

**Material and Energy Balances**  
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**Module No # 08**  
**Lecture No # 38**  
**Mechanical to Energy Balances**

Welcome to today's lecture on mechanical energy balances in the previous lecture we looked at some of the introduction aspects of energy balances. We primarily focused on chemical processes today we will talk about mechanical processes which are seen in any chemical and biochemical industry.

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## Chemical vs. Mechanical Processes

- **Chemical**
  - Reactors, distillation columns, evaporators, heat exchangers etc.
  - Shaft work, KE and PE are not important
  - $Q = \Delta U$  (closed systems) and  $Q = \Delta H$  (open systems)
- **Mechanical**
  - Compression of gases, pumping of liquids
  - Shaft work, KE and PE are important
  - Heat flows and internal energy are secondary

What is the difference between chemical and mechanical process? A chemical process would be something like a reactor or distillation column, evaporator, heat exchanger etc. In such systems shaft work, kinetic energy and potential energy changes are not very critical however you would have heat and change in internal energy or enthalpy which play a major role when it comes to energy balance calculations.

In one of the earlier problems we tried to identify the change in kinetic energy and also incorporate that while we perform energy balance coactions. But if you remember we saw that the effect of change in kinetic energy was very small and it was about 100 of the effect of change in enthalpy. So such processors where change in enthalpy and heat or change in internal energy

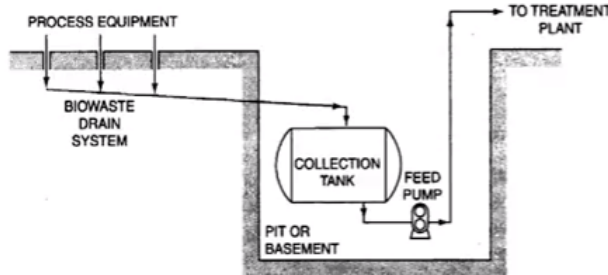
and heat play a major role are called as a chemical processes. So you would use internal energy for closed system and enthalpy changes for open system. There are another type of process which is called as the mechanical process.

Processors like compressor of gases a pumping of liquids or mechanical process here shaft work change in kinetic and potential energies are much more crucial. Heat flow in internal energy changes are secondary this is because here usually there is not a big change in temperature there might be some change in temperature but that is not very significant.

So for this reason for these processes which are called as mechanical processes we are more concerned about accounting for shaft work and change in kinetic and potential energies. Let us look at some of these systems and let us try to derive how an energy balance equation would look like for such mechanical processes.

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## Mechanical Energy Balance



- What size pump should be used?
- What pressure drop occurs between collection tank drain and the pipeline to the treatment plant?

What you see here is a mechanical process there is a collection tank which basically collect the bio waste from the drain and this bio waste is then pumped using the feed pump to the treatment plant. You need to perform mechanical energy balances to calculate what size pump should be used what should be the capacity of the pump and what would be the pressure drop which are occurs between the collection tank drain and the pipe line to the treatment plant.

So to understand know these values is crucial for designing appropriate equipment for this reason we need to fully understand how to mechanical energy balances.

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## Mechanical Energy Balance Equation: Derivation

$$\begin{aligned} I &= 0 \\ \Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p &= \dot{Q} - \dot{W}_s \\ \Delta \dot{U} + \Delta(P\dot{V}) + \frac{m\dot{u}^2}{2} + m\dot{g}\Delta Z &= \dot{Q} - \dot{W}_s \\ \text{Dividing by } \dot{m}, & \\ \Delta \hat{U} + \Delta \hat{P}\hat{V} + \frac{\Delta u^2}{2} + g\Delta Z &= \frac{\dot{Q} - \dot{W}_s}{\dot{m}} \\ \frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta Z + \left( \Delta \hat{U} - \frac{\dot{Q}}{\dot{m}} \right) &= \frac{-\dot{W}_s}{\dot{m}} \end{aligned}$$

What is a mechanical energy balance equation? We already had energy balance equation for open and close system those who are energy balance equations. Now let us convert them to mechanical energy balance where focus is primarily on shaft work, kinetic energy and potential energy changes. For an open system at steady energy balance equation becomes input – output. From the general energy balance equation for an open system we have  $\Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p = \dot{Q} - \dot{W}_s$ .

This  $\Delta \dot{H}$  can be written in terms of internal energy and specific volume. So let us do that the  $\Delta \dot{H}$  value you have can actually written in terms of internal energy. So then the equation would become  $\Delta \dot{U} + \Delta(P\dot{V}) + \dot{E}_k$  which would be  $M \dot{\Delta U} + \frac{M \dot{u}^2}{2} + \dot{E}_p = \dot{U} \dot{Q} - \dot{W}_s$ . This entire equation can be divided by  $M \dot{\Delta U}$ .

So when we do this what you end up with is  $\Delta \hat{U} + \Delta \hat{P}\hat{V} + \frac{\Delta u^2}{2} + g\Delta Z = \frac{\dot{Q} - \dot{W}_s}{\dot{m}}$ . Rearranging this equation we can get  $\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta Z + \Delta \hat{U} - \frac{\dot{Q}}{\dot{m}} = \frac{-\dot{W}_s}{\dot{m}}$ . What I have done here is  $\Delta P \times V_{cap}$  I have converted to divided by  $\rho$ .  $V_{cap}$  is specific volume which is equal to  $1/\rho$  so by substituting density in

terms of specific volume we get  $\Delta P$  divided by  $\rho$  instead of  $\Delta PV$  cap. So the rest of the terms are the same except for some rearrangements in the equation.

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## Mechanical Energy Balance Equation: Derivation

$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta Z + \hat{F} = -\frac{\dot{W}_s}{\dot{m}} \quad \text{--- (1)}$$

$$\hat{F} = 0, \dot{W}_s = 0$$

$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta Z = 0 \quad \text{--- (2)}$$



The term within the bracket which is  $\Delta U$  cap –  $Q$  dot divided by  $M$  dot is also called as friction factor which can be written as  $F$  prime. So the equation then becomes  $\Delta P$  divided by  $\rho$  +  $\Delta U$  square by 2 +  $G$   $\Delta Z$  +  $F$  cap =  $WS$  dot /  $M$  dot. In situations where frictions factor = 0 such as smooth pipes something like that this  $F$  prime or  $F$  cap can actually be equated to 0 so friction factor can be 0 in certain conditions.

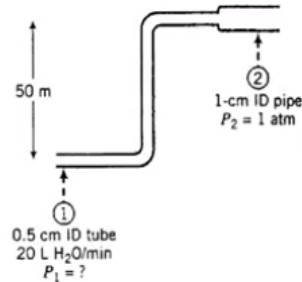
If shaft work is also 0 then  $WS$  dot will also be equal to 0 giving the equation  $\Delta P$  divided  $P$   $\rho$  +  $\Delta U$  squared / 2 +  $G$   $\Delta Z$  = 0. The first equation we have here is  $\Delta P$  divided by  $\rho$  +  $\Delta U$  squared +  $G$   $\Delta Z$  +  $F$  cap = -  $WS$  dot /  $M$  dot is called as the general mechanical balance equation. The second equation which I have at the end which is  $\Delta P$  divided by  $\rho$  +  $\Delta U$  squared by  $\rho$  +  $G$   $\Delta Z$  = 0 is the vernalize equation.

It is the simplified form of mechanical energy balance equation applicable to specific system where there is no friction or shaft work. So with this derivation we can move on to performing some mechanical energy balance problems.

**(Refer Slide Time: 07:18)**

# Example #1

- Water flows through the system shown here at a rate of 20 L/min. Estimate the pressure required at point 1 if friction loss is negligible.



Here is the first example problem water flow through the system shown here at the rate of 20 liters per minute estimate the pressure required at point 1 if friction loss is negligible. So you have been given a system where water is flowing you have also been told that friction loss is negligible so using this system let us perform mechanical energy balance calculations.

(Refer Slide Time: 07:43)

## Example #1



$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta Z + \hat{F} = -\frac{W_s}{M}$$

$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta Z = 0$$

$$u_1 = \frac{20 \frac{\text{L}}{\text{min}} \times \frac{1 \text{ m}^3}{1000 \text{ L}} \times \frac{1 \text{ min}}{60 \text{ s}}}{3.14 (0.25)^2 \frac{\text{m}^2}{4} \times \frac{1 \text{ m}^3}{1000 \text{ L}}}$$

$$u_1 = 17 \text{ m/s}$$

$$u_2 = 4.24 \text{ m/s}$$

$$\Delta u^2 = u_2^2 - u_1^2 = (4.24)^2 - 17^2 = -271 \text{ m}^2/\text{s}^2$$

The general mechanical balance equation would be  $\Delta P$  divided by  $\rho$  +  $\Delta U$  squared by 2 +  $G \Delta Z$  +  $F_{cap} = -WDS \text{ dot} / M \text{ dot}$ . In this system we have been told that there is no friction which means the friction factor can be equated to 0 and we also know that there is no shaft work because there are no moving parts in the system which means  $W_s$  also goes to 0. So for this system we can use the Bernoulli equation for performing mechanical energy balances.

So Bernoulli equation is  $\Delta P / \rho + \Delta U^2 / 2 + G \Delta Z = 0$  we have been asked to calculate the pressure at point 1 we also have the information about pressure at point 2 which is 1 atmosphere we have information about  $\Delta Z$  because we know the difference in height between point 1 and point 2 is 50 meters. The value we need to calculate is  $\Delta U^2$  for this we need to know the velocity of water flowing at point 1 and point 2.

As the cross sectional areas are different the velocities also be different based on the volumetric flow rate which is 20 liters per minute we can calculate the velocity or the linear velocity as volumetric flow rate divided by cross sectional area. So let us try and do that linear velocity at point U1 would be equal to 20 liters per minute and If you want to convert this liter to meter it would be meter cube it would be 1 meter cube contains 1000 liters and we also have to convert minutes to seconds which would be 1 minute contains 60 seconds.

So this is now converted to meter cube per second you want to divide this by cross sectional area which would be  $\pi R^2$   $\pi$  is 3.14  $R$  here is 0.25 squared this point 25 is the radius of the pipe at point 1 so this is in terms of centimeters . So we need to convert this to meters so this is actually centimeter squared so we want to convert this to meter square so this would be 1 meter squared contains 10 power 4 centimeter squared.

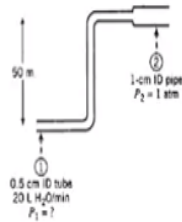
So in this equation you have centimeter squared getting cancelled minute getting cancelled meter squared getting cancelled and you end up with meters per second which is the linear velocity which is U1 is actually equal to 17 meters per second similarly we can calculate U2 which should be equal to 4.24 meters per second you actually do not have to perform all the calculations which we did to calculate U2 what you see is the cross sectional area 0.2 is going to be 4 times the cross sectional area at point 1.

This is because you have the radius which is twice the radius of 0.1 at 0.2 so the radius at 0.2 is two times radius at point 1 which means the cross sectional area at 0.2 will be 4 times cross sectional area at point 1. This means the linear velocity will be 1 fourth of linear velocity at point 1. So therefore we get  $U_2 = 4.24$  meters per second. So we need to calculate that  $\Delta U^2$  which would be  $U_2^2 - U_1^2$  which is  $4.24^2 - 17^2$  giving a change in velocity squared as -271 meter squared by second squared.

We can substitute this value to verbalize equation and we will be able to calculate the pressure drop.

(Refer Slide Time: 12:10)

### Example #1



$$\Delta P = P_2 - P_1 = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$$

$$\Delta Z = 50 \text{ m}$$

$$\Delta u^2 = -271 \text{ m}^2/\text{s}^2$$

$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g \Delta Z = 0$$

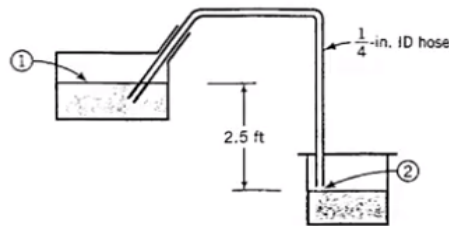
$$P_1 = 4.56 \text{ bar}$$

We have delta P which is equal to P – P1 and we have delta Z = 50 meters and we have delta U squared equal to -271 meter squared per second squared. So here we already know that the value for P1 which is 1 atmosphere which is equivalent to 1.013 times 10 power 5 pascals. Now using this value we can substitute all of the them into the verbalize equation which is delta P / divided by Rho + delta U squared by 2 + G delta Z = 0 substituting all these values we can get the value for pressure at 0.1 as 4.56 bar.

(Refer Slide Time: 13:12)

## Example #2: AE units

- Gasoline ( $\rho = 50.0 \text{ lb}_m/\text{ft}^3$ ) is to be siphoned from a tank. The friction loss in the line is  $0.80 \text{ ft}\cdot\text{lb}_f/\text{lb}_m$ . Estimate how long it will take to siphon 5.00 gal, neglecting the change in liquid level in the gasoline tank during the process and assuming that both point 1 and point 2 are at 1 atm.



Problem adapted from Felder and Rousseau, Elementary Principles of Chemical Processes, 3rd edition Wiley-India

Please perform appropriate unit conversion to verify if the value I have is correct moving on to the next problem you will try and solve this example problem where all the parameters are given to us in American engineering units. If you are not familiar with American engineering units I recommend that you re-watch the video where we discussed units and dimensions through the aspects related American engineering system.

We will try and apply those principles here in these problems and you will be able to understand what is unique to the American engineering system. Gasoline with a density of 50 pound mass per foot cube is to be siphoned from a tank. The friction loss in the line is 0.8 feet pound force per pound mass. Estimate how long it will take to siphon 5 gallons, neglecting the change in liquid level in the gasoline tank during the process and assuming that 0.1 and 0.2 are at 1 atmosphere.

So here you have a friction term that uses pound force as I mentioned right now you should go back and look up the video on units and dimension to understand the inherent problems with the pound force term. You should try to remember and recollect what GC which is the conversion factor represents we will apply that conversion factor while we are solving this problem.

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The diagram shows a pipe system with a tank at point 1 and a lower tank at point 2. The pipe has a diameter of 1/4 inch. The vertical distance between the two tanks is 2.5 feet. Handwritten equations and calculations are as follows:

$$\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g \Delta z + \hat{F} = -\frac{w \dot{s}}{M \dot{m}}$$

$$\Delta u^2 = u_2^2$$

$$\Delta z = -2.5 \text{ ft}$$

$$\hat{F} = 0.8 \frac{\text{lb}_f \cdot \text{ft}}{\text{lb}_m}$$

$$\left(\frac{u_2^2}{2}\right) \frac{\text{ft}^2}{\text{s}^2}$$

$$(\Delta z g) \frac{\text{ft}}{\text{s}^2} \cdot \text{ft} = \frac{\text{ft}^2}{\text{s}^2}$$

$$g_c = 32.174 \frac{\text{lb}_m \cdot \text{ft}}{\text{lb}_f \cdot \text{s}^2}$$

The general mechanical energy balance would be  $\Delta P$  divided by  $\rho$  +  $\Delta U$  squared by 2 +  $G \Delta Z$  +  $F_{cap}$  = -  $W S$  dot divided by  $M$  dot. So here we have been told that the pressure at 0.1 and 0.2 are both one atmosphere which means  $\Delta P$  would be equal to 0. We also do not have any moving in this system between shaft work will be equal to 0 we do have a change in height there is a 2 and half feet change in height, and we also would have a change in velocity because of 0.1 water is stationary inside the tank and at point 2 you actually have water flowing out of the pipe.

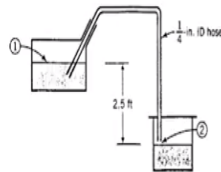
So there would be some velocity at 0.2 so there is going to be at  $\Delta U$  squared  $F_{cap}$  is also been given. Now let us try to calculate each of these parameters  $\Delta U$  squared would be equal to  $U_2$  squared because  $U_1$  squared is 0 and  $\Delta Z = -2.5$  feet and you also been given  $F_{cap} = 0.8$  pound force times feet divided by pound mass. Now let us look at each of the terms and try to understand what units we have been in.

We ultimately have to have all the units in terms of feed pound force per pound mass for the equation to be valid that is the rule for dimensional homogeneity. So taking that into account the  $U_2$  squared divided by 2 which you would calculate would actually be in terms of feet squared by second squared. This now needs to be converted to feet times pound force divided by pound mass. The second term which you calculate is in terms of  $\Delta Z$  times  $G$ .

So G is acceleration due to gravity which would be feet per second squared and delta Z would again be feet. So therefore units for delta Z times G would also be feet squared by second squared this also needs to be converted to pound force times feet divided by pound mass. If you remember the term conversion factor is GC we know that GC is numerically equal to the acceleration due to gravity at 45 degree latitude. So this would be 32.174 if you remember the units and that units for GC would be pound mass times feet divided by pound force time second squared. B

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### Example #2: AE units



$$g_c = 32.174 \frac{\text{lb}_m \cdot \text{ft}}{\text{lb}_f \cdot \text{s}^2}$$

$$\frac{U_2^2}{2} \frac{\text{ft}}{\text{s}^2} \times \frac{1}{g_c} \frac{\text{lb}_f \cdot \text{s}^2}{\text{lb}_m \cdot \text{ft}} + 32.174 \frac{\text{ft}}{\text{s}^2} \times (-2.5) \text{ft} \times \frac{1}{g_c} \frac{\text{lb}_f \cdot \text{s}^2}{\text{lb}_m \cdot \text{ft}} + F = 0$$

$$U_2 = 10.5 \text{ ft/s}$$

By using this GC appropriately in each of these terms we can convert all these terms to the same units. So what you would have is  $U_2^2 / 2$  times feet squared per second squared / GC which would give you units of pound force divided by pound mass times feet second squared so this would be the units or GC.

So here you would have second squared cancelled feet cancelled and you would be having final units as feet times pound force divided by pound mass, which is the same units as the friction factor that has been given + you have 332.174 which is the G value in the G delta Z the unit or this would be feet per second squared times delta Z which is - 2.5 feet in this again divided by GC the units being pound force second squared divided by pound mass feet.

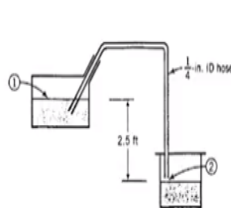
So you would have feet and second square cancelled and you would have the same units as  $F_{cap} + F = 0$ . So what happens is you would have to calculate  $U_2$  squared and you would be able

to solve this problem we already know the value for GC is 32.174 pound mass feet per pound force second square. Substituting for value of GC into this equation and also accounting for F cap you will be able to calculate the value for U2 which is the velocity at point 2.

U2 to be equal to 10.5 feet per second so this is the linear velocity we have been asked to calculate the time taken for siphoning 5 gallons which means we need to know the volumetric flow rate. So we can convert the linear velocity to volumetric flow rate by multiplying with the cross sectional area. the cross sectional area for the pipe can be calculated using the inner diameter of the pipe.

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### Example #2: AE units



$$V = 10.5 \frac{\text{ft}}{\text{s}} \times 3.14 \times (0.125 \text{ in})^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in}}\right)^2$$

$$V = 3.58 \times 10^{-3} \text{ ft}^3/\text{s}$$

$$\text{Time} = \frac{\text{Volume}}{\text{Vol. flow rate}} = \frac{5 \times 0.1337}{3.58 \times 10^{-3}} = 187 \text{ s}$$

$$\text{Time} = 187 \text{ s}$$

Volumetric flow rate would be equal to velocity of 10.5 in term of feet per second times cross sectional which is 314 or pipe times R squared R is 0.125 inch squared and this inch can be converted to feet as 1 feet is basically 12 inches and using this we would get volumetric flow rate as 3.58 times 10 power -3 feet cube per second. This gives us the volumetric flow rate in terms of feet cube per second. We now need to calculate the time taken for training 5 gallons of gasoline.

So time taken would be equal to volume divided by volumetric flow rate however here volume is given in terms of gallons and flow rate in terms feet cube per second we need to know the conversion factor for one gallon to feet cube so 1 gallon = 0.1337 feet so using that you now 5 gallons would be 0.1337 times 5 divided by 3.58 times 10 power -3 which is the volumetric flow rate giving the total time taken as 187 seconds.

So it takes approximately 187 seconds to drain 5 gallons of gasoline from this system with this you would have seen how American engineering system is used how the conversion factor GC has to be applied while we perform involving the term pound force. We also looked at conversion factor from gallon to volumetric flow rate in terms of feet cube so converting gallon to feet cube we have to use the conversion factor which was not a number to remember.

Unlike what we usually use in SI unit with this we have come to conclusion of mechanical energy balances we will perform some tutorial problems in the next lecture until then thank you and good bye.