Material and Energy Balances
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Module No # 01
Lecture No # 03
Fundamentals of Material Balances

Hello everybody in the last class we looked at the different process variables and the different process parameter which are used to define chemical and bio chemical process. Today we will be talking about the fundamentals which are required for performing material balances. So before we start performing material balances we need to understand the terminologies which are associated with material balance calculations.

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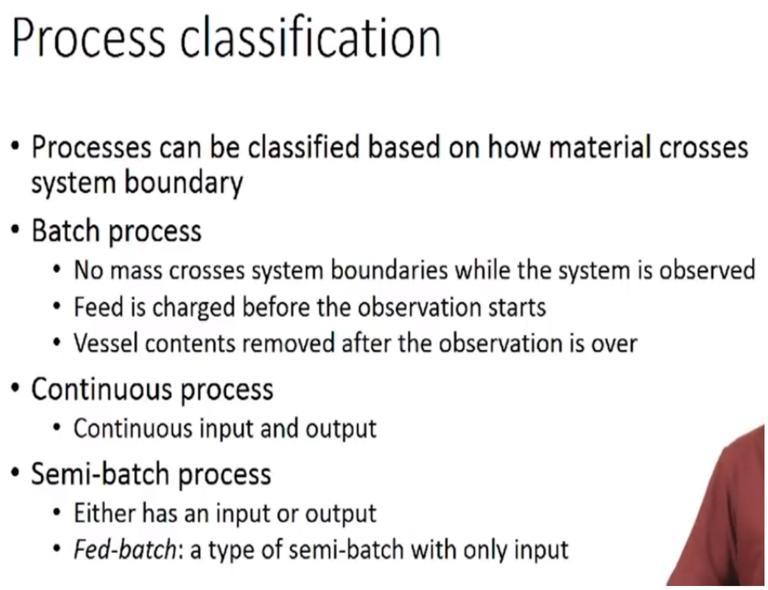

Systems and surroundings are the terms which are always used to define the system of interest and whatever comes around us. A system is the defined as any arbitrary portion of a whole process or the entire process itself that you are interested in studying the limits are establish by something called the some boundaries. Types of systems can be classified as close system or open systems when no mass exchange happens between the system and its surroundings it is called the closed system.

When materials cross the system boundary then the system is called an open system surroundings are everything that is outside the system boundary. The boundary is the arbitrary line which is drawn to enclose the system which we are observing. Processes can be classified based on material crosses the system boundary or not a batch process is one where no mass crosses the system boundary while the system is being observed.

So any material which is required for the system is fed before the observation starts and after the process completes the final output is taken out. So during the time of the observation materials do not cross the system boundary you also something called a continuous process where there is a continuous input to a system and a continuous output which is taken out of the system you can also have semi batch processes which either have an input or an output.

Fed batch is the special type of semi batch process where you have only the input no output.

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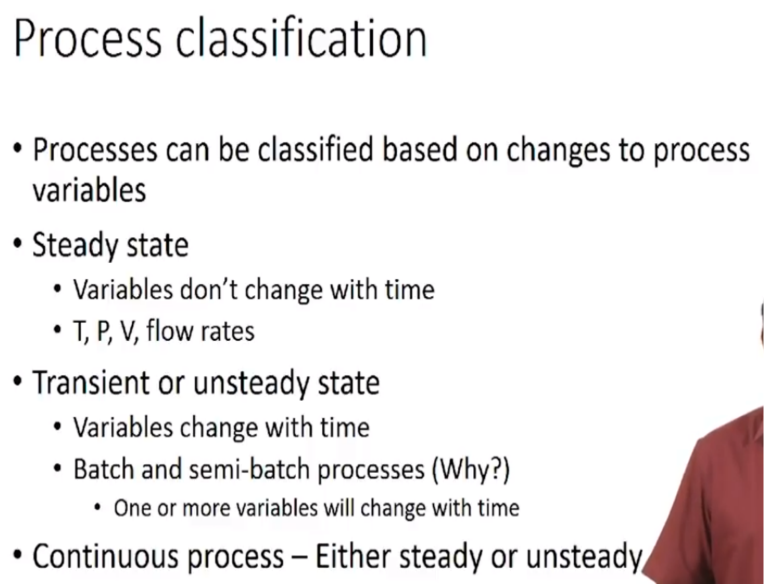

Processes can also be classified based on changes to process variables when the process variable do not change with respect to time the system is said to be at study state the process variables which are looked at or temperature or pressure, volume and flow rates. You can also have transient or unsteady state processes where the variable will change with respect to time.

Batch and semi batch processes are almost always transient or unsteady state process can you think of a reason why? Because in batch or semi batch process one or more of the variable will change with respect to time. For example in a semi batch process you will be having only an input or an output which means the volume will have to change or the pressure will have to change. You have continuous process operating either as steady state or as unsteady state process.

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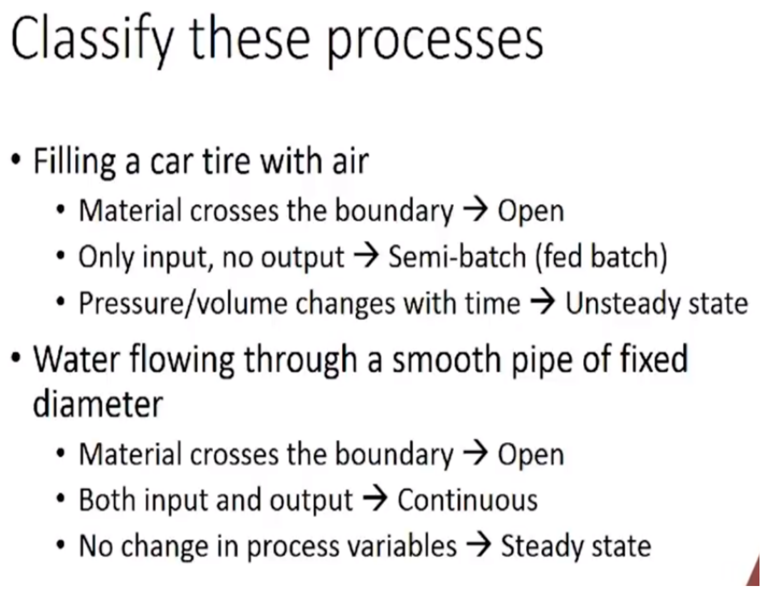

Let us look at a few example problems which can be classified based on the classification with we looked at. So the first process is filling a car tire with air so here you have air entering in the tire so a system as a boundary which is crossed by the material so which means the system is open system and you have only a input you do not have an output which means the system would be a fed batch which is a special case of semi batch process.

Pressure or the volume will be changing with respect to time both of them could also be changing so the process is unsteady state process. Let us look at a second example problem where you have water flowing through a smooth pipe of a fixed diameter. Here again material is crossing the system boundary so the system is an open process you have both an input and output which means it is an continuous process and this is a steady state process as there are no changes to the process variables.

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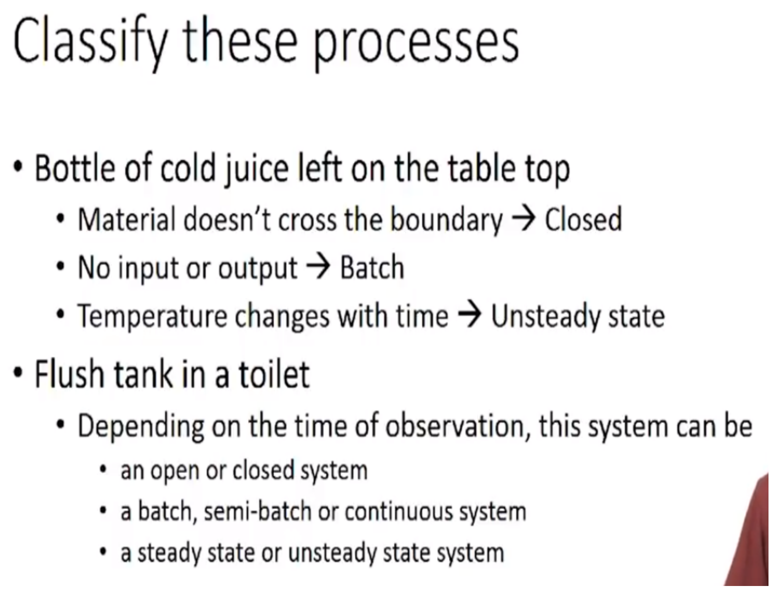

Let us look at a third example which is a bottle of clod juice left on the table top. So here material bottle does not cross the system boundary which means it is a closed system. Again you do not have any input or output which means the process is the batch process when you have changes with respect to time with temperature because it is a cold bottle of juice which is left outside at room temperature.

The temperature of this juice is going to keep increasing therefore the process is an unsteady state process. Okay let us look at a little more complicated example which is a flush tank in the toilet. Based on the time of observation this system can be actually exist as open or closed a batch or a semi batch or a continuous process and a steady state or a unsteady state system.

So this is because when you observe it the system could actually have different output or inputs and there could be change in volume or there could be steady state operation. When you turn the flush tank on the water keeps coming out which means the system becomes and open system and there is also a semi batch system. During this time of flushing you could also have water coming in which would make it a continuous system.

However after the flush tank emptied you only have the refilling then it could be a fed back system. So once the flush tank is full it could be just a batch system where no material is loss the system is boundaries. So the system can also exist as steady state or unsteady state depending on range in volume with respect to time or change in time temperature with respect to time. So why

I choose the example is for you to understand that a system could exist in different states depending on when it is being observed.

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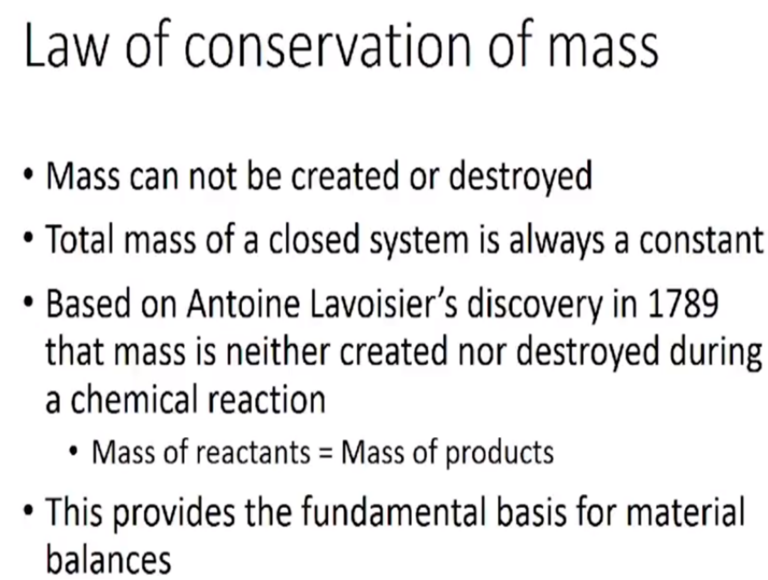

So the fundamental concept on which the material balances are build is the law of conservation of mass. Mass cannot be created or destroyed it is only be change from one form or another form total mass of closed system will always have to be constant this is based on Antoine's Lavoisier's discovery in 1789 shows that mass is neither created nor destroyed during a chemical reaction.

He observed that the mass of reactants is equal to the mass of products so this provides the fundamental basis for performing in any material balances problem.

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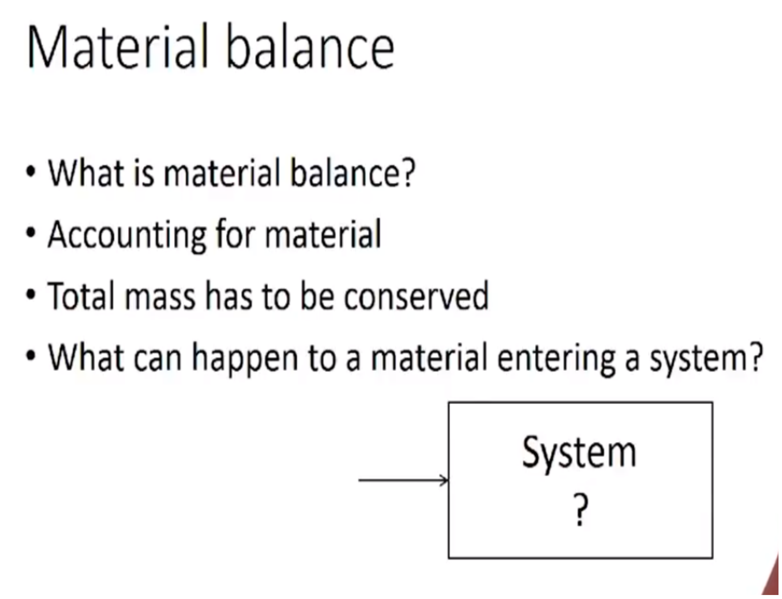

What is a material balance I keep talking about performing material balances what do I mean by? Material balances is nothing but writing an accounting for a material we know that total mass of the system as to be conserved. So we need to understand what could happen to the material which enters to the system so that we can write a balance sheet for materials.

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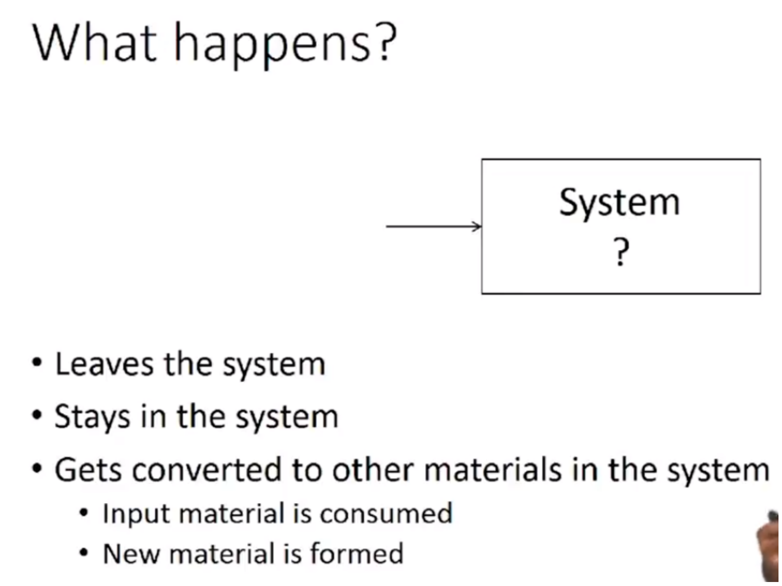

Let us say what can happen when material enter into the system it would either leave the system or I could stay back in the system or it could get converted to other materiel in the system. In this case the input material will be consumed and new material would be formed.

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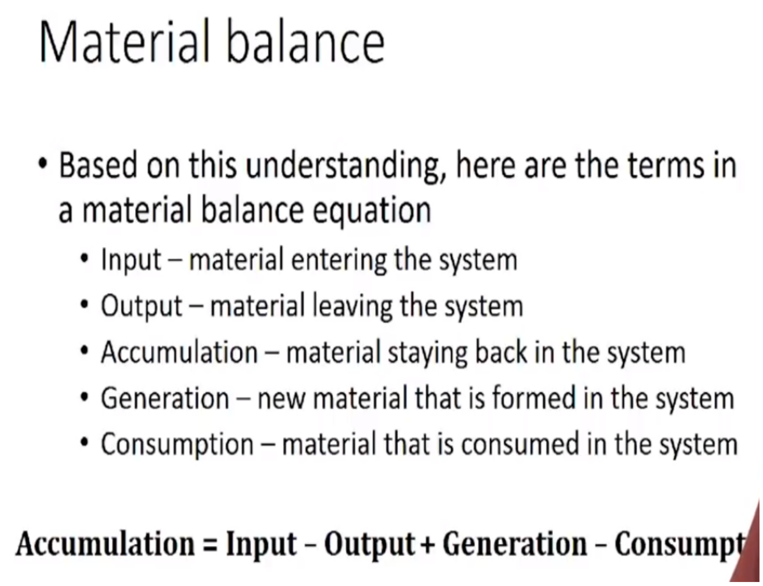

So based on this understanding here are the terms which we are using in the material balance equation input which is material entering the system output which is the material leaving the system and accumulation which is the material which is staying back in the system and generation which is the new material which is being formed in the system and you also have consumption which is the material that is consumed in this system.

So we need to come up with equation which will correlate these five terms so this general equation which fixed for both material and energy balances is as follows. Accumulation = input – output + generation – consumption so this equation forms the basis for performing all the material and energy balances which we will repeat throughout this course.

As part of these course this is the fundamental equation which you will try to apply to simple systems and then try to understand how the same equation can be applied for much more complicated systems and using systemic approach to solve the problem you would be able to break down even the most complex processes into very simple steps by applying these particular equation and performing material and energy balance.

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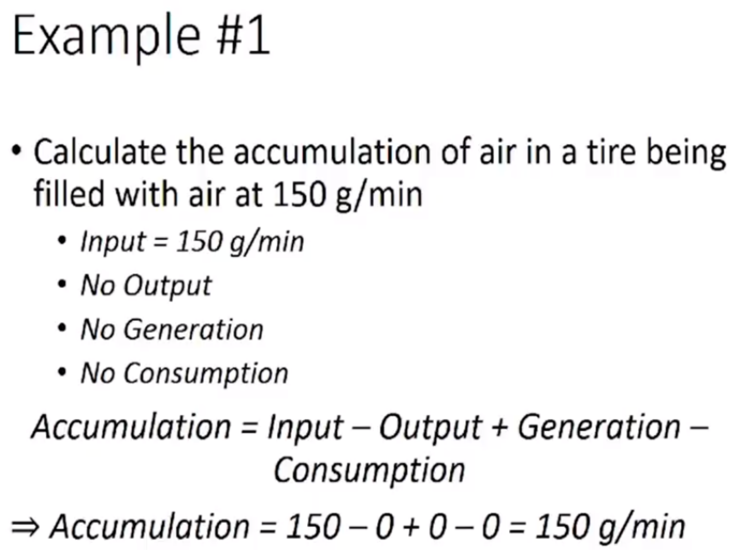

So to start with we will look at how to perform material balances here is an example of a very simple system this is the same system which is used as an example to classify the process which we are talking about. Calculate the accumulation of air in a tire which is being filled with air at 150 grams per minute. For this system the input is at 150 grams per minute you do not have any output there is no generation or consumption because there are no reaction is happening in the system.

So your accumulation equation is accumulation equals in input – output + generation – consumption which means your accumulation = input which is 150 grams per minute. So this is a simple example problem which shows how you can account for material for which is entering in the into a system.

Let us look at a problem which seems a little more complicated however while using a same problem the same principles we will be able to solve this problem with the same level of ease which we had for the previous example.

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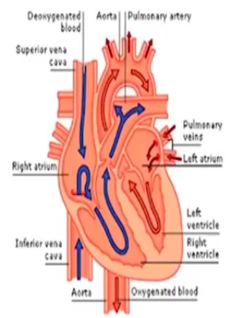
Example #2

- The heart is divided into two sides, each with two chambers. Blood enters the left atrium from the pulmonary veins and drains to the left ventricle, where the blood is pumped out at an average rate of 70 beats per minute. The stroke volume (the volume of blood discharged from the ventricle to the aorta) is 70 mL for each contraction. Assuming that no reactions with blood occur in the heart and heart chambers do not accumulate blood, determine the inlet and outlet mass flow rates for the left side of the heart. Assume ρ_{blood} = 1 g/mL.



Outlet mass flow rate = Stroke volume × Beats per minute × ρ_{blood}

= 70 mL/beat × 70 beats/min × 1 g/mL = 4900 g/min

No reactions ⇒ Generation = 0; Consumption = 0

Also given: Accumulation = 0

⇒ Inlet mass flow rate = Outlet mass flow rate = 4900 g/min

The problem statement goes as shown here the heart is divided into two sides each with two chambers blood enter the left atrium from the pulmonary veins and brings to the left ventricle where the blood is pumped out at the average rate of 70 beats per minute the stroke volume which is defined as the volume or blood discharged from the ventricle to the aorta is 70 ML for each contraction.

Assuming that more reaction with blood occurs in the heart and the heart chamber do now accumulate blood determine the inlet and output mass flow rates for the left side of the heart you can assume the density of the blood to the 1 gram per ML. So using this we can calculate the mass flow rate for the outlet stream it has been told that the stroke volume which is the volume of blood which is discharged from the ventricle to the aorta is 70 per beat.

So using this we can actually calculate the volume of blood which is pumped out per minute and convert the volume to the mass of blood which is pumped out from the heart per minute. So output mass flow rate is equal to stroke volume times beats per minute times density of blood which is 70 ML per beat times 70 beats per minute times 1 gram per ML giving you a mass flow rate of 4900 per minute.

Now going back to fundamentally equation we know that we have identify if there are generation consumption or accumulation terms before we can calculate the input point. The question clearly states that there are no reaction are happening which means generation and consumption terms are 0 the question also states that blood is not accumulating in the chambers so accumulation is also 0.

Plugging these values back into the fundamental balance equation we get input equals output which is 4900 grams per minute. So both the inlet and outlet flow rates are 4900 grams per minute and these problem could be solved at same ease as we solve the problem with the air being filled for a tire. Although the problem looks complicated breaking them down into the simpler tem which forms the material balances makes it very simple problem to solve.

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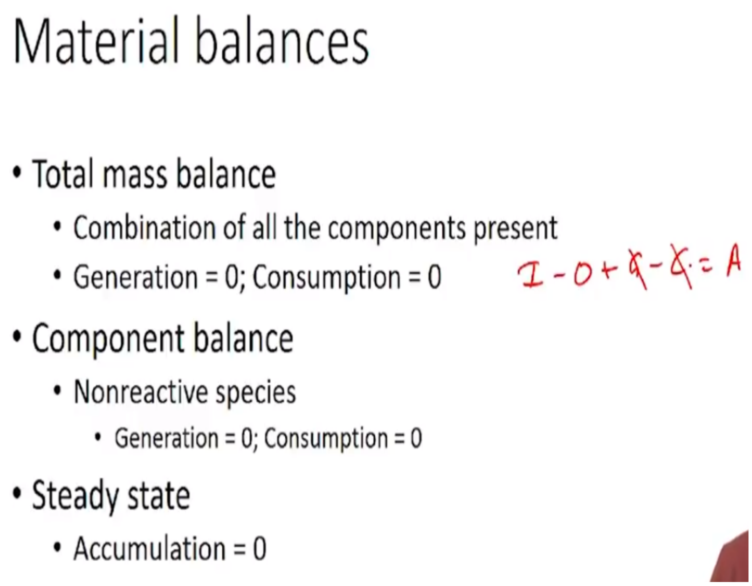

Now that we have looked some of the examples problem let us see how the procedure for performing material balance would look. What type of material balances can write? For any given system we can write the total mass balance the total mass balance is basically the combination of all the component which is present in the system so all the materials which are entering all the material which are leaving are taken into account and the balance equation is written.

So if were to write a balance equation for this system we start with input – output + generating – consumption = accumulation. As total mass cannot be created or destroyed you would have generation and consumption terms going to 0 thereby you will end up with input – output equals accumulation. You can also write components balances so these are basically for individual components which are entering and leaving the systems.

You might have multiple components which are entering and leaving the system and we can write separate balances for each of these components some of these components can be non-reactants when they are non-reactor you do not have any generation or consumption of that component. However if the reaction is taking place and the components are taking part in the reactions then you will have to account for the generation and consumption terms of these components.

Last but not least for the process would be at steady state accumulation would be equal to 0 when we described steady state we mention that the parameters do not change with respect to time when accumulation happens what would happen is some of the components which would actually being building up in the systems which means there will be a change with respect to time.

So under steady state condition accumulation would be zero and these is an assumption you will consistently make throughout the course for most of the problems we will assume the system to be at steady state. This is a reasonable assumption able to make because most of the times processes happening in an industrial scale are happening at steady state unsteady state processes usually happen as transient processes for very short period of time such as startup and at the time of ending the process.

Other times you actually have steady state operation for this reason for all the calculation which we perform for the material balances and even for energy balances we will assume the system to be at steady unless it is explicitly stated otherwise. So please remember this whenever I perform calculation in the subsequent lecture I will be assuming accumulation to be 0 even if it is not stated in the problem and the explicitly because we will be assuming steady state for performing the calculation.

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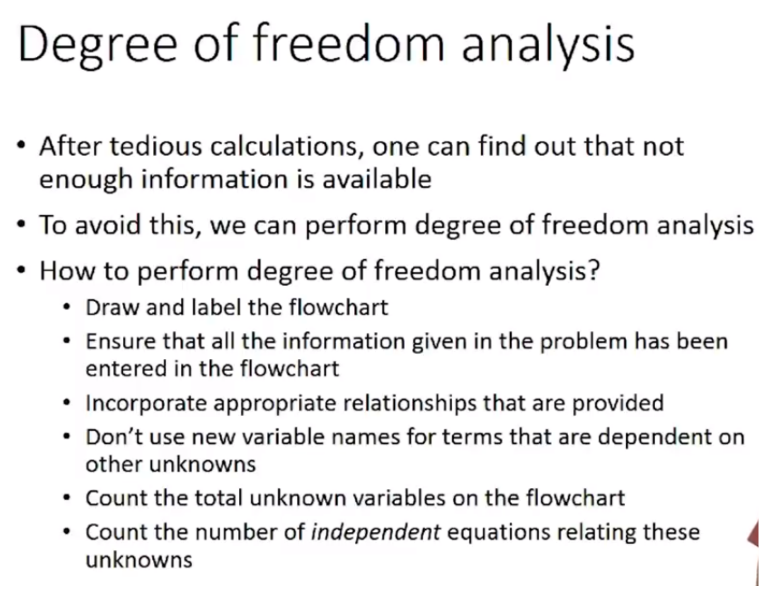

So when we perform material balance calculation one thing we come across is there is lot tidies calculation that are associated with each problems and many other times after performing tidies calculation we realize that there is not enough information in the problem will for us to arrive at the final answer. To avoid this we can performs something called the degree of freedom analysis.

How do we perform degree of freedom analysis so there is a systematic way which you can take to make sure that the problem is actually solvable. First you would draw a label flow chart and then you will ensure that all the information is given in the problem as been entered in the flow chart then you would go about incorporating the appropriate relationship that are provided when you have two variable which are related by some correlation given in the problem.

Do not use new variables instead write the depend equations as the unknown count the total number of unknown variable in the flowchart and then count the number of independent equations that you can write which relates these unknown variables.

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Degree of freedom analysis

- Calculate the degrees of freedom (n_{df}) as

$$n_{df} = n_{unknowns} - n_{indep\ eqns}$$

- If $n_{df} = 0$, there are equal number of unknowns and independent equations and the problem can be solved
- If $n_{df} > 0$, there are more unknowns than independent equations
  - at least n_{df} variables need to be fixed to solve the problem
  - check if you have missed any correlations or equations
- If $n_{df} < 0$, there are more independent equations than unknowns
  - either the flowchart is missing some unknowns or the problem is overspecified with redundant or inconsistent equations

You can actually calculate the degree of freedom as number of unknown – the number of independent equations. If the degree of freedom = 0 then you have the same number of unknown and same number of independent equations which means the problem can be solved there is going to be only one unique solution to be the problem. If then degree of freedom is greater than 0 then actually have more than one unknown which cannot be obtained.

So you have more unknowns than independent equation so you need at least NDF number of variables that need to be fixed before you can solve the problem or you need NDF number of equations to solve he problem. So when you get something like this many a times it could be because of an over side.

So it is important to check if you have missed it any correlation or equations and ensure that all the data which is provided to you in the problem has actually been entered into the flow chat that you have prepared. If NDF is less than 0 then there are more independent equation than unknown although this does not happen commonly in some case you would see these happening.

This could either mean that some of the unknowns has been missed while preparing the flow chart or it also possible that the problem is over specified which could mean either there are reluctant equations and information provided to you or it would also mean that there are inconsistent equations which will lead to confusion while solving the problem. So performing the degree of freedom analysis can actually ensure that the problem we are working on is the actually solvable.

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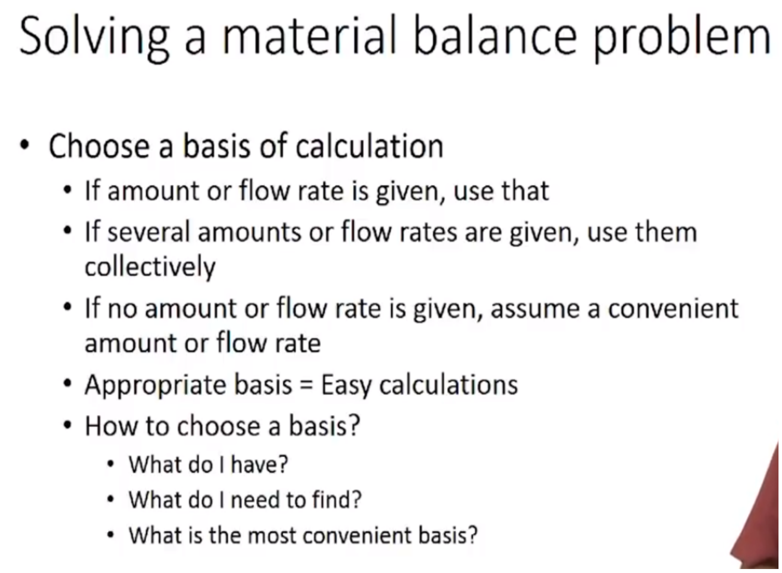

Now after establishing that we have a solvable problem how do we go about performing the material balance problem itself. First we need to choose something called base calculation what is a basis? A basis is a number to choose to make a calculation easier if amount of low rate is given then you would use as your basis. For example if you have problem which states that 1 kilo of substance of A enters into the system and then you would choose that one kilogram of A as your basis.

If you have the several amount of flow rates that are given then also use them collectively if no amount of flow rate is given when you would have to assume a continent amount of flow rate as the basis you can choose either 1 or 100 which your make your calculations easier. Choosing an appropriate base will ensure that the calculation you perform are easy to work how do you choose a base?

When you need to calculate something you need know what you already have what you need to find and what would be most convenient basis answering these three questions will actually given you a best basis for performing the calculation,

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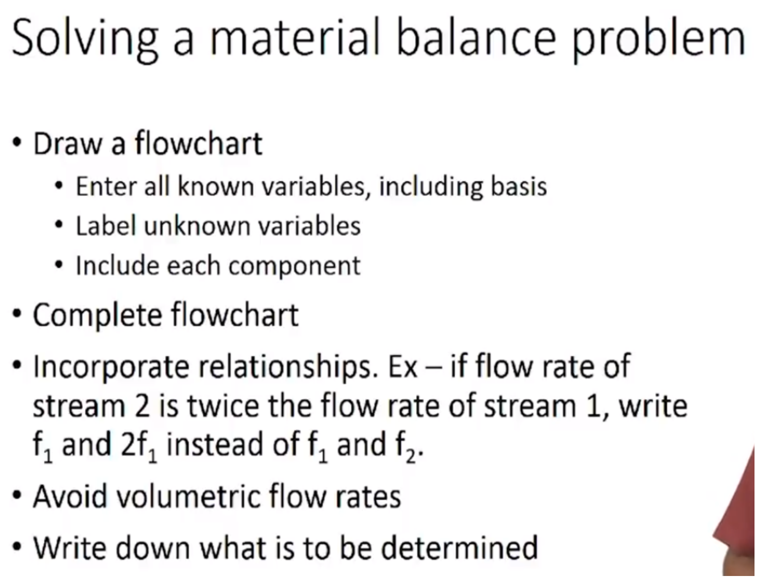

Once you have decided on the basis you start drawing the flow chart this flow chart should have all the known variables which includes the basis which we have chosen even if the basis is an assumption we have made please ensure that you enter this value in the flow chart label all the unknown variable and write them in aspirate places of the flow chart. Include each component which is there in the system on the flow chart itself here role is now to complete the flow chart.

So what you try to do is you will incorporate all the relationships for example if the flow rate of stream two is given as twice that of the flow rate of stream 1 do not write F1 and F2 instead you can write F1 and 2 F1 so this helps you understand that there is an dependency of these two variables and this help you solve the problem. Whenever you are preparing these flowcharts ensure that you do not use volumetric flow rates the reason is volume is not concerned.

Mass is the only thing which is concerned so if you have a change in phase then can actually be a significant change in volume without mass being changed at all write down what as to be determined. So once you have all the data from the question which is put into your flow chart make sure that you also should note down what information needs to be calculated.

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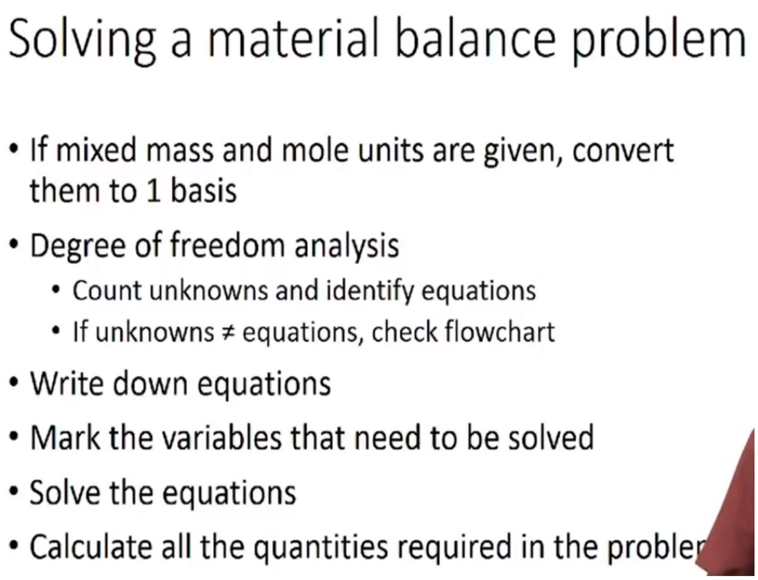

If the problem gives you two different basis for example that could be two stream with one stream given in terms of mass units and other stream in terms of moles. Then convert all of them into 1 basis make sure that all of them either exist as mass or moles so that you can actually perform the calculation effectively.

Perform degree of freedom analysis basically you count all the unknowns and identify the equations and make sure that unknown and the number of independent equation are the same if they are not please check the flow chart again once you have performed degree of freedom analysis write down all the equations that are correlating these unknown variables mark the variables that need to solve for and finally solve these equation this will give you the desired answer.

Calculate finally solve these equations this will give you a desired answer calculate all the quantity that need to be required that need to be calculated in the problem. So read the problem carefully and ensure that you have all the values that I have not asked for in the problem.

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Solving a material balance problem

- Check if you had used the values given in the problem for your basis. If basis was different from the given values **(not a good idea)**, then adjust answers proportionately

Incase due to some unfortunate reason you had chosen a number which is different form the value which is given in the system as the basis then you should make sure that you should go back and convert these values use a values which is not given in the problem as a basis if you have a flow rate or mass or mole are presented to you in the problem you should ensure that you choose that values as your basis incase you do not do that you will actually have trouble with converting the values back and this could lead to unnecessary errors in your calculation.

Please avoid this however in some rare situation where you might have over seen what was given in your problem and ended up choosing a different you any way confirm where these values where the one which are the asked and whether the values that are the once it have been provided and then convert them back to appropriately. With these fundamental you are now equipped to perform material balances for simple systems.

So in the next class we will look at single unit processes without reaction which are the some of the simplest processes for performing materials balances. We will look at different processes can exist and we will also perform a many example problems which will help in understand these system better.