

Material and Energy Balances
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Module No # 04

Lecture No # 17

Material Balance Calculations for Multiple Units With Reaction – Part 2

Hello everybody welcome to today's lecture on material balance calculation for multi-unit processes with reactions. As I had promised in the previous lecture today we will be looking at an example problem where the word problem needs to be converted into a flow chart before we perform the material balance calculations.

(Refer Slide Time: 00:37)

Drawing a flowchart

- Converting word problems to flowcharts
- More complicated for multiple unit processes

It is important to convert word problems into flow charts and gain the experience in the skill set which is required for understanding the information that has been provided to us and then drawing flow chart based on that. So this will help us interacting with management as an engineer we should be able to grasp what is being conveyed by the technical staff and the managers and try to convert into technical information that can still be processed further.

This is easy when we talk about single unit process so we have done this before when we did single unit processes we identify the number of inputs and outputs and we also drew the flow chart before we started performing the material balances this is very straight forward in the single unit processes.

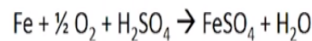
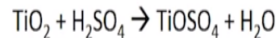
However it is lot more complicated with multiunit processor because we need to understand how the stream is flowing in one of the unit to the other there could be stream which are leaving the system altogether whereas there could be other streams which are being taken to the next unit for further processes. So understanding this and converting the flowchart sorry converting the word problem into the flow chart is the skill set which is gain ed over a period of time.

So here we will look at an example problem which is reasonably complicated problem which multiple units and you will try to flow chart as much as we can and compare with what we have in the what you have been given in the text book and try to perform material balances for that particular system.

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Example

- Sorel slag containing 70% TiO_2 , 8% Fe and 22% inert silicates is fed to a digester and reacted with H_2SO_4 , which enters as 67% aqueous solution. The reactions in the digester are as follows:



Both reactions are complete. The theoretically required amount of H_2SO_4 for the Sorel slag is fed. Pure oxygen is fed in the theoretical amount for all the Fe in the Sorel slag. Scrap iron (pure Fe) is added to the digester to reduce the formation of ferric sulfate to negligible amounts. Thirty-six kg of scrap iron are added per kg of Sorel slag. The products of the digester are sent to the clarifier, where all the inert silicates and unreacted Fe are removed.



So the example problem states that several slag containing 70% titanium dioxide 8% iron and 22% inert silicates is fed to a digester reacted with sulphuric acid which enters as 67% aqueous solution the reactants in the digester are given where titanium dioxide reacts with sulphuric acid and iron also reacts with sulphuric acid both reaction go to completion the theoretically required amount of sulfuric acid for the sorral slag is fed pure oxygen is fed in the theoretical for all the iron in the sorrel slag.

Scrap iron which is pure iron is added to the digester to reduce the formation of ferric sulphate in negligible amounts 36 kilogram of scarp iron is added per kilogram of sorrel slag which is fed

the products of digester are sent to the clarifier where all the inert silicates and unreacted iron are removed the solution of TiOSO_4 and FeSO_4 and from the clarifier is cooled crystallizing FeSO_4 which is then completely removed by a filter.

The product TiOSO_4 solution from the filter is evaporated down to a slurry it contains 82% TiOSO_4 the slurry is sent to a hot air dryer from which a product of pure hydrate which is $\text{TiO}_4\text{S}_2\text{O}$ is obtained. The hydrate crystals are sent to a direct fired rotary kiln where the pure TiO_2 is produced according to the following reaction where the hydrate is converted to TiO_2 and S_2SO_4 .

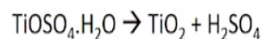
This reaction also goes to completion using the basis of 100 kilograms of sorrel slag feed you have been asked to calculate the kilogram of water removed by the evaporator the exit kilogram of water per kilogram of dry air from the dry air if the air entered contain 0.036 moles of water per mole of dry air and the air rate is 18 moles of dry air per 100 kilogram of sorrel slag and you have also been asked to calculate the kilograms of product TiO_2 produced.

So before we start with material balances we now need to draw a flow chart as you saw there are multiple process and we need to read this problem one more time carefully for each unit identify what are the inputs and output each of outputs are taken for the processing and which of the outputs are leave the system. So let us do that by drawing the flowchart.

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Example

- The solution of TiOSO_4 and FeSO_4 from the clarifier is cooled, crystallizing FeSO_4 , which is completely removed by a filter. The product TiOSO_4 solution from the filter is evaporated down to a slurry that is 82% TiOSO_4 . The slurry is sent to a hot air dryer from which a product of pure hydrate $\text{TiOSO}_4 \cdot \text{H}_2\text{O}$ is obtained. The hydrate crystals are sent to a direct fired rotary kiln, where the pure TiO_2 is produced according to the following reaction:



This reaction is also complete. On the basis of 100 kg of Sorel slag feed, calculate:

- The kilograms of water removed by the evaporator.
- The exit kg of H_2O per kg dry air from the dryer if the air enters having 0.036 moles H_2O per mol dry air and the air rate is 18 kmol of dry air per 100 kg of Sorel slag.
- The kilograms of product TiO_2 produced.



In the first sentence of the problem it is stated that sorrel slag containing 70% TiO_2 8% iron and 22% silicates is fed to a digester so which means the first process or the first unit we have in the process is a digester so let us first draw our digester so this is the digester so this digester is fed with sorrel slag so this is the first information we have so this sorrel slag reacts with sulfuric acid which enters as 67% aqueous solutions this means that we have another input which is 67% aqueous solution of H_2SO_4 .

So that would be an additional input for the system which is 67% H_2SO_4 which is aqueous solutions and then what else we have the reactions given and then it is also told that the theoretically required amount of sulphuric acid or sorrel slag is fed and pure oxygen is fed in the theoretical amount for all the iron in the sorrel slag. So in addition to these inputs you also have another input which is the pure oxygen which is fed to the digester.

So you have pure oxygen fed to the digester then it is also fed that scrap iron which is pure iron is added to the digester to reduce the formation of the ferric sulphate to the negligible amounts. So you also have scrap iron added to the digester so it has been told that 36 kilo grams of scrap iron is added per kilogram of sorrel slag. The products of digester are then sent to the clarifier everything that comes out of the digester then enters into a clarifier everything that comes out of the digester then enters the clarifier.

So this means whatever we have out of the digester enters into the clarifier so here all the inert silicates and unreacted iron are removed. So this means these components are leaving the system inert silicates and unreacted iron are actually removed from the system so which would remove the over system. So the next step is the solution of TiOSO_4 and FeSO_4 from the clarifier is cooled which causes the crystallization of FeSO_4 .

So this means whatever is remaining here the solution TiOSO_4 and FeSO_4 is cooled so you have a heat exchanger and a crystallizer which cooled it and crystals are formed so this slurry is then fed to a filter so where all the crystallized FeSO_4 is removed and only the solution of TiOSO_4 leaves. So you have a filter so this would be filter and from the filter all the FeSO_4 is leaving the system and you have the solution of TiOSO_4 then taken to the evaporate.

So it goes to an evaporator so what is given here is this solution is the evaporated down to the slurry that contains 92% TlOSO_4 so which means water is removed from the system so you have water leaving as the water vapor in the system and you have a slurry which is then sent to a dryer from which you have the pure hydrate formed. So this has been also given as the hot air dryer so this enters into a hot air dryer. So a hot air dryer is where you have hot air entering the dryer in addition to the wet component and this air is help you in drying the component.

So in addition to this you also have air entering and you would have air carrying water leaving the system so this would be air which is entering and you would have air which is wetter leaving the system. So the slurry which is now slightly dried is then taken to rotary kiln where it is dried completely to remove the H_2SO_4 based on the reaction. So hydrate crystals are converted to TlO_2 and H_2SO_4 so this goes to a rotary kiln and from the rotary kiln you take out TlO_2 which is product and H_2SO_4 also leaves the system.

So this gives you the flowchart for performing calculation related to any material balance for this system. So let us now compare it with what is given with the text so if you look at the system it is very similar to what I have drawn it clearly explains the process. Now let us go about drawn and this clearly explains the process now let us go about solving this problem trying to identify which system to start with and what basis and so on.

So the basis for problem as already been given in the problem statement we have been told that 100 kilograms of sorrel slag is entering into the system and you have been asked to calculate different thing using this basis. So we will start the problem with the basis of 100 kilograms sorrel so basis is 100 kilograms of sorrel slag.

Now if you look at the aspect which we need to calculate we need to look at the kilograms of water which is removed from the evaporator which is this stream and we also need to calculate the exit kilograms of water per kilogram of dry air from the hot air dryer so which is basically this stream and we need to calculate the kilograms of product TlO_2 produced which is this stream.

So these are the three component which we need information on so our problem statement ask us to find information about these three streams. So this means we do not care about what

information is present in this stream or this or this and so on. So as we do not need to know about this streams what we can do is we can choose a system which is engulf these particular stream. So taking that into account we want the streams for water air + water and TiO_2 to be leaving the system.

So to solve the first part of the system we need to make sure that this particular stream is crossing the system the other stream in between do not have to cross the system boundary. So based on that we will first choose the system as this particular system so when you do this what happen is you have sorrel slags scrap iron H_2SO_4 pure oxygen all being fed to the system and you have $FeSO_4$ inert silicates unreacted iron and water leaving the system along with the slurry for $TiOSO_4$.

So using this particular system we can perform material balance calculation to get the information regarding this particular stream which is water which is evaporated in the evaporator as we need to use the stoichiometric given in the reactions we will convert the mass into moles so that we can directly use the stoichiometric coefficient and perform calculation to get the generation and consumption terms. Taking that into account we first need to know the molecular weight of titanium and iron.

So molecular weight of titanium is 47.9 kilograms per kilo moles and molecular weight of iron is 55.8 kilograms per kilo mole. So now that we have is we have calculate the number of moles of TiO_2 and iron and H_2SO_4 that has are being fed. So we know that 100 kilograms of sorrel slag is entering so this contains 70 kilograms TiO_2 and 8 kilograms of iron and the rest is inert silicates.

So now this 70 kilograms TiO_2 would then be $70 / 47.9 + 32$ giving you the value of 0.876 kilo moles the number of moles of 8 kilograms of iron would be $8 / 55.8$ giving you a value of 0.143 kilo moles of iron. The problem statement lets us that H_2SO_4 is fed based on the theoretical requirement of sulfuric acid as per the sorrel slag this means the amount of sulfuric acid which is required to react with TiO_2 and iron has been fed.

So based on that the stoichiometric looking at stoichiometric we know that 1 mole of we know that one mole of TiO_2 reacts with 1 moles of H_2SO similarly you have one mole of iron also

reacting with 1 mole of H_2SO_4 . So this means H_2SO_4 fed would be equal to 0.876 kilo moles 0.143 kilo moles which is the number of moles of TIO_2 and iron fed in the form of sorrel slag. So using this we get the amount of TIO_2 sorry amount of H_2SO_4 fed is 1.02 kilo moles and this would be 100 kilograms.

We also know that pure oxygen is fed so we need to calculate how much of pure oxygen is fed pure oxygen is fed at theoretical requirement of iron which is fed through the sorrel slag. So we know that 0.143 kilo moles of iron is fed and one mole of iron reacts with half a mole of oxygen based on the stoichiometry of the reaction given. So this means 0.143 kilo moles of iron would require 0.143 kilo moles of iron will be require 0.143 divided by kilo moles o oxygen.

So pure oxygen fed would be equal to 0.143 times 0.5 times 32 which is 2.29 kilo grams so we also have an information that 36kilograms of pure iron is fed per kilogram of now that we have the information about the sorrel slag H_2SO_4 and O_2 the component is fed is the scarp iron we have been told that scarp iron or pure iron is fed at a rate of 36 kilo grams per kilo grams of sorrel slag.

So this means pure iron fed would be equal to 36 times 100 giving you 3600 kilograms of pure iron being fed. In the problem there have been two reactions which are taking place in the digester which are $\text{TIO}_2 + \text{H}_2\text{SO}_4$ gives $\text{TIOSO}_4 + \text{H}_2\text{O}$ and the other reaction is $\text{FE} + \text{half } \text{O}_2 + \text{H}_2\text{SO}_4$ gives $\text{FESO}_4 + \text{H}_2\text{O}$. So taking the first reaction the amount of TIOSo_4 that would be produced is one mole of TIO_2 will be produced one mole of TIOSO_4 if the reaction goes to completion.

The problem statement tells us that both these reaction goes to completion in the digester this means the number of moles of TISO_4 formed would be equal to the number of moles of TIO_2 which is fed. So we know that 0.876 kilo moles of TIO_2 is fed this implies 0.876 kilo moles of TIOSO_4 is generated. Looking at the system that we have chosen which is basically this TIOSo_4 leaves only through the stream TIOSO_4 does not leave through the other streams.

This stream as only unreacted iron and inert silicates and this particular streams as only water this particular streams has only FESO_4 so this means TIOSO_4 leaves only the particular stream and based on the information form the problem we know that this is 82% TIOSO_4 solution. So

the rest as to be water so we need to calculate the amount of water which is leaving through the system.

So we now have information about water which is present in this particular system particular stream and we know that there is some water entering here through to the H₂SO₄ stream and water is not leaving through the inter silicates or through the FESO₄ water is leaving only through this but these two particular streams. So based on these information we can now start writing a balance equation for water so which is exactly what we have to do now.

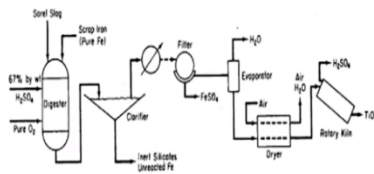
Based in this information we can start writing the water balance equation so the water balance equation for the system which is chosen would start from input - output + generation - consumption = accumulation at steady there would not be any accumulation water is not being consumed by any of the reactions which is happening in the digester that means consumption terms will go to 0 so you are input output and generation.

So we need to calculate the output through the water vapor stream so this output as two stream one is TISO₄ slurry stream and other is your H₂O stream which is the water vapor. So input water needs to be calculated first input is from the H₂SO₄ stream we know that 100 kilograms of sulfuric acid is fed and this forms a 67% solution so in this implies 100 kilogram of H₂SO₄ forms a 57% solution.

So water in this stream would be 100 divided by 0.67 times 0.33 so this gives you a value of 49.25 kilo grams. So 49.25 kilo grams of water is fed through the H₂SO₄ stream now we need to calculate the amount of water which is leaving the TIOSO₄ slurry stream.

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Example



H_2O Output 1 = ?

$$0.876 \text{ kmol of } TiOSO_4 \approx 140.07 \text{ kg}$$

$$\Rightarrow \text{Water in } TiOSO_4 \text{ slurry} = \frac{140.07}{0.82} \times 0.18 = 30.75 \text{ kg}$$

H_2O Gen = ?

$$1.02 \text{ kmol of } H_2SO_4 \approx 1.02 \text{ kmol of } H_2O$$

$$\Rightarrow H_2O \text{ Gen} = 1.02 \times 18 = 18.36 \text{ kg}$$

$$W_o = I - O_1 + G$$

$$= 49.25 - 30.75 + 18.36$$

$$W_o = 36.86 \text{ kg}$$

So let us call that output 1 water output 1 is which is equal to the amount of water leaving through the $TiOSO_4$ slurry. So based on the calculations we have performed we know that 0.876 kilo moles of $TiOSO_4$ was produced so this 0.876 kilo moles of $TiOSO_4$ would weight 140.07 kilo grams so this 140.07 constitutes the 82% or the slurry which is leaving the evaporator.

So this implies water here in $TiOSO_4$ slurry so water in $TiOSO_4$ slurry will be equal to 140.07 divided 0.82 times 0.18 giving you a value of 30.75 kilo grams. Now we need to calculate amount of water which is generated so water generated = what so based on this stoichiometric for every mole of H_2O H_2SO_4 consumed there is one mole of H_2O produced there are two equation both of which follows the same stoichiometric ratios.

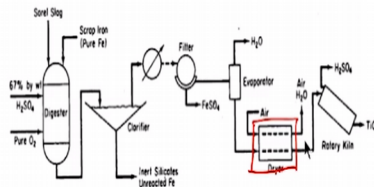
So this means for every mole of H_2SO_4 consumed of digester one mole of H_2O would have been produced. Based on the information we have given to us we know that 1.02 kilo moles of H_2SO_4 has been consumed. S 1.02 kilo moles of H_2SO_4 consumed would have resulted in formation of 1.02 kilo moles of water. So this implies water generate would be equal to 1.02 times 18 giving you a value of 18.36 kilograms.

So now that we have all these terms we can calculate the amount of water leaving in the form of water vapor from the evaporator. So let us call that W_o out would be input – output 1 + generation so this is input value is 49.25 output is 20.75 and your generation is 18.36 giving you a total

amount of water leaving the system as 36.86 kilograms of water vapor leaving the evaporator. So with this we have calculated the first part of the problem.

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Example



$$\text{Air input} = \frac{0.036 \text{ mol H}_2\text{O}}{\text{mol DA}}$$

$$\text{Air fed} = 18 \text{ kmol DA} / 100 \text{ kg SS}$$

Water balance

$$I - O + G - C = \Delta$$

$$I = 0$$

Input:

$$\text{H}_2\text{O Slurry} = 30.75 \text{ kg} = 1.708 \text{ kmol}$$

$$\begin{aligned} \text{H}_2\text{O inlet air} &= (18 \times 0.036) \text{ kmol} \\ &= 0.648 \text{ kmol} \end{aligned}$$

In the second part of the problem we have been asked to calculate the exit kilograms of water per kilogram of dry air from the hot air dry air if the air entering contains 0.036 moles of water per mole of dry air and it is also been given air rate is 18 kilo moles of dry air per 100 kilograms of sorrel slag which is fed. So we need to basically calculate the amount of water which is leaving through the stream.

So based on the information we have we know that the air entering here the air input has 0.036 moles of H₂O per mole of dry air and air is fed at a rate of 18 kilo moles of dry air per 100 kilograms of sorrel slag in the problem and the calculation we have performed we know that water is entering into the dry air through the slurry here and water is leaving as evaporated water vapor along the air in addition to that water is also accompanying FeSO₄ in the form of water of hydration with the TiO₂ So₄ crystals which is entering into the rotary kiln.

As we need to calculate the information about the stream here the system we chose would be only the dry air, Taking the dry air as the system let us now calculate the water which is leaving along with the air. Now water balance across dry air would be basically start from input – output + generation – consumption equal to accumulation at steady state there is accumulation you have

input for water both through the air and through the slurry and you have output for water through and the water for hydration you do not have any generation you do not have consumption.

So it would be input = output so now we need to calculate the amount of water which is leaving in the form of hydration and the amount of water which is entering through these two streams. So let us first calculate the input water streams so input for water in the form of slurry how much water is being fed we know that 82% TiOSO₄ present in the slurry and we have already calculated that the remaining amount would be water and that value is 30.75 kilo grams.

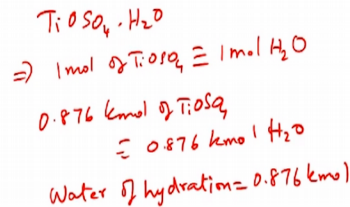
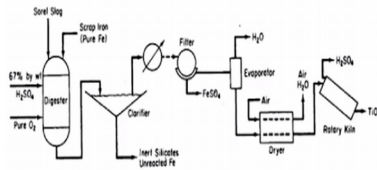
So water entering through the slurry is 30.75 kilograms the water in the inlet air has been given in terms of moles per mole of dry air so we know that there is 0.36 moles of water per mole of dry air and we have also been told that 18 kilo moles of dry air is fed for 100 kilo grams of sorrel slag as we have 100 kilograms of sorrel slag as the basis we know that 18 kilo moles of dry air has been fed water in the inlet air would be equal to 18 times 0.036 kilo moles.

So this equals 0.648 kilo moles so this is 30.75 kilograms the water in the inlet air has been given in terms of mole per dry air. So we know that there is 0.36 moles of water per mole of dry air and we have also been told that 18 kilo moles of dry air is fed for 100 kilo grams of sorrel slag as we have 100 kilo grams of sorrel slag as the basis we know that 18 kilo moles of dry air has been fed water in the inlet air would be equal to 18 times 0.036 kilo moles.

So this equals 0.648 kilo mole so this 30.75 kilo grams can also be converted to moles and by dividing it with the molecular weight and we end up with 1.708 kilo moles. So we have now calculated the amount of water which is entering into the system we now need to calculate the amount of water which is leaving the system in the form of water of hydration.

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Example



Water in inlet air + Water in slurry
 = Water in exit air
 + Water of hydration

$$0.648 + 1.708 = W_A + 0.876$$

$$W_A = 1.48 \text{ kmol}$$

$$W_A \equiv 26.64 \text{ kg}$$

$$\text{Mass of DA} = (18 \times 28.84)$$

$$= 519.12 \text{ kg}$$

$$\therefore \text{H}_2\text{O} / \text{kg DA} = \frac{26.64}{519.12} = \frac{0.0513 \text{ kg}}{\text{kg}}$$

Based on the stoichiometric for the water hydration we know that $\text{TiOSO}_4 \cdot \text{H}_2\text{O}$ is the hydrated crystals this implies 1 mole of TiOSO_4 uses up one mole of water as the water of hydration we know that 0.876 kilo moles of TiOSO_4 has been formed and it is entering into the dry air so this would use 0.876 kilo moles of water in the form of water of hydration. So water leaving as water of hydration would be equal to 0.876 kilo moles.

So using all this information and substituting them back into the equation we can get the amount of water using all these information and substituting them back into the water balance equation we calculate the amount of water leaving with the air. So that would be water in inlet air + water in slurry would be equal to water in exit air + water of hydration so this means it has $0.648 + 1.708$ equals water in exit air + 0.876.

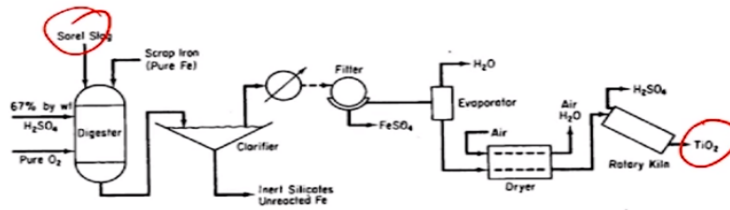
So you have water in exit air as 1.48 kilo moles which equivalent to 26.64 kilograms now that we know 26.64 kilograms of water is leaving the air we still need to calculate the kilograms of water per kilogram of dry air which is leaving the system. So we know 18 kilo moles of dry air is fed so that means the mass of dry air fed would be 18 kilo moles times molecular of dry air which is 28.84 if we assume 79% nitrogen and 21% oxygen the molecular weight for dry air would be 28.84 and using this mass of dry air would be equal to 519.12 kilograms.

So for 519.12 kilograms you have 26.64 kilo grams of water leaving so water kilograms of water leaving per kilo gram of dry air would be 26.64 divided by 519.12, So this would be equal to

0.0513 kilo grams per kilo grams. So with this we have the information for the second part of the problem.

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Example



$$T_i : I = 0$$

Mass of TiO_2 product

$$= \text{Mass of } TiO_2 \text{ fed}$$

$$= \boxed{70 \text{ kg}}$$

So the third part of the problem ask us to calculate the kilograms of TiO_2 which is leaving the system so this is very simple and straight forward and we need to do here is choose the overall system we know that titanium is not leaving the system titanium is not leaving the system through any stream other than this particular stream. This means all the TiO_2 which is entering in to the system in the form of the sorrel slag would only be leaving the system through the TiO_2 stream leaving the rotary kiln knowing that input = output would be the simplest equation we use.

So taking that into account for TI which is elemental problem or titanium sorry element balance for titanium we know that whatever the titanium is entering we are leaving in the form of TiO_2 again so taking this into account the amount of TiO_2 leaving the system mass of TiO_2 product would be equal to mass of TiO_2 fed which is 70 kilo grams.

So with this we have solved the problem so we started with the word problem and drew the flow chart based on the information given and we have performed all the material balances required to calculate all the information that has been asked for with this we will come to the conclusion for standard reactions in multiunit processes.

In next lectures we will look at certain specialize reactions such as bio chemical reactions which is suitable for people with biotechnology background and also look at combustion reaction which are very critical in chemical industries and students with the chemical engineering background might be interested in learning that we will look at these two types of reaction and see how to perform material balances for such reactions thank you.