Material and Energy Balances
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Lecture - 54
Mixing and Solution: Tutorials - 2

So here is the next example problem.

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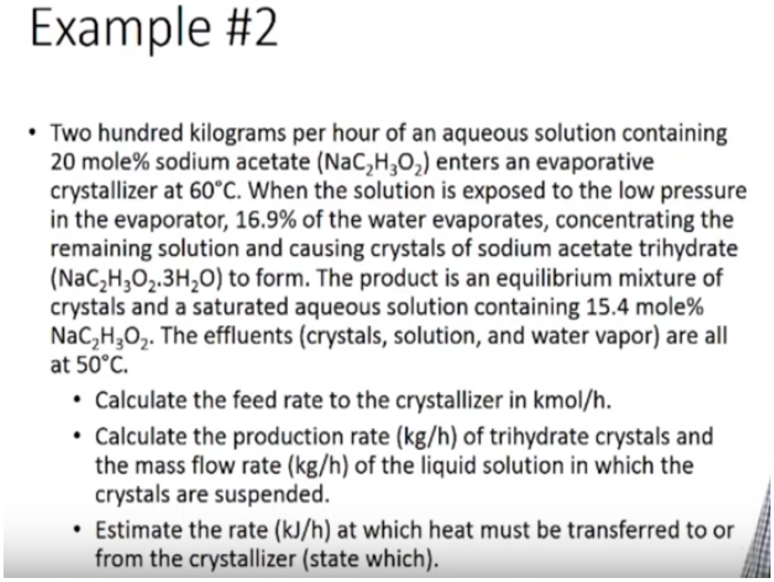

Example #2

- Two hundred kilograms per hour of an aqueous solution containing 20 mole% sodium acetate ($NaC_2H_3O_2$) enters an evaporative crystallizer at 60°C. When the solution is exposed to the low pressure in the evaporator, 16.9% of the water evaporates, concentrating the remaining solution and causing crystals of sodium acetate trihydrate ($NaC_2H_3O_2.3H_2O$) to form. The product is an equilibrium mixture of crystals and a saturated aqueous solution containing 15.4 mole% $NaC_2H_3O_2$. The effluents (crystals, solution, and water vapor) are all at 50°C.
  - Calculate the feed rate to the crystallizer in kmol/h.
  - Calculate the production rate (kg/h) of trihydrate crystals and the mass flow rate (kg/h) of the liquid solution in which the crystals are suspended.
  - Estimate the rate (kJ/h) at which heat must be transferred to or from the crystallizer (state which).

200 kilograms per hour of an aqueous solution containing 20 mole percent sodium acetate enters an evaporative crystalizer at 60 degree Celsius. When the solution is exposed to the low pressure in the evaporator, 16.9% of the water evaporates concentrating the remaining solution and causing crystals of sodium acetate trihydrate to form. The product is an equilibrium mixture of crystals and a saturated aqueous solution containing 15.4 mole percent of sodium acetate.

The effluents which are the crystals, solution, and water vapor are all at 50 degree Celsius. Calculate the feed rate to the crystallizer in kmol/h and calculate the production rate as kg/h of trihydrate crystals and mass flow rate of liquid solution in which the crystals are suspended. Estimate the rate at which heat must be transferred to or from the crystallizer. We need to also state whether heat needs to be supplied or removed. Now let us look at how we would solve this problem.

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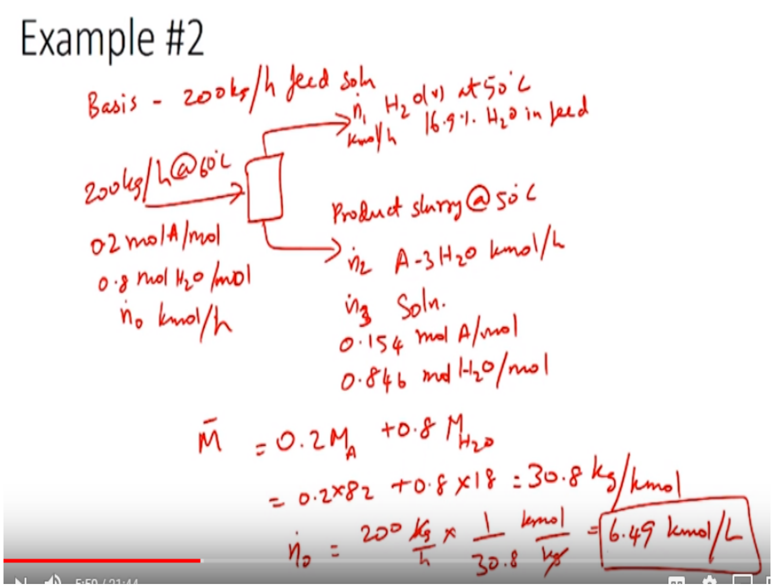

Example #2

Basis - 200kg/h feed soln

$$\bar{M} = 0.2M_A + 0.8M_{H_2O}$$

$$= 0.2 \times 82 + 0.8 \times 18 = 30.8 \text{ kg/kmol}$$

$$\dot{n}_0 = 200 \frac{\text{kg}}{\text{h}} \times \frac{1}{30.8} \frac{\text{kmol}}{\text{kg}} = \boxed{6.49 \text{ kmol/L}}$$

First we need to perform the required material balances. For performing the required material balances we need to identify the basis. What would be the basis for this system? We have been told that 200 kg/h of feed solution enters. So the basis for this process would be 200 kg/h of feed solution. Now let us draw the flowchart. So you have a process where 200 kg/h at 60 degree Celsius is coming in.

So this contains 0.2 mole fraction of A per mole of the mixture and the rest is 0.8 moles of water per mole of the mixture. A is the sodium acetate. So from here you have two streams which are leaving. One is the product slurry. Product slurry is leaving at 50 degree Celsius and you have two things in this slurry. You have crystals which are present which are the trihydrate crystals and you have a solution which is accompanying the crystals.

So we will have A 3H2O crystals which are leaving us and you also have a solution, aqueous solution which is containing the sodium acetate and water. So we know that molar composition of this is 0.154 moles of A per mole of solution and 0.846 moles of water per mole of solution. So the other stream is the water vapor. So you have water vapor which is leaving the system, also at 50 degree Celsius.

And it has been told that 16.9% of water in feed is actually getting evaporated and leaving through the stream. So let us first label this flowchart and identify the parameters which need to be calculated. So we have mass flow rate. However, we need molar flow rate because all the compositions are given in terms of mole fractions or mole percentages. So let us call this n naught kmol/h of feed which is coming in.

And we will call n 1 kilo moles of water per hour which is water vapor that is leaving the system and we will also have n 2 dot kilo moles of the trihydrate crystals leaving and we will have this solution as n 3 dot. So we need to calculate all these parameters which we can do by using simple material balance. The first part of the problem asks us to calculate the molar flow rate of the feed.

So that would mean we would have to calculate, we would have to convert this 200 kg/h into kmol/h which is n naught dot. Let us try and do this. So for this we can calculate the average molecular weight of the mixture which would be 0.2 times molecular weight of sodium acetate + 0.8 times molecular weight of water. So this would be 0.2 times 82 + 0.8 times 18 giving a value of 30.8 kg/kmol. So this is the average molecular weight for the mixture.

So the molar flow rate n naught can be calculated as 200 kg/h divided by molecular weight which would be 1/30.8 kmol/kg. So kilogram gets cancelled and you would end up with 6.49 kmol/h. So this is the molar flow rate of the feed which is entering the system. So this is the first part. Now for the second part we would have to perform material balance calculations.

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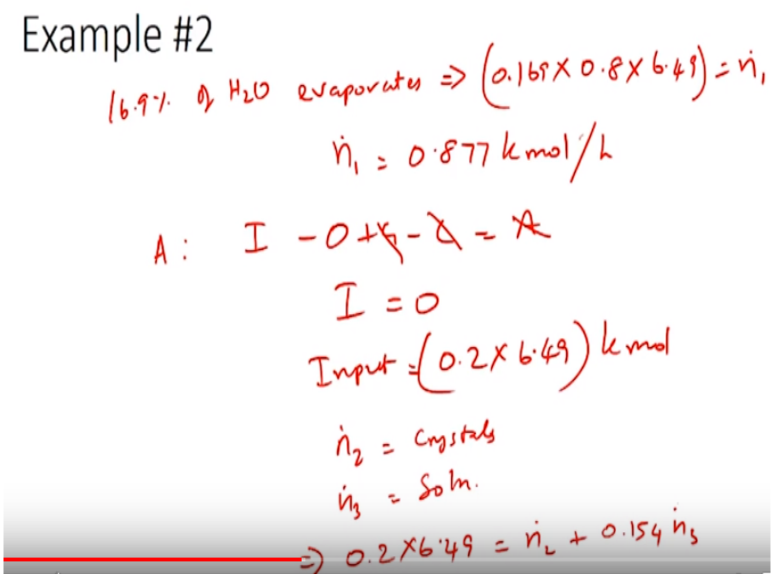

Example #2

16.9% of H_2O evaporates $\Rightarrow (0.169 \times 0.8 \times 6.49) = \dot{n}_1$

$\dot{n}_1 = 0.877$ kmol/h

A : $I - O + \cancel{G} - \cancel{C} = \cancel{A}$

$I = O$

Input $= (0.2 \times 6.49)$ kmol

$\dot{n}_2$ = Crystals

$\dot{n}_3$ = Soln.

$\Rightarrow 0.2 \times 6.49 = \dot{n}_2 + 0.154\,\dot{n}_3$

The first information we have from the problem is 16.9% of water evaporates. So this implies 0.169 times 0.8 times 6.49 would be the number of moles of water which is evaporated. So this would be equal to n 1 dot. So n 1 dot would be equal to 0.877 kmol of water vapor per hour. So then we can write the sodium acetate balance. Sodium acetate balance would be input – output + generation – consumption = accumulation. It is not taking part in any reaction.

So generation and consumption go to 0 and that steady state accumulation goes to 0 giving you only input equals output. So there are two components of sodium acetate which are leaving the system and there is only one component of sodium acetate entering the system. The component entering which is input would be equal to 0.2 times 6.49. This is the number of kilo moles of sodium acetate entering the system.

We now have to identify how many moles of kilo moles of sodium acetate is leaving the system in terms of crystals and how much is leaving along with the solution. So we have two components which is the n 2 dot and n 3 dot. So n 2 dot represents the number of moles of crystal which is leaving. So that is we will call that n 2 dot is the crystals and n 3 dot is the number of moles of the solution which contains the sodium acetate.

So we have also calculated that the sodium acetate concentration in the solution is 0.154 mole fraction. So substituting these values we would have the balance equation become 0.2 times 6.49 = n 2 dot + 0.154 n 3 dot. Similarly, we can write a balance equation for water.

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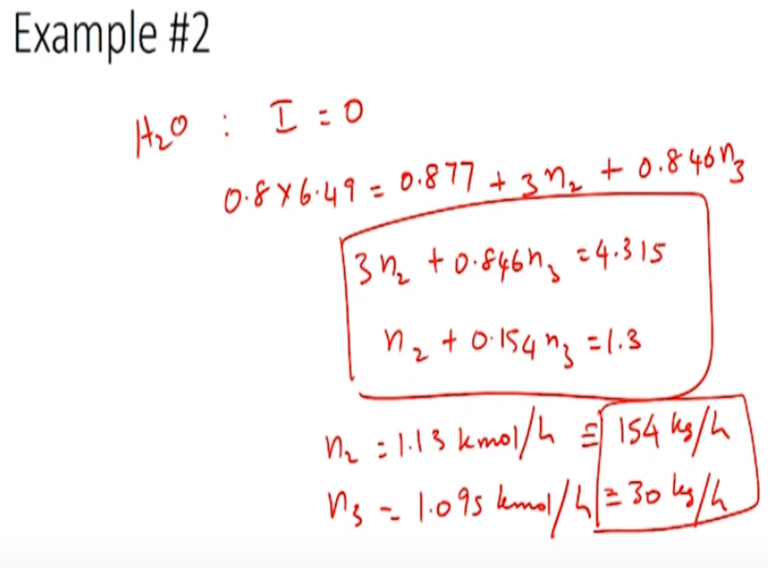

Example #2

$H_2O : I = O$

$0.8 \times 6.49 = 0.877 + 3n_2 + 0.846n_3$

$$\boxed{\begin{aligned} 3n_2 + 0.846n_3 &= 4.315 \\ n_2 + 0.154n_3 &= 1.3 \end{aligned}}$$

$n_2 = 1.13 \text{ kmol/h} \equiv \boxed{154 \text{ kg/h}}$

$n_3 = 1.095 \text{ kmol/h} \boxed{= 30 \text{ kg/h}}$

So the water balance would also be input equals output. So water is coming in through the solution and it is leaving as part of the solution, as part of the water vapor and also as water of hydration. So we need to account for all the three components. We have already calculated the amount of water leaving as vapor. We know the amount of water which is entering. We just need to now write the balance equation for the other two. So let us write this equation.

So water inlet would be 0.8 times 6.49 and water outlet would be 0.877 kmol which is leaving as water vapor + 3 times n 2 moles of the crystals because for every mole of the crystal which is leaving, 3 moles of water is accompanying with it. So that would mean 3 times n 2 would be the number of moles of water leaving with n 2 moles of the crystals which is leaving. So for the water which is leaving along with the solution we would have it as 0.846 times n 3 where 0.846 is the mole fraction of water in the solution.

So now we have an equation which simplifies to 3 n 2 + 0.846 n 3 = 4.315. So we already had an equation where it was n 2 + 0.154 n 3 = 1.3. So using these two equations we can solve for n 2 and n 3 when we solve this equation simultaneously. So n 2 would be equal to 1.13 kmol/h and n

2 would be equal to 1.095 kmol/h. We have been asked to calculate the mass of the crystals and the solution which is accompanying the crystals.

So we can convert these to masses by using the molecular weight. So the molecular weight of the crystal would be 136 kg/kmol because we have 3 moles of water along with 1 mole of acetic acid. Molecular weight of sodium acetate is 82 and molecular weight of water is 18 giving us the molecular weight of the sodium acetate crystals which are the trihydrate crystals as 136. So using that we can calculate this to be weighing 154 kg/h and the solution would basically weigh 30 kg/h.

So this gives us the flow rates, mass flow rates of the solution and the crystals which are leaving the system. So the solution would contain sodium acetate and water and based on the mass fractions we can actually find the average molecular weight and use that for calculating the mass of the solution which is leaving the system. So these values give us the total mass of the components which is leaving the system. So you would have the rest of it leaving as water vapor. So that would give us the total balance for the mass.

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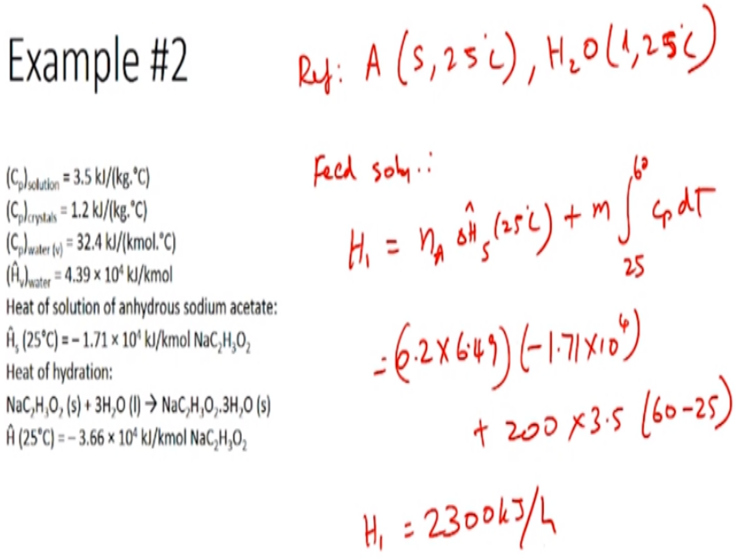

Example #2

$(C_p)_{solution} = 3.5$ kJ/(kg.°C)
$(C_p)_{crystals} = 1.2$ kJ/(kg.°C)
$(C_p)_{water\,(v)} = 32.4$ kJ/(kmol.°C)
$(\hat{H}_v)_{water} = 4.39 \times 10^4$ kJ/kmol
Heat of solution of anhydrous sodium acetate:
$\hat{H}_s\,(25°C) = -1.71 \times 10^4$ kJ/kmol $NaC_2H_3O_2$
Heat of hydration:
$NaC_2H_3O_2\,(s) + 3H_2O\,(l) \rightarrow NaC_2H_3O_2.3H_2O\,(s)$
$\hat{H}\,(25°C) = -3.66 \times 10^4$ kJ/kmol $NaC_2H_3O_2$

Ref: A (s, 25°C), H_2O (l, 25°C)

Feed soln.:

$$H_1 = n_A\,\Delta\hat{H}_s(25°C) + m\int_{25}^{60} C_p\,dT$$

$$= (6.2 \times 6.49)(-1.71\times10^4) + 200 \times 3.5\,(60-25)$$

$$H_1 = 2300\ \text{kJ/h}$$

So with this we can move on to the next part which is the energy balance associated with solutions. So as I have always established, it is important that we understand the first step of any energy balance is material balance. That is why I keep doing problems where I spend quite a lot of time trying to do the material balance before I start the energy balance. So most of the times in real life scenario when you actually perform energy balances the flow rates and masses will not be directly given.

So instead you would have to perform all the calculations required to complete the material balance and then identify the energy balance problem. So let us do that. The third part where we are doing the energy balances will be starting here. For the third part, we have been asked to calculate the amount of heat that needs to be supplied or removed from the system. So let us do that. So how would we go about doing this? So first thing is to identify the reference states.

The reference states for the two different chemical enthalpies which are present are sodium acetate and water. So sodium acetate solid at 25 degree Celsius and water liquid at 25 degree Celsius can be used as the reference states for our calculations. So here I am going to slightly deviate from what we have been doing till now. Until now, we have been building enthalpy tables.

So this has been simple to use because the enthalpy we get would always be either in terms of kilo joules per mole of a substance or kilo joules per mole of a solute and so on. However, here we have different solutions and we cannot use one term that can be multiplied with the specific enthalpy. So we need to look at the process in a more comprehensive fashion.

So let us look at the individual components and try to identify the change in enthalpy for the individual components. The first component we have in the system is the feed solution. So let us start with identifying the change in enthalpy for the feed solution. So the change in enthalpy for the feed solution can be n H cap or can just be called as H 1.

And this would be equal to n A times delta H s cap at 25 degree Celsius which is the number of moles of sodium acetate times the heat of solution associated with converting these solid and liquid water, so solid sodium acetate and liquid water pure components into aqueous solution. So this would be the first component and this particular solution is then heated to form a 60 degree temperature, to reach 60 degree temperature.

So the C p value for the solution, feed solution and the product solution have been given as 3.5 kJ/kg degree Celsius. So we will multiply it with the mass of the solution which is m integral 25 to 60 C p dT. So you would have this become 0.2 times 6.49 which is the number of moles of sodium acetate in the solution times the heat of solution which is given as -1.71 times 10 power 4 kJ/kmol of sodium acetate.

So that is why we are using the number of moles of sodium acetate here. So this times -1.71 times 10 power 4. So this would be the first term and the second term becomes 200 times 3.5 times 60 – 25. So this can actually be simplified to get the final value as roughly 2300 kJ/h. So this is the change in enthalpy for the feed solution. We now need to identify such change in enthalpy for the other components in the process.

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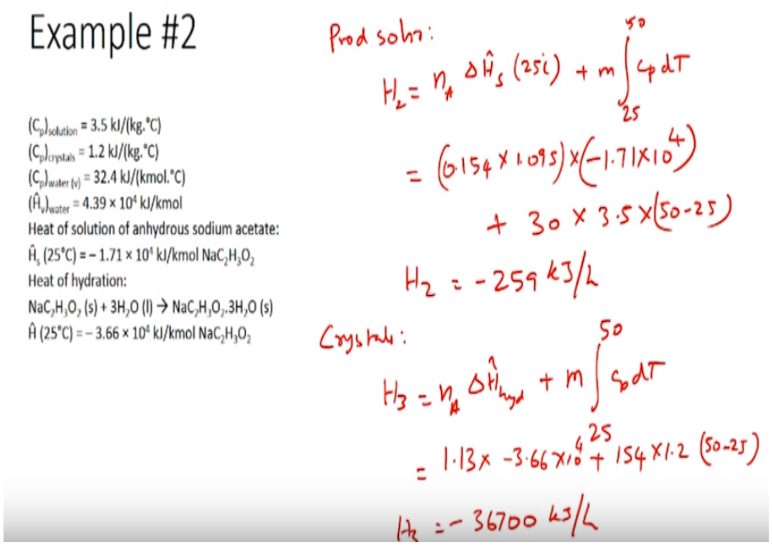

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Heat of hydration:
$NaC_2H_3O_2\,(s) + 3H_2O\,(l) \rightarrow NaC_2H_3O_2.3H_2O\,(s)$
$\hat{H}\,(25°C) = -3.66 \times 10^4$ kJ/kmol $NaC_2H_3O_2$

Prod soln:

$$H_2 = n_A\,\Delta\hat{H}_s\,(25c) + m\int_{25}^{50} C_p\,dT$$

$$= (0.154 \times 1.095) \times (-1.71 \times 10^4) + 30 \times 3.5 \times (50-25)$$

$$H_2 = -259 \text{ kJ/h}$$

Crystals:

$$H_3 = n_A\,\Delta\hat{H}_{hyd} + m\int_{25}^{50} C_p\,dT$$

$$= 1.13 \times -3.66 \times 10^4 + 154 \times 1.2\,(50-25)$$

$$H_3 = -36700 \text{ kJ/h}$$

So let us look at the next component which is the product solution. For the product solution, we would have to calculate the change in enthalpy, let us call it H 2 as n A times delta H s cap 25 degree Celsius which is the first step where the pure components are converted into the solution at 25 degree Celsius and from here it would have to be heated to 50 degree Celsius. So it will be m C p dT m integral C p dT.

Then it is L going from 25 to 50 C p dT. The C p for all the solutions has been given as 3.5 kJ/kg. So we will write this and this would give us 0.154 times 1.095 which is the number of moles of sodium acetate in the product solution times the heat of solution which is -1.71 times 10 power 4 + 30 times 3.5 times 50 - 25. So 30 is the mass of the product solution leaving the system. So this will give you a value of H 2 = -259 kJ/h.

So the next component you have to look at are the crystals. So for the crystals you have the pure components which are liquid water and solid sodium acetate forming trihydrate crystals. So the first step is the hydration process itself. So we have the heat of hydration given. So we will account for that as the first step. The first step is, so for H 3 would be equal to n A times delta H cap of hydration. So the value is given here.

H cap for hydration is given in terms of per kilo mole of sodium acetate so that is why we have to use the number of moles of sodium acetate to be multiplied with the specific enthalpy to get the actual enthalpy value + m integral C p dT where the hydrate crystals are heated from 25 degree Celsius to 50 degree Celsius.

And this would be number of moles of sodium acetate in the crystals which is leaving would be 1.13 kilo moles times the enthalpy of hydration which is -3.66 times 10 power 4 + the mass flow rate of the stream of crystals which is 154 times C p of the crystals which is given as 1.2 times the change in temperature which is 50 to 25, 50 -25. So this would mean H 3 would be equal to -36700 kJ/h.

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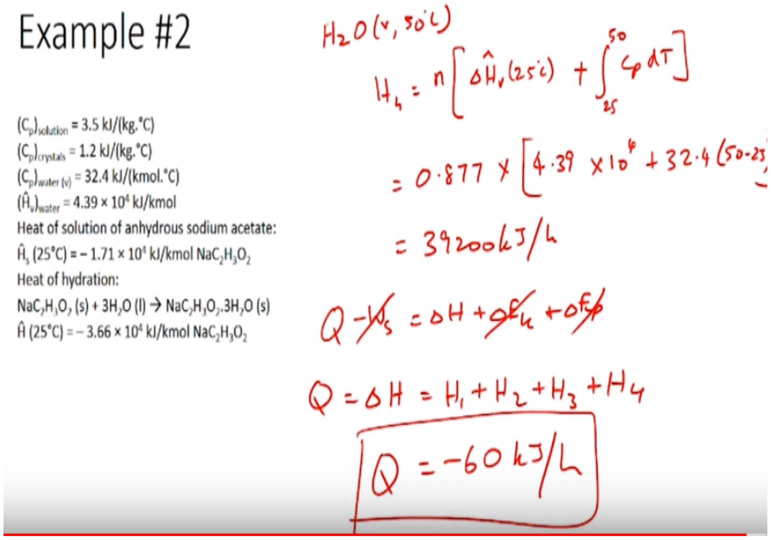

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Heat of hydration:
$NaC_2H_3O_2\ (s) + 3H_2O\ (l) \rightarrow NaC_2H_3O_2.3H_2O\ (s)$
$\hat{H}$ (25°C) = -3.66×10^4 kJ/kmol $NaC_2H_3O_2$

$H_2O(v, 50°C)$

$$H_4 = n\left[\Delta\hat{H}_v(25°C) + \int_{25}^{50} C_p\, dT\right]$$

$$= 0.877 \times \left[4.39 \times 10^4 + 32.4(50-25\right.$$

$$= 39200 \text{ kJ/h}$$

$$Q - \cancel{W_s} = \Delta H + \cancel{\Delta E_k} + \cancel{\Delta E_p}$$

$$Q = \Delta H = H_1 + H_2 + H_3 + H_4$$

$$\boxed{Q = -60 \text{ kJ/h}}$$

So the last component which we need to account for is water vapor at 50 degree Celsius. So the H 4 for that, the change in enthalpy for that would be n which is the number of moles of water times delta H cap v which is the heat of vaporization at 25 degree Celsius for water + integral C p dT 25 to 50 C p of water vapor dT and using this equation we can get these values for the heat of vaporization at 25 degrees from the steam tables and the C p value for water vapor from some any handbook or textbook.

Using these values we would have this become 0.877 times 4.39 times 10 power 4 + 32.4 times 50 - 25 and this would be equal to 39200 kJ/h. So now the energy balance equation we have would be Q – W s = delta H + delta E k + delta E p. So E k, E p and W s will go to 0. So Q would be equal to delta H. So which would be equal to the summation of all these three, all these enthalpies which would be H1, H 2, H 3 and H 4 giving Q as -60 kJ/h.

So this means heat needs to be removed from the system for the process to happen. That is why you have a negative sign. The heat needs to be removed at a rate of 60 kJ/h. So with this we have performed an energy balance calculation for a process where there is solutions. Thank you and see you in the next lecture.