v)} \arrow[r] \arrow[d] & \underset{H_2O}{(90^\circ C, l)} \\ (100^\circ C, v) \arrow[r] & (100^\circ C, l) \arrow[u] \end{tikzcd}$$

So using this we can calculate H 1 cap as follows. H 1 cap would be equal to integral 150 to 111 C p of toluene vapor dT + delta H v cap negative of it for toluene. So this we are using the negative of heat of vaporization because you have condensation reaction happening, condensation process happening here not vaporization. So it will be the negative of heat of vaporization plus integral 111 to 90 C p of toluene liquid dT.

So now that we have this as the values for H 1 cap, let us calculate H 1 cap. So H 1 cap = integral 150 to 111 times 1.3 dT. So we are getting the C p values from the table which was given to us in the problem minus 230. So the value of heat of vaporization can also be obtained from the table plus integral 111 to 90 C p of toluene liquid is 1.7 dT. So using this we would end up with H 1 cap as -316.4 kJ/kg. So this gives us H 1 cap. Now we need to identify H 2 cap.

So H 2 cap is again the change in enthalpy for water from the reference state which is 150 degree Celsius and vapor. From there water goes to, this is water at this condition goes to the outlet condition which is 90 degree Celsius liquid water. So from here to here you have both change in temperature and phase. Again you build a similar hypothetical path where it is first cooled to the boiling point of water which is 100 degree Celsius and vapor and from here it changes the phase to form 100 degree Celsius liquid.

So there is a condensation process and from here it is further cooled to get 90 degree Celsius and liquid water. So the first step is delta T, second step is delta phi and the third step is delta T. So this is the process we can build.

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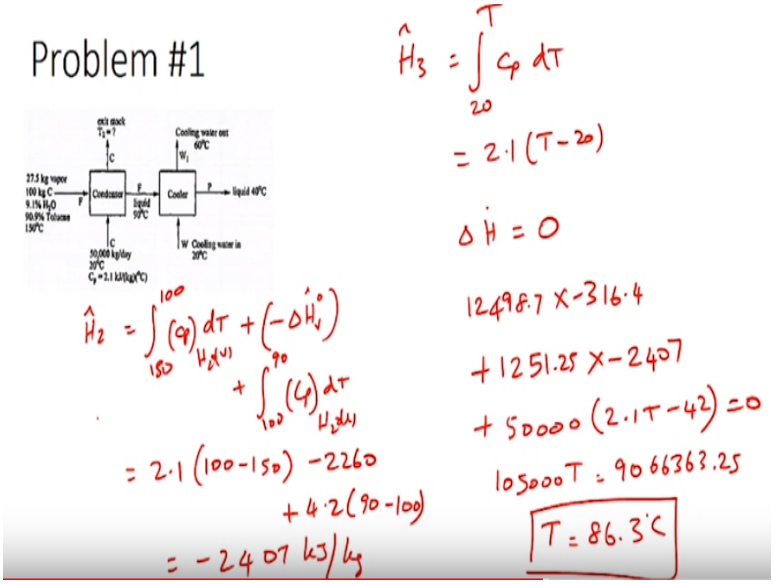

Problem #1

$$\hat{H}_2 = \int_{150}^{100} (C_p)_{H_2O(v)}\, dT + (-\Delta \hat{H}_v^{\circ}) + \int_{100}^{90} (C_p)_{H_2O(l)}\, dT$$

$$= 2.1(100-150) - 2260 + 4.2(90-100)$$

$$= -2407 \text{ kJ/kg}$$

$$\hat{H}_3 = \int_{20}^{T} C_p\, dT$$

$$= 2.1(T-20)$$

$$\Delta \dot{H} = 0$$

$$12498.7 X - 316.4$$

$$+ 1251.25 X - 2407$$

$$+ 50000(2.1T - 42) = 0$$

$$105000 T = 9066363.25$$

$$\boxed{T = 86.3^{\circ}C}$$

So using this, we would again be able to calculate the H 2 cap. So H 2 cap can be calculated as follows. H 2 cap = integral 150 to 100 C p dT + -heat of vaporization for water + integral 100 to 90 C p dT. So the first C p would be for water vapor and the second C p would be for water liquid. So now we have the values of C p for water vapor and water liquid from the problem. So using that we will get 2.1 times 100 – 150 - 2260 + 4.2 times 90 -100.

So this would give us a value of -2407 kJ/kg. The next thing to calculate is H 3 cap. So for H 3 cap you have the coolant which is actually being cooled sorry which is being heated from 20 degree Celsius to temperature T 2. There is no phase change for the coolant which has been given. So we will only look at the change in temperature. So it is a single step process where it is heated from 20 degree Celsius to a temperature of T. So C p of coolant dT.

So C p of coolant has been given as 2.1. So this would just be 2.1 times T - 20. So we have the general energy balance equation simplified to delta H dot = 0. So now we can substitute the values for these H 1, H 2, ad H 3 and we already know the masses of the toluene water and coolant which are coming in. So your equation for delta H dot will actually become 12498.7 times – 316.4 + 1251.25 times -2407 + 50000 times 2.1T – 42 = 0.

So this equation will then simplify to form 105000 times T = 9066363.25. So from here temperature T is calculated as 86.3 degree Celsius. So the coolant is actually heated from 20 degree Celsius to 86.3 degree Celsius so that the vapor which is coming in at 150 degree Celsius can be cooled to form a condensed liquid at 90 degree Celsius. So now that we have the first part of the problem solved, let us move on to the second part of the problem.

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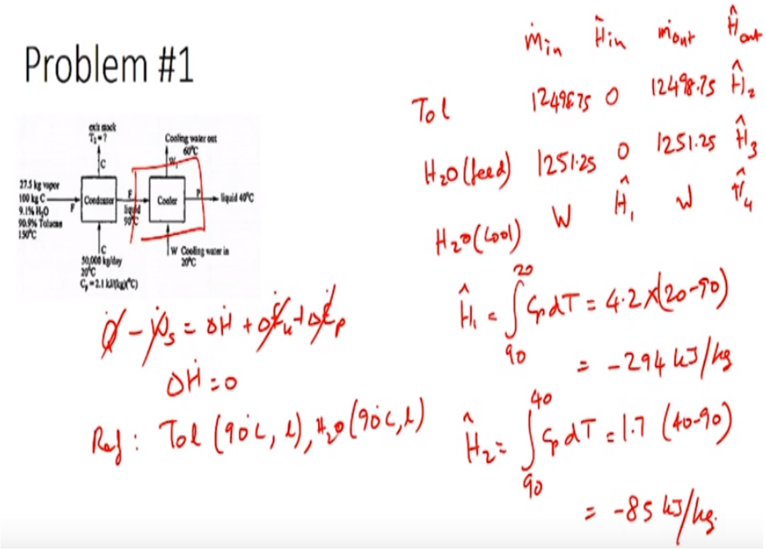

Problem #1

	$\dot{m}_{in}$	$\hat{H}_{in}$	$\dot{m}_{out}$	$\hat{H}_{out}$
Tol	12498.75	0	12498.75	$\hat{H}_2$
H_2O (feed)	1251.25	0	1251.25	$\hat{H}_3$
H_2O (cool)	W	$\hat{H}_1$	W	$\hat{H}_4$

$$\cancel{\dot{Q}} - \cancel{\dot{W}_s} = \Delta\dot{H} + \cancel{\Delta\dot{E}_k} + \cancel{\Delta\dot{E}_p}$$

$$\Delta\dot{H} = 0$$

Ref: Tol (90°C, l), H_2O (90°C, l)

$$\hat{H}_1 = \int_{90}^{20} C_p\,dT = 4.2 \times (20-90) = -294 \text{ kJ/kg}$$

$$\hat{H}_2 = \int_{90}^{40} C_p\,dT = 1.7\,(40-90) = -85 \text{ kJ/kg.}$$

For the second part of the problem we have been asked to calculate W which is the mass of water which is fed in the cooler. So we already have the information about the liquid which is entering the cooler and it would be the same as the liquid which is leaving the cooler as well. So having that information we can now start writing the energy balance equations.

So again here the equation would be for an open system Q dot – W s dot = delta H dot + delta E k dot + delta E p dot. Again, the equation will simplify to form delta H dot = 0 because you do not have any change in kinetic or potential energies or shaft work and as we already discussed the process does not have any energy exchange between the system we have chosen which is the cooler and the surroundings of the system. So this means delta H dot becomes 0.

So we now have to choose the reference states for which we can build the enthalpy table. So let us choose the reference state. So here I have chosen the inlet conditions for the liquid which is entering as the feed for the cooler as the reference states. So this means toluene at 90 degree Celsius and liquid and water at 90 degree Celsius and liquid have been used as the reference states. So you also have water which is coming in at 20 degree Celsius.

So we will have to account for that separately. So here you would have to build an enthalpy table. So what we need to do here is we actually have to list water twice because you have water coming in and leaving in 2 different streams and the conditions are different. So we need to write water entering through the liquid feed for the cooler as one stream and water which is entering through the cooler as a coolant as another stream.

So what happens is the table would actually look like this. So you would have toluene and you would have water which is the feed and you would have water which is the coolant. So these three are the components which are there in the system and you can have m in dot H cap in m out dot and H dot out. So m in dot for toluene would be 12498.75. Water through the feed liquid would be 1251.25.

Water which is coming in through the cooler would be, through the coolant stream would be W and the same would be the values for the outlet streams as well. So you have 12498.75, 1251.25 and W as the mass flow rates out. So now let us look at which of the enthalpies would go to 0. So we had taken the toluene at 90 degree Celsius and water at 90 degree Celsius as the reference states which means these two will go to 0.

So the stream where water is entering as the cooling water would be having some enthalpy difference from the reference state. So, so would the other streams. So we would have to calculate 4 different enthalpies here. However, here there is no phase change which we have. So we would not have to build any hypothetical paths. We just have to account for change in temperatures.

So once we have H 1 cap, so H 1 cap is basically water liquid at 90 degree Celsius which is the reference state is being cooled to the cooling water condition which is water at 20 degree Celsius. So this process would just be a change in temperature. So you have C p dT, integral C p dT from 90 degree Celsius which is the reference state to 20 degree Celsius which is the inlet condition and this would be = 4.2 times 20 – 90 so giving you a value of -294 kJ/kg.

So H 2 cap would be = integral C p dT from 90 to cooling to, toluene is leaving at 40 degree Celsius. So you have 40 as the temperature for the outlet stream. So this would be = 1.7 times 40 – 90 giving you a value of -85 kJ/kg.

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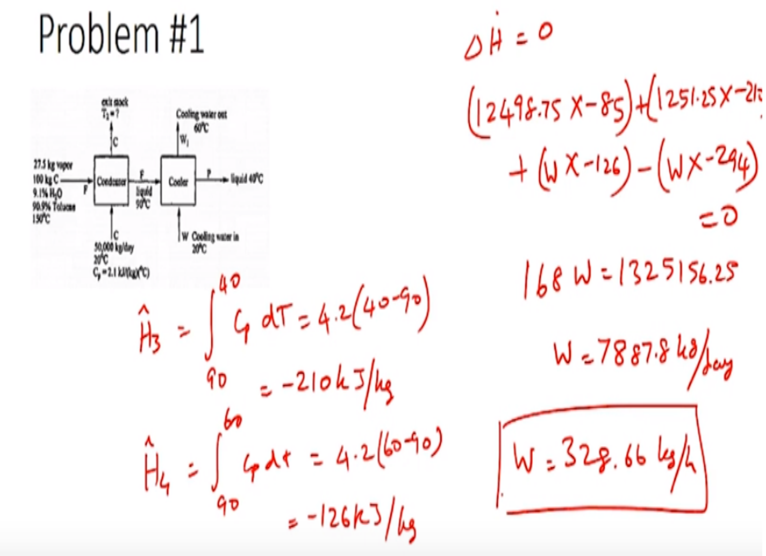

Problem #1

$$\Delta \dot{H} = 0$$

$$(12498.75\,X - 85) + (1251.25\,X - 21$$

$$+ (W\,X - 126) - (W\,X - 294) = 0$$

$$168\,W = 1325156.25$$

$$W = 7887.8\ \text{kg/day}$$

$$\boxed{W = 328.66\ \text{kg/h}}$$

$$\hat{H}_3 = \int_{90}^{40} C_p\,dT = 4.2(40-90) = -210\ \text{kJ/kg}$$

$$\hat{H}_4 = \int_{90}^{60} C_p\,dT = 4.2(60-90) = -126\ \text{kJ/kg}$$

So now we can calculate H 3 cap. So H 3 cap is for the process where water at reference state of 90 degree Celsius and liquid is being cooled to the water liquid which is leaving through the product stream P at 40 degree Celsius. So the process is integral 90 to 40 C p of water dT. So you have a cooling from 90 degrees which is the reference state to 40 which is the exit condition for water. So this would be 4.2 times 40 – 90 giving you a value for H 3 as -210 kJ/kg.

H 4 cap would be integral 90 to 60 C p of dT with C p being the C p of water liquid. So here you are looking at water which is the cooling water leaving the system. So now we have taken the reference state for water as 90 degree Celsius. So from there it is being cooled to the outlet stream which is 60 degree Celsius. So you have integral 90 to 60 C p dT. So this becomes 4.2 times 60 - 90 so which = -126 kJ/kg.

So we had already simplified the general energy balance equation as delta H dot = 0. So substituting all these values for H caps and the masses which we have you would have 12498.75 times -85 + 1251.25 times -210 + W times -126. So these are the enthalpies for the outlet stream and for the inlet stream you only have the cooling water which is entering so which would be –W times -294. This is equal to 0.

So this equation can then be simplified to get 168 W = 1325156.25 giving a value for W as 7887.8 kg/day. The problem asks us to calculate the water mass which needs to be fed per hour. So dividing this by 24 we would get W = 328.66 kg/h. So this is the rate at which water needs to be supplied to the system for getting the required cooling. With this we have actually looked at a simple example problem where we had a multiunit process where both the processes had energy balances that can be performed and we had a system where there was no reaction as such.

There was only phase change which was happening and we were looking to perform the energy balances required to calculate the temperature of an exit stream and also the mass which is required for an inlet stream to get the desired levels of cooling. So with this we have performed one example problem for the energy balance review. In the next lecture, we will look at another example problem which will again try to apply the fundamentals which we looked at during the review and we will perform this calculations in the next lecture. Thank you.