

**Material and Energy Balances**  
**Prof.Vingesh Muthuvijayan**  
**Department of Biotechnology**  
**Indian Institute of Technology – Madras**

**Module No # 08**  
**Lecture No # 37**  
**Introduction to Energy Balances: Tutorials**

Hello everybody, welcome to today's class on introduction to energy balances we have looked at some of the fundamental aspects for performing energy balance calculations in open and closed systems, and also learnt about thermodynamic tables in the previous few lectures. Today we will perform some calculations and problems as a tutorial session so that we get completely familiarized with the concepts that we have learnt and we would feel comfortable solving the problems which would be given in terms of energy balances.

**(Refer Slide Time: 00:55)**

## Energy Balances on an Open System

- Since blood is refrigerated for storage, it is warmed before contact with a patient to prevent hypothermia. Calculate the rate of heat required to continuously warm 10.0 L/min of blood from 30°C to 37°C using an electric heater. A stirrer adds work to the system at a rate of 0.50 kW. Assume the heat capacity of blood is a constant at 4.185 J/g.°C and the density of blood is 1.0 g/mL. Working volume of the tank is 1.0 L.



Let us first start with the first problem for the tutorials since blood is refrigerated for storage it is warmed before contact with patient before hypothermia. Calculate the rate of heat required to continuously warm 10 liters per min of blood from 30 degree Celsius to 37 degree Celsius using an electric heater a stirrer adds work to the system at a rate of 0.5 kilowatts.

Assume the heat capacity of blood is a constant at 4.185 joules per gram degree Celsius the density of blood is 1 gram per ML. Working volume of tank is 1 liter so we have been given all the information that we need to perform the energy balances for this problem.

(Refer Slide Time: 01:43)

## Warming up blood

$$\Delta H + \Delta E_k + \Delta E_p = \dot{Q} - \dot{W}_s$$

$$\dot{Q} = \Delta H + \dot{W}_s$$

$$\dot{Q} = \Delta H - (0.5) \text{ kW}$$

$$\Delta H = m \int_{T_1}^{T_2} C_p dT = 10 \frac{\text{L}}{\text{min}} \times 1000 \frac{\text{g}}{\text{L}} \times \frac{1 \text{ min}}{60 \text{ s}} \int_{30}^{37} 4.185 dT$$

$$\Delta H = 4.882 \text{ kW}$$

Let us start solving the problem with the flow chart so here we have blood flowing 10 liters per minute. So you also have been given that work is done on the system in the form of stirrer which is given as the shaft work  $W_s$  0.5 kilo watts would this work is positive or negative? According to our convention we have assumed that work done by the system is positive and work done on the system is negative if that is going to be the case here work is done on the system which means this would be considered to be negative work.

So  $W_s$  is given as 0.5 negative 0.5 kilowatts now we also know that heat is supplied to the system in the form of  $\dot{Q}$  which is the heat and then we can calculate this  $\dot{Q}$  using our general energy balance equation for an open system. For an open system the general energy balance equation is  $\Delta H \dot{+} \Delta E_k \dot{+} \Delta E_p \dot{=} \dot{Q} - \dot{W}_s$ .

So in this equation we can ignore kinetic and potential energies because there is actually no change in height which has been given to us and there is also no change in the velocity of the blood which is flowing because of this the equations simplifies to  $\dot{Q} \dot{=} \Delta H \dot{+} \dot{W}_s$  so now we need to calculate  $\Delta H \dot{+} \dot{W}_s$  is already given to us. So  $\dot{W}_s$  can be entered -0.5 so the value for  $\dot{Q}$  would be  $\Delta H \dot{-} 0.5$  kilowatts.

So if we can calculate  $\Delta H \dot{+}$  in terms of kilo watts we can substitute the value here and find the value for heat which is being supplied. So what would be  $\Delta H \dot{+}$  so we have been given

the CP value for blood so using this we know that the delta H dot can be calculated as M dot integral CPDT initial to T5. So here M dot would be 10 liters per minute times 1000 grams per liters.

So this would give us grams per minute so if we need to convert to grams per second we would multiple it by 1 minute contains 60 seconds and then this is the M dot value + integral of CP has been given as 4.185DT with temperature going from 30 degree Celsius to 37 degree Celsius. So this value can be calculated as 4.882 kilowatts so delta H dot = 4.882 kilowatts.

**(Refer Slide Time: 04:55)**

## Warming up blood

$$\begin{aligned}\dot{Q} &= \dot{\Delta H} - 0.5 \text{ kW} \\ &= 4.882 - 0.5 \\ \dot{Q} &= 4.382 \text{ kW}\end{aligned}$$

As we have Q dot equals delta H dot – 0.5 kilowatts so we can substitute the value for delta H dot here which is 4.882 – 0.5 giving you a value of 4.382 kilowatts so the heat that needs to be added to the system would be 4.382 kilowatts. So this was the simple energy balance problem for an open. Let us more of these example problems to get us familiarize with performing energy balances.

**(Refer Slide Time 05:29)**

## Energy Balances on an Open System: Steam Tables

- To sterilize a fermenter, two streams of water are fed. Feed 1 is 120 kg/min at 30°C and feed 2 is 175 kg/min at 65°C. The pressure inside the fermenter is 17 bar (absolute). The exiting steam leaves the fermenter through a 6-cm ID pipe. Calculate the required heat input to the fermenter in kJ/min if the steam leaving is saturated at the fermenter pressure. Neglect kinetic energies of the liquid inlet streams.



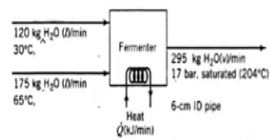
Here is the second system again you have a open system however here we will steam table instead of using M integral CPDT. So let us look at the question to sterilize the fermenter two steams of water are fed Feed 1 is 120 kilograms per minute at 30 degree Celsius and feed 2 is 175 kilograms per minute at 65 degree Celsius the pressure inside the fermenter is 17 bar absolute the existing steam leaves the fermenter through a 6 centimeter inner diameter pipe.

Calculate the require heat input to the fermenter in terms of kilojoules per minute if the steam leaving is saturated at the fermenter pressure you can neglect kinetic energy of the liquid inlet steam. If you read the question carefully we have all the information about the system which are coming in and leaving the system we know that we have two feeds one at 120 kilograms and other at 175 kilograms per minute.

You also know the temperature at which they are coming in and they are states we also have information about the pressure inside the fermenter and the exiting steams state. We have been told that existing steam is saturated and we also know that the existing steam is leaving through a 6 centimeter inner diameter pipe. The problem gives us that we have to ignore kinetic energy of the inlet stream which means we would have to account for the kinetic energies of the exist steam.

**(Refer Slide Time: 07:11)**

## Fermenter Sterilization



$$\Delta \dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p = \dot{Q} - \dot{W}_s$$

$$\dot{Q} = \Delta \dot{H} + \Delta \dot{E}_k$$

$$\Delta \dot{H} = ?$$

$$\hat{H} \text{ for } H_2O(l) \text{ at } 30^\circ C = 125.7 \text{ kJ/kg}$$

$$\hat{H} \text{ for } H_2O(l) \text{ at } 65^\circ C = 271.9 \text{ kJ/kg}$$

$$\hat{H} \text{ for sat. vap. } H_2O(v) \text{ at } 17 \text{ bar} \\ = 2793.4 \text{ kJ/kg at } 204^\circ C$$

$$\Delta \dot{H} = \sum_{\text{out}} \dot{m}_i \hat{H}_i - \sum_{\text{in}} \dot{m}_i \hat{H}_i \\ = (295 \times 2793.4) - (120 \times 125.7 + 175 \times 271.9)$$

$$\Delta \dot{H} = 7.61 \times 10^5 \text{ kJ/min}$$

So using this information let us try to solve this problem so here is the flow chart which describes the process we have 120 kilograms of water liquid at 30 degree Celsius at 175 kilograms of water liquid at 65 degree Celsius coming in heat is supplied to the system and we have 295 kilograms of water vapor leaving at saturated steam at 17 bar. So based all this information we will have to perform energy balance calculation.

So this being an open system the general energy balance equation would be  $\dot{H} + \Delta \dot{E}_k + \Delta \dot{E}_p = \dot{Q} - \dot{W}_s$ . Now we cannot ignore the kinetic energy term because we have been told that only the inlet energy can be ignored. So we have to calculate the outlet kinetic energies so this term would exist obviously  $\Delta \dot{H}$  will exist because you have change in enthalpy due to phase change which is happening and also due temperature change which is happening.

So we can ignore potential energy changes because here is no change in position or height in steam which has been given to us. So we can assume that this is negligible at it is 0  $\dot{Q}$  and  $\dot{W}_s$  have to be looked at so  $\dot{Q}$  is the heat which is supplied for the change in enthalpy and the kinetic energy  $\dot{W}_s$  would go to 0 because there are no moving parts we do not have any shaft or piston which is there.

So which means  $\dot{W}_s$  can also go to 0 so the equation simplifies to  $\dot{Q} = \Delta \dot{H} + \Delta \dot{E}_k$  so let us first calculate  $\Delta \dot{H}$ . For calculating  $\Delta \dot{H}$  what do we need? We need

about the inlet enthalpies and the outlet enthalpies. So this we can identify by using steam tables so please go back and refer to steam table from one of the text books or if you access to some other steam tables.

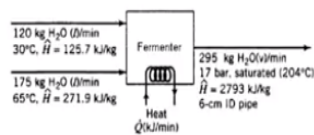
Please refer to the steam tables which you have to identify the enthalpies for the inlet steam which is liquid water at 30 degree Celsius and liquid water at 65 degree Celsius you would also have to identify the enthalpy specific enthalpy for saturated vapor at 17 bars which is your exit conditions. Using these steam tables we can identify the specific enthalpy for water liquid at 30 degree Celsius = 125.7 kilojoules per kilogram.

Similarly the specific enthalpy for water liquid at 65 degree Celsius = 271.9 kilojoules per kilogram so looking at saturated steam tables we would be able to get the specific enthalpy value for saturated vapor which is water at 17 bar as 2793.4 joules per kilogram and the temperature is 204 degree Celsius. So having these values we can calculate  $\dot{H}_{out} - \dot{H}_{in} = \sum \dot{m}_{out} \hat{H}_{out} - \sum \dot{m}_{in} \hat{H}_{in}$ .

So this would be 295 times 2793.4 – 120 times 125.7 + 175 times 271.9 so this value would be  $\dot{H} = 7.61 \times 10^5$  kilojoules per minute. So this gives you the change in enthalpy for the process.

**(Refer Slide Time: 11:36)**

## Fermenter Sterilization



$$\hat{V} \text{ for exit stream} = 0.1166 \text{ m}^3/\text{kg}$$

$$\text{Vol. flow rate} = (0.1166 \times 295) \text{ m}^3/\text{min}$$

$$A = \pi r^2 = \pi \times 3^2 \times \frac{1}{10^6} = 2.83 \times 10^{-3} \text{ m}^2$$

$$\text{Velocity} = \frac{\text{Vol. flow rate}}{A} = \frac{295 \times 0.1166 \text{ m}^3/\text{min}}{2.83 \times 10^{-3} \text{ m}^2} \times \frac{1}{60}$$

$\text{Velocity} = 202 \text{ m/s}$

So the next term we need to calculate is the change in kinetic energy as we are assuming the kinetic energy of the inlet steams are 0. Calculating the kinetic energy of the exist steam will give us the total change in kinetic energy. So the for calculating the kinetic energy of the exit steam we need to calculate the velocity so for that we need to be able to calculate the volumetric flow rate divided by cross sectional area.

So first thing is we need to identify volumetric flow rate how can we convert the mass flow rate that we have into volumetric flow rate. The steam tables gives us information not just about internal energy and enthalpy it also gives us the specific volume using specific volume we can convert the mass flow rate which we have into volumetric flow rate. So let us first identify the specific volume for the exit steam.

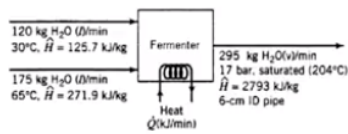
For specific volume for the exit steam would be equal to 0.1166 meter cube per kilogram so this is the specific volume for the exit steam based on the steam tables now we need to multiply this specific volume with the mass with volumetric flow rate so this the volumetric flow rate would be equal to 0.1166 times 295. So this would give us volumetric flow rate in terms of meter cubes per minute.

Now we have to divide this volumetric flow rate by cross sectional area so cross sectional are A can be calculated using the inner diameter which has been given. So cross sectional area is pie R squared which would be pie times inner radius of pie. So this would give us the area cross sectional area in centimeter cube and converting into meter cube we would have to divided it by 10 power 2.

So this gives us the value of 2.83 times 10 power -3 meter squared so this is the cross sectional area. So we can get velocity as volumetric flow rate divided by cross sectional area which gives us 295 times 0.1166 meter cube per minute divided by 2.83 times 10 power -3 meter squared and we would also like to convert this minutes to second so we divided by a 60. So giving us the velocity as 202 meters per second.

**(Refer Slide Time: 14:50)**

## Fermenter Sterilization



$$\begin{aligned}\dot{E}_k &= \frac{1}{2} \dot{m} V^2 \\ &= \frac{1}{2} \times 295 \frac{\text{kg}}{\text{min}} \times \left(202 \frac{\text{m}}{\text{s}}\right)^2 \\ &= 6.02 \times 10^3 \text{ kJ/min} \\ \dot{Q} &= \dot{H} + \dot{E}_k = 7.61 \times 10^5 + 6.02 \times 10^3 \\ \dot{Q} &= 7.67 \times 10^5 \text{ kJ/min}\end{aligned}$$

So this is the velocity of this exist now that we have the velocity we can calculate kinetic energy which is  $\Delta E_k$  dot as half  $MV$  squared. So  $M$  dot is 295 kilograms per minute and we have velocity in terms of meter per second given as 202 so this kilogram meter square per second squared will be converted to kilo joules as a derived units so we get a value of 0 to 2 times 10 power 3 kilojoules per minute.

So substituting this back into the equation for heat we get  $Q$  dot =  $\Delta H$  dot +  $\Delta E_k$  dot which is 7.61 times 10 power 5 + 6.02 times 10 power 3 giving us  $Q$  dot T equals 7.67 times 10 power 5 kilo joules per minute. So this is the heat which needs to be supplied to the system. What I want you understand from the calculation which we did is even while we ignored the kinetic energy for the inlet steams and accounted only for kinetic energy of the exit steams we got a value which was 100 value which we would have got in for the change in enthalpy.

So the change in enthalpy was in the order or 10 power 5 whereas the change in kinetic energy was in the order of 10 power 3 this indicates that if for a thermodynamic process where there is temperature and phase the change in specific enthalpy or change in enthalpy is a lot more significant then change in kinetic energy that is why in many cases we tend to ignore change in kinetic energy while perform calculations.



So here we have been able to calculate the total change in heat even by accounting kinetic energy which accounted for only about 1% of the final answer. Now that we have clear understanding of this let us move on to the next tutorial problem.

**(Refer Slide Time: 17:02)**

## Simultaneous Material and Energy Balances

- Saturated steam at 1 atm is discharged from a turbine at a rate of 1150 kg/h. Superheated steam at 300°C and 1 atm is needed as a feed to a heat exchanger; to produce it, the turbine discharge stream is mixed with superheated steam available from a second source at 400°C and 1 atm. The mixing unit operates adiabatically. Calculate the amount of superheated steam at 300°C produced and the required volumetric flow rate of the 400°C steam.



Until this point we have been performing energy balance calculations only in this problem we will look at simultaneous material and energy balances to obtain the values which we are looking for. The problem statement says saturated steam at 1 atmosphere is discharged from a turbine at a rate of 1150 kilograms per hour. Superheated steam at 300 degree Celsius and at 1 atmosphere is needed as a feed to a heat exchanger.

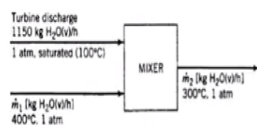
To produce it the turbine discharge steam is mixed with superheated steam available from second source at 400 degree Celsius to 1 atmosphere. The mixing unit operate adiabatically calculate the amount of superheated steam at 300 degree Celsius produced and the required volumetric flow rate of the 400 degree Celsius steam. So here unlike the previous problem which we looked at from energy balances we have not been asked calculate change in enthalpy or heat.

However we have been asked to calculate the mass of superheated steam produced and the volumetric flow rate of superheated steam which is required to be fed. So these are usually terms which we calculate for material balances however if you were to write material balances for this problem you would find that the degree of freedom is equal to 0. So to solve this problem we

also need to write the energy balance equation let us see how we go about solving such a problem.

**(Refer Slide Time: 18:37)**

## Simultaneous Material and Energy Balances



$$1150 + \dot{m}_1 = \dot{m}_2 \quad \text{--- (1)}$$

$$\Delta H + \Delta E_k + \Delta E_p = \dot{Q} - \dot{W}_s$$

$$\Delta H = 0$$

$$\begin{aligned} \Delta H &= \sum_{\text{out}} \dot{m}_i \hat{H}_i - \sum_{\text{in}} \dot{m}_i \hat{H}_i \\ &= \dot{m}_2 \hat{H}_2 - (\dot{m}_1 \hat{H}_1 + \dot{m}_0 \hat{H}_0) \end{aligned}$$

$$\hat{H} \text{ H}_2\text{(v) Sat at } 100^\circ\text{C} = 2676 \text{ kJ/kg}$$

$$\hat{H} \text{ H}_2\text{(v) at } 400^\circ\text{C} = 3278 \text{ kJ/kg}$$

$$\hat{H} \text{ H}_2\text{(v) at } 300^\circ\text{C} = 3074 \text{ kJ/kg}$$

$$\Delta H = 3074 \dot{m}_2 - 3278 \dot{m}_1 - 2676 \times 1150$$

Here is the flow chart we describes the process you have the turbine discharge coming in at 1150 kilograms per hour the condition that are given. You also have a 400 degree Celsius superheated steam being sent to the mixture and the final outlet steam is the mixed steam containing both these steams and this is a superheated steam at 300 degree Celsius at 1 atmosphere.

For performing calculations the values for enthalpies can be obtained only from steam tables so let us look at how this problem would be what are the material balances we can write? So here the only material balance we can write would be the total material balance because you have only one component and it is a mixture. So the total balance material for this problem would be  $1150 + \dot{M}_1 = \dot{M}_2$  where 1150 is the mass of water vapor entering as turbine discharge.

As  $\dot{M}_1$  is the mass of superheated steam being fed to the mixture and  $\dot{M}_2$  is the mass of superheated steam which is produced. So this 1 equation and you have to 2 unknowns so this is why you have the degree of freedom which is greater than 0. So you would have one degree of freedom. So you cannot calculate both these values using only material balance. So here we would use energy balance to the rescue so the energy balance equation for this open system would be  $\Delta H + \Delta E_k + \Delta E_p = \dot{Q} - \dot{W}_s$ .

Considering that this system does not have any acceleration or change in position we can ignore to the kinetic and potential energy changes. So these two terms go to 0 as we do not have any moving parts in the system WS also goes to 0. In addition to this the problem statement tells us that the process is adiabatic which means there is no energy supplied upon the system to the system from the surrounding in the form of heat.

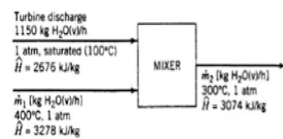
So there is no heat exchange between the system and surroundings meaning  $\dot{Q}$  is also equal to 0. Now we need to calculate  $\dot{H}$  in terms of the mass flow rate which we have that would help us with the second equation which can be used along with the material balance equation to get the mass flow rates of the inlet and the outlet streams. Let us try doing that so  $\dot{H}$  would be equal to  $\sum \text{outlet streams } \dot{M}_i H_{i, \text{cap}} - \sum \text{inlet streams } \dot{M}_i H_{i, \text{cap}}$ .

So the outlet streams are  $\dot{M}_2$  times let us call that as  $H_{2, \text{cap}} - \dot{M}_1$  times  $H_{\text{cap}} +$  let us call  $\dot{M}_0$  80 cap. So where  $\dot{M}_0$  is 1150 and 80 dot is the enthalpy associated with the inlet steam. So looking the steam tables we can identify these values so enthalpy  $H_{\text{cap}}$  for specific enthalpy for water vapor which is saturated at 100 degree Celsius is given = 2676 kilojoules per kilogram.

Specific enthalpy of water vapor at 400 degree Celsius so this is the superheated steam would be 3278 kilojoules per kilogram. Specific enthalpy of water vapor which is superheated at 300 degree Celsius is 3074 kilojoules per kilogram. So substituting this values here we get  $\dot{H} = 3074 \text{ times } \dot{M}_2 - 3278 \dot{M}_1 - 2676 \text{ times } 1150$ .

**(Refer Slide Time: 23:31)**

## Simultaneous Material and Energy Balances



$$\Delta \dot{h} = 0$$

$$1150 \times 2676 + \dot{m}_1 \times 3278 = \dot{m}_2 \times 3074 \quad (2)$$

$$\dot{m}_1 = 2240 \text{ kg/h} \quad \Rightarrow \hat{v} = 3.11 \text{ m}^3/\text{kg}$$

$$\dot{m}_2 = 3390 \text{ kg/h}$$

$$V = 2240 \times 3.11$$

$$V = 6980 \text{ m}^3/\text{h}$$

Substituting these value of H dot into the equation we had you would have delta H dot = 0 which means the equation we got would become 1150 times 2676 + M1 dot 3278 = M2 times dot 3074 so this equation is the equation which again as M1 and M2 using the material balance and this equation we can solve them simultaneously to get M1 dot and M2 dot. So M1 dot from here would be 2240 kilogram per hour and M2 dot would be 3390 kilograms per hour.

So M2 dot is the mass of the final superheated steam which is produced from the mixture so 3390 kilograms per hour at superheated steam at 300 degree Celsius is produced from this mixture we have also been asked to calculate the volume volumetric flow rate of the superheated steam at 400 degree Celsius needs to supplied to the mixture. So we have the mass flow rate we can again convert this through the volumetric flow rate using the specific volume data which is available in these team tables.

The specific volume data for this particular condition is 3.11 meter cube per kilogram so using this specific we can calculate the volumetric flow rate as  $ZV = \text{mass flow rate}$  which is 2240 times the specific volume which is 3.11 giving volumetric flow rate as 6980 meter cube per hour. So this would be the volumetric flow rate s with that we come to the conclusion of basic introduction related to energy balances.

In the subsequent classes we will look at different types to energy balances so whatever we have looked at till now or for chemical processers we will also look at mechanical process where have

to perform mechanical energy balances later on we will move on to performing energy balance calculations on more complicated systems until the next class thank and good bye.