Material and Energy Balances
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Module No # 08
Lecture No # 39
Mechanical Energy Balances: Tutorials

Hello everybody in the previous lecture we looked at performing mechanical energy balance calculations. In today's lecture we will actually perform more example problems so that we can familiarize ourselves with how to calculate mechanical energy and how to perform balance this for mechanical processes. So this will be tutorial session on this topic let us first start with the first problem.

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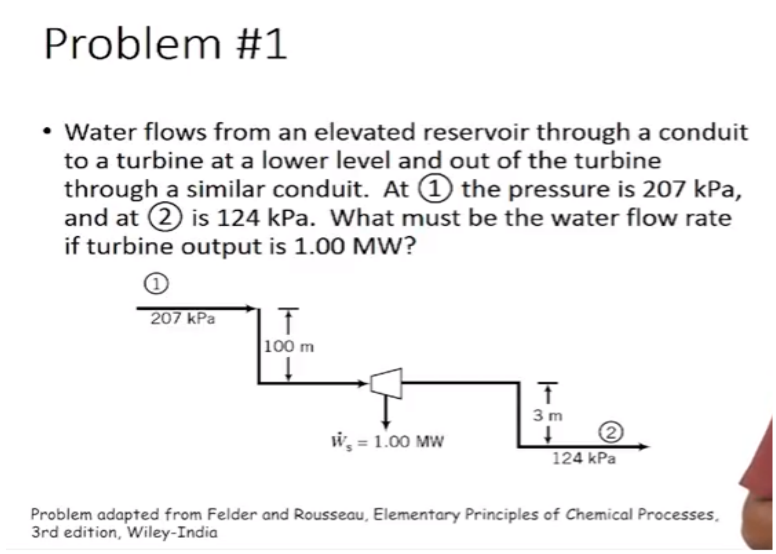

Here is the first example problem water flows from the elevated reservoir through a conduit to a turbine at a lower level and out of the turbine through a similar conduit. At one which is the point 1 which is the pressure is 207 kilopascals and at point 2 the pressure is 124 kilopascals what must be the water flow rate if the turbine output is 1 megawatts. So the setup is given so you have 0.1 which is certain elevated level form here the water flows down to the turbine and then it again falls back to the second point.

So you have initial drop of 100 meters and the second level of a drop is 3 meters you have also been given that the work shaft work is 1.00 megawatts, so having this information let us try to perform mechanical energy balance calculations.

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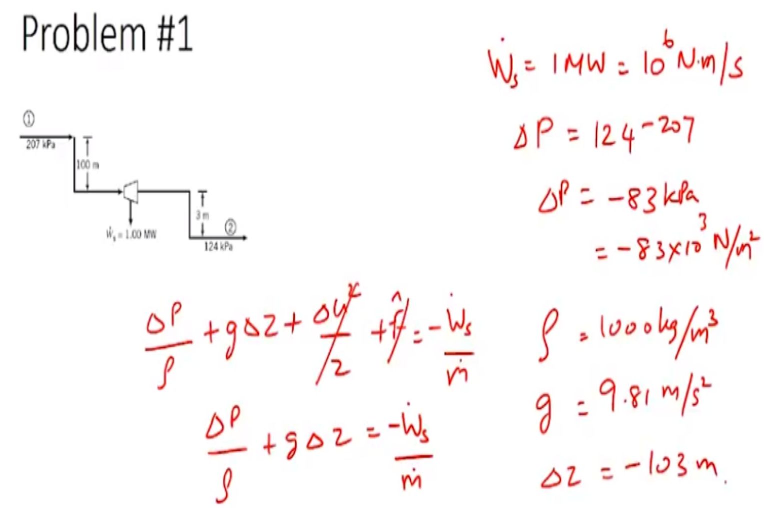

The general equation for the mechanical energy balances can be written delta P divided by row + G delta Z + delta U squared by 2 + F cap = -WS dot divided by M dot. So this would be general mechanical energy balance equation. In this system we can ignore friction factor because here is no friction factor which has been provided to us however we can ignore shaft to work we can also ignore change in velocity because we have not been given that the conduit is changing in diameter.

We have been told that conduits are about are similar which means we can assume that the cross sections are the same which would indicate that the linear velocity would also be the same. So the equation here simplifies to delta P divided by row + G delta Z = - WS dot divided by M dot the shaft work has been given as WS dot is 1 megawatt which is equal to 10 power 6 newton meter per second and we have been told that the pressures are given for 0.1 and 0.2.

So delta P would be final pressure – initial pressure which is 124 – 207 giving a value of delta P as – 83 kilopascals so this can be written as -83 times 10 power 3 newton per meter square and then density is the density water which could be 1000 per gram meter cube and G which is acceleration due to gravity could be 9.81 meter per second square and delta Z which is the total change in height would be equal to -103 meters.

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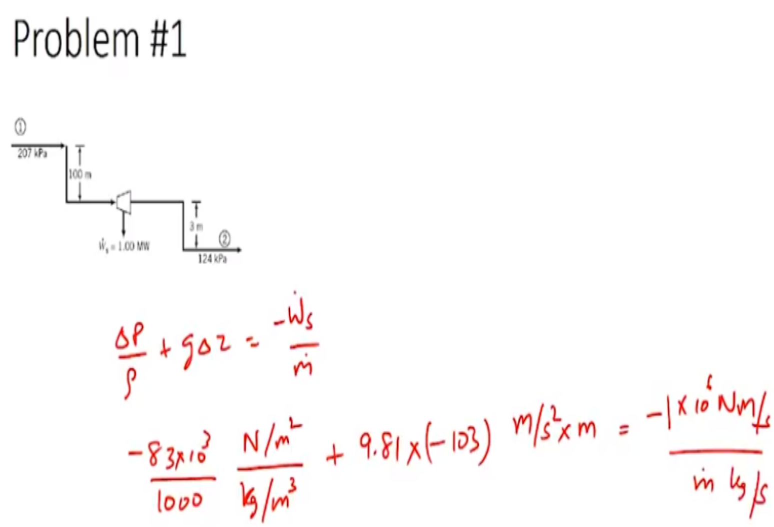

With this we can actually substitute all the values back into the mechanical energy which we had so which is delta P divided by rho + G delta Z = - WS dot / M dot so delta P we have calculated as -83 times 10 power 3 divided by density which is 1000 unit would be newton per meter square divided by kilograms per meters cube + G which is 9.81 times delta Z which is – 103.

So the units would be meters per second squared times delta Z is again meters the term on the right hand side is –WS dot divided by M dot our – WS dot has been given as 1 times 10 power 6 newton’s meters per second divided by M dot which would be in terms of kilo grams per second.

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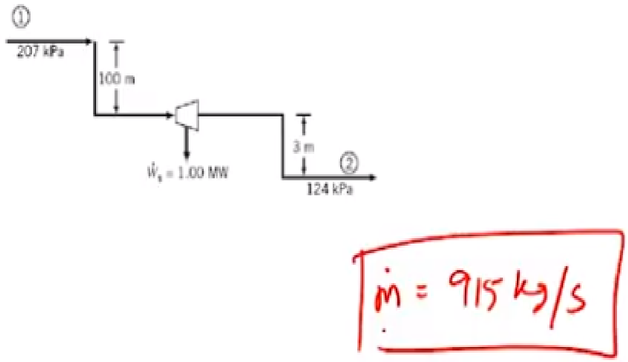
Problem #1



In this equation you can verify if the units are correct just try to substitute the actual values which are the basic units from SI units for Newton and Pascal and confirm that units for individual in terms are correct and if you do that you finally end up with dot 915 kilograms per second.

So when 915 of water flow through the turbine per second you would be able to generate 1 megawatt of shaft work. So with this we would have solved this problem let us look at another tutorial problem to look at fully understand and get into the habit of solving mechanical energy balance calculation.

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Problem #2

- One thousand liters of a 95% glycerol-5% water solution is to be diluted to 60% glycerol by adding a 35% glycerol solution pumped from a large storage tank through a 5-cm ID pipe at a steady rate. The pipe discharges at a point 23 m higher than the liquid surface in a storage tank. The operation is carried out isothermally and takes 13 min to complete. The friction loss is 50 J/kg. Calculate the final solution volume and the shaft work in kW that the pump must deliver, assuming that the surface of the stored solution and the pipe outlet are both at 1 atm. Data – ρ_{gly} = 1.26 kg/L and ρ_{water} = 1.00 kg/L.

In this problem we have given a mixture which contains glycerol and water and we have also been told that the volume of mixture is 1000 liter and need to finally prepare a mixture which would be containing 60% glycerol instead of 95% glycerol. So for identifying how much of each of these components which have to be fed we first need to know that density of these mixtures.

We only have the volumes and we have density for the pure components so the first for the problem would be to calculate the density of the mixtures so that we get the mass of the mixtures which is being fed. For this we first need to make an assumption the assumption would be the volume of the mixture would be equal to the summation of the volume of the pure component by that what I imply is when two things are added glycerol and water are added there are no interaction which results in change in volume.

Or in other words volume is additive with that assumption in mind let us first try to calculate the density of the mixture so that we can calculate the mass of the mixtures.

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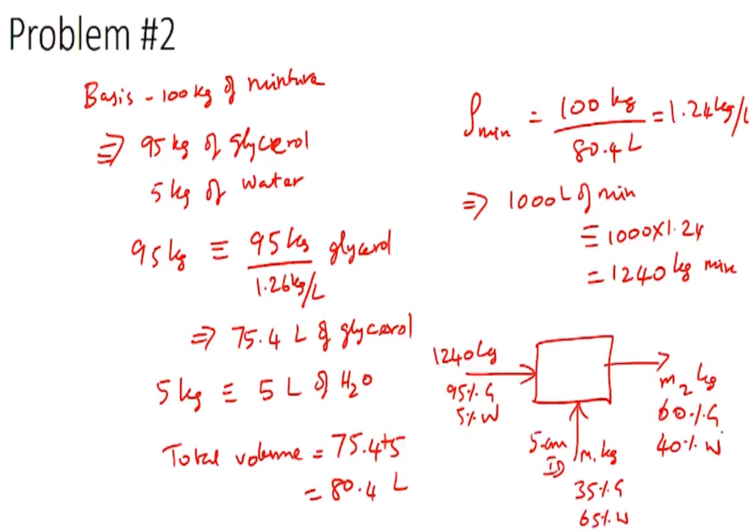

To calculate the density of the mixtures let us first calculate for the 95% for calculating the density of the 95% glycerol mixture we will assume a basis of 100 kilograms of mixture. So this 100 kilograms of mixture will contain 95 kilograms glycerol and 5 kilograms of water we know the density of glycerol and water those values are given to us. So this 95 kilograms of glycerol would actually occupy 95 kilograms divided by 1.26 kilograms per liter of glycerol.

So this would be the volume so this is mass divide by density so this value would be equal to 75.4 liters of glycerol. So what this means is 75.4 liter of glycerol would weight 95 kilograms similarly 5 kilograms of water would basically occupy a volume of 5 liter of water. So with this we know that total volume of the mixture would be equal to 75.4 + 5 giving you 80.4 liters as the total volume of 100 kilograms mixtures.

So from here we can calculate the density of the mixture as mass of the mixtures which is 100 kilograms divided by volume of mixture which is 80.4 liters giving us the density of 1.24 kilograms per liter. This implies that 1000 liter of mixtures would actually weigh 1000 times 1.24 which is equal to 1240 kilograms of mixture.

So the process flow chart would something like this you would have 1240 kilograms of 95% glycerol 5% water coming in you also have M1 kilograms 35% glycerol 65% water coming in through a pipe with an inner diameter of 5 centimeters. So the final stream is let us call it M2 kilograms and this contains 60% glycerol and the rest 40% is water.

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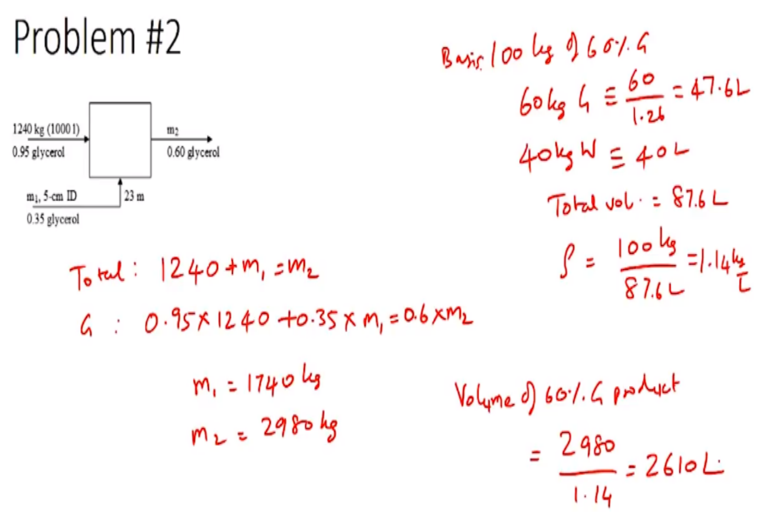

Now that we have calculated the mass of the mixture which is coming in we can now perform simple material balance calculations to identify the mass of 35% glycerol which needs to be fed and the mass of 60% glycerol which will be produced. So for that we can write a total mass balance which would be 1240 + M1 = M2 we can also write a glycerol balance which would be

0.95 times 1240 + 0.35 times M1 = 0.6 times M2. So you have 2 equations and 2 unknowns you can solve for M2.

So the value for M1 would be equal to 1740 kilogram and the value for M2 = 2980 kilograms the problem requires is not just to calculate the mass of 35% glycerol and 60% glycerol solution it has been asked that we calculate the volume of the final mixture which is 60% glycerol for that we need to know the density of the final solution. So using the same technique which we did earlier we will be able to calculate the density of final mixture which is 60% glycerol.

So what you would do is again assume a basis of 100 kilograms of 60% glycerol so this would be the basis for calculation of density the 100 kilogram of 60% of glycerol would contains 60kg of glycerol and it would also contains 50 kilograms of water. So the volume of occupied by 60 kilogram of glycerol would be 60 divided by density which is 1.26 and 40 kilogram of water would occupy of 40 liters of water.

So the volume of 60 kilogram of glycerol would be equal to 60 divided by 1.26 which is 47.6 liters. So the total volume would be equal to 87.6 liters so your density would be equal to 100 kilograms divided by 87.6 liters giving you a final value of 1.14 kilograms per liter. So using this we can calculate the volume of the 60% glycerol product. So you have the mass and the density so volume would be mass divided by density which is 2980 divided by 1.14 giving a final volume of approximately 2610 liters.

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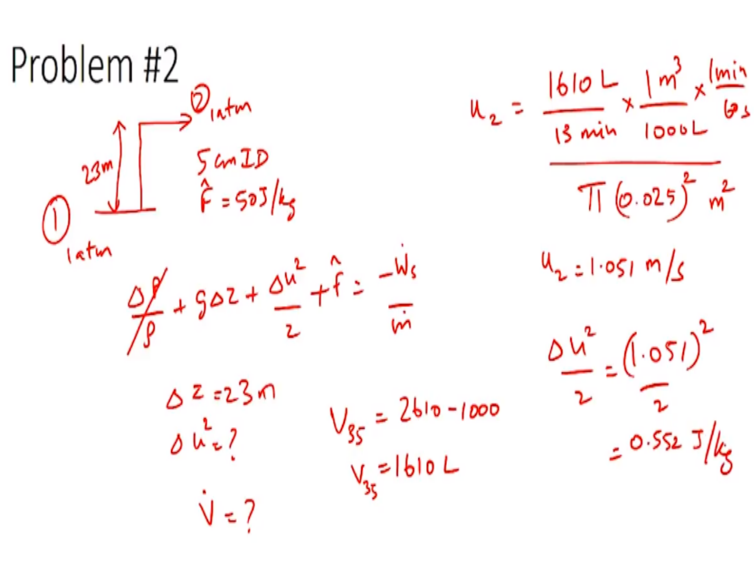

The next part of the problem expects to perform the mechanical energy balance calculations what are the information's that we have regarding the mechanical balance calculations. We know that there is a pump which needs to deliver the liquid at a height which is 23 meters above the point at which it is being discharged. The problem statement tells us that the pipe discharge us the liquid 23 meters above the liquid level which means this is the liquid level let us call these as point 1.

And from here the discharge happens at a height of 23 meters above the liquid and the discharge through a pipe which is 5 centimeter inner diameter and the fraction and the friction factor = 50 joules per kilogram we also know that at 0.1 and 2 the pressure is 1 atmosphere with all the information let us write the general mechanical energy balance equation and solve for the shaft work that needs to be delivered.

So we have delta P divided by rho divided by delta G + delta U square by 2 + F cap = - WS dot divided by M dot. As pressure change is 0 the first system can go to 0 you would have G delta Z where delta Z = 23 meters and we need to calculate delta U squared so we have the diameter of the pipe using that would be able to calculate the velocity at the point discharge we can assume that velocity of liquid which is in the storage tank would be 0 because it is as stationary liquid in a tank so that means we just to calculate the velocity at the point of discharge.

Friction factor value as been given to us so we can actually substitute the values we can calculate the shaft work to calculate the linear velocity at the point of discharge we have the diameter of pipe which means we can calculate the cross section of the pipe cross sectional area of the pipe using the volumetric flow rate we will be able to calculate the velocity which is the linear velocity.

Volumetric flow rate divided by cross sectional area will give you the velocity to calculate the volumetric of the 35% glycerol solution we need to know the total volume of the 35% glycerol solutions which has been supplied. We already know that the process happens in 13 minutes so we know the time frame so if we know the total volume we can calculate the volumetric flow rate.

As we already have assumed that volumes are additive we know that the total volume of the final 60% mixture and the initial volume of the 95% glycerol mixture so we can calculate the volume of the 35% glycerol mixture using these two values. So the volume of 35% percent glycerol mixture would be 2610 – 1000 giving a value of 1610 liters so this is the volume which we are adding.

We know that this is being added over a period of 13 minutes from which we can calculate the volume of volumetric flow rate from the volumetric flow rate we can calculate the linear velocity by divided it using the cross sectional area. So linear velocity of 0.2 would be = 1610 liters divided by 13 minutes times 1 meter cube per 1000 liters just for converting this to meters and the minutes can be converted to seconds as 1 minute contains 60 seconds.

So this now gives you a volumetric flow rate in terms of meter cubes per seconds and this can be divided by the cross sectional area which would be Pie R squared R here would be 0.025 meters. It is 5 centimeters inner diameter so which would mean 2.5 centimeter radius an converting that to meters you would have 0.25 meters. So from this we can calculate the velocity as 1.051 meters per second.

So we can calculate delta U squared by 2 as 1.051 whole squared divided by 2 which is equal to 0.552 joules per kilogram.

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Problem #2

$$g\Delta z = 9.81 \times 23 = 225.6\ J/kg$$

$$\hat{F} = 50\ J/kg$$

$$\dot{m} = \frac{1740}{13 \times 60} = 2.23\ kg/s$$

$$\frac{-\dot{W}_s}{\dot{m}} = \left(\frac{\Delta u^2}{2} + g\Delta z + \hat{F}\right)$$

$$\Rightarrow \dot{W}_s = -2.23\,(0.552 + 225.6 + 50)$$

$$\boxed{\dot{W}_s = -0.62\ kW}$$

As we already knew the value for delta Z we can calculate G delta Z as G is 9.81 times delta Z is 23 giving a value of 225.6 joules per kilogram. We already know that F cap is 50 joules per kilogram we can calculate the mass flow rate from the mass we already calculated from the material energy balances. So it is 1740 kilograms supplied in 13 minutes so interns of kilograms per second it would be 2.33 kilograms per grams per second.

The equation would be –WS dot divided by M dot is equal to delta E squared by 2 + G delta Z + F cap so this would mean WS dot would be equal to -2.23 times 0.552 + 225.6 + 50 giving a value for WS dot as -.62 kilowatts. So the shaft work which would be delivered the pump would be 0.62 kilowatts.

So with this we have performed a different types of mechanical energy balance calculations hopefully you have familiarized yourselves with performing this calculations try more problems along these lines so that you are familiar with the calculation steps and also you can understand how the concepts need to be applied with that I would like to thank you for today's lecture and meet you in the next class.