Material and Energy Balances
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Module No # 01
Lecture No # 01
Fundamental of Engineering Calculations

Hello everybody welcome to this NPTEL course on material and energy balances and I am Doctor MuthuVijayan I am a faculty in department of biotechnology IIT Madras. During this course you will look at some of the most fundamental aspect of material and energy balances and look at how these can be applied eventually into higher level courses. Before we dealt into basics of material and energy balances we first need to understand the fundamental associated with any engineering calculation.

This will be the first lecture of this course the most important aspect of any fundamental calculation which is being performed would be the units and dimensions.

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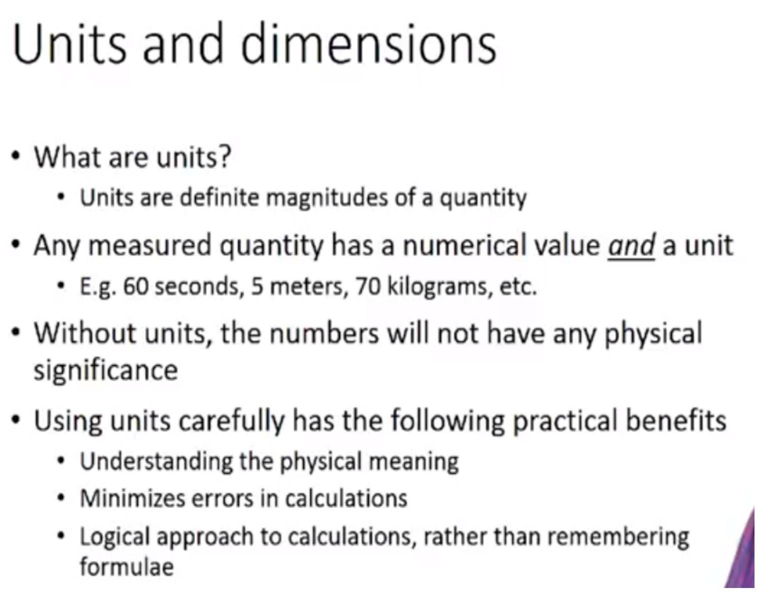

What are units? Units a are basically definite magnitude of any quantity, any measurable quantity would have two thing one as a numerical value and other is the units 60 seconds represent time 5 meters would represent distance or length and 70 kilogram would represent mass.

Without units these number do not have any physical significance they will just stand as pure numbers. Using units carefully has lot of practical advantages, it ensures that understand the physical meaning of the number which is being given I do not have to clearly state that the time taken was 60 seconds. If I just say it took 60 seconds you would automatically know it is time with that I am talking about.

So the physical meaning of the number is clearly understood when the units are actually associated with the number. It minimizes errors and calculations when you ensure that all the numerical values which you are using in your calculating are accompanied by the units you will also make sure that units tally with each other and you use appropriate conversions there by you can minimize the calculation errors which usually come due to failure to convert 1 unit to other.

It also gives a way to establish a logical approach towards calculation rather than memorizing formulae.

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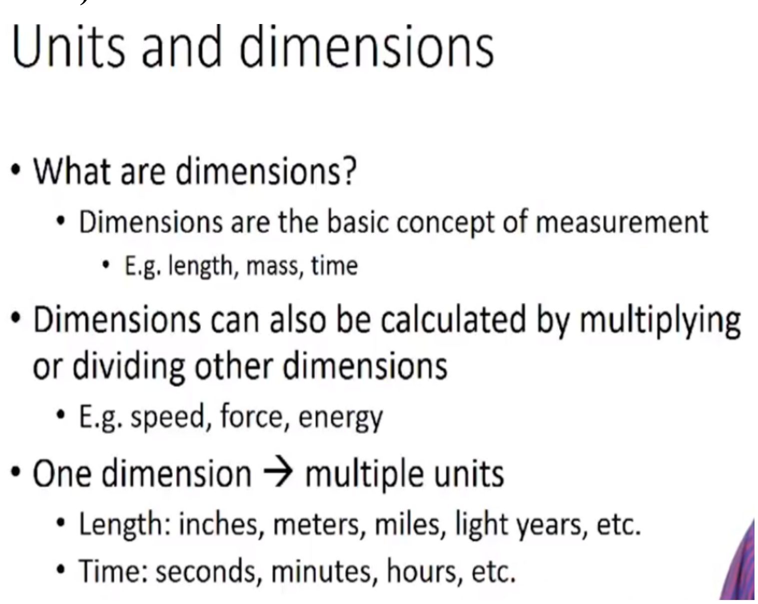

What are dimensions? Dimension are the basic concepts of measurement examples would be length, mass, time etc. Dimensions can be calculated by multiplying other dimensions or divided two dimensions examples would be speed, force, energy, acceleration and so many other things. One dimension can actually have multiple units for example length can be measured in terms of inches, meters, miles, light years and so many other terms.

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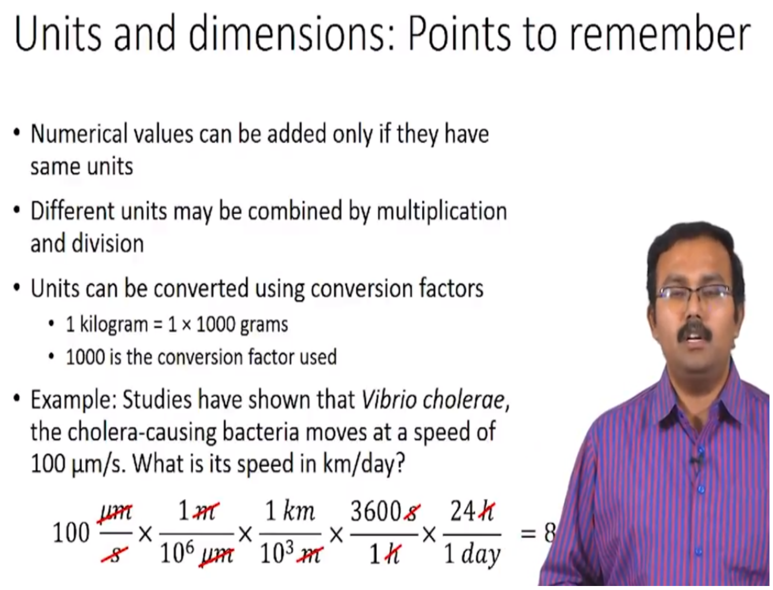

So similarly time can be measured as second, minutes, hours, days, weeks and so on when you are talking about units and dimensions there are certain points which you need to remember numerical values can be added only when they have the same units this different units can be combined by multiplication or division. Units can be converted from 1 unit to another using conversion factors.

I kilo gram can be converted to grams by multiplying with conversion factor of 1000 here is an example where we try to perform these kind of unit conversion let us look at doing a systematic approach to ensure that we minimize errors and calculation. The problem statement is studies have shown that VIBRIO cholerae the cholera causing bacteria moves at a speed of 100 micrometers per second.

What is it is speed in kilometers per day so the information which as given to you is in terms of micro meters per second and they want this converted to kilo meters per day. So the micrometer needs to be converted to the kilometer and the second needs to be converted to day. So let us go about solving this 100 micrometers per second is the data given we know that 1 micro meter is 10 power -6 meters.

So 1 meter divided by 10 power 6 micrometers is multiplied to the given value now this meter can be converted to kilometer as 1 kilometer contains 1000 meters. So now we have technically converted micrometers to kilo meters we now need to convert the second to say we know that 1 hour contains the 3600 second and 1 day contains 24 hours using these values you would finally get kilo meters per day.

We can even verify these so the micro meter given in the speed cancels of with the conversion factor which is given and the meter taken in the conversing again cancels of with the second conversion factor the second cancels of with the first conversion factor and the hours cancels off in the second conversion factor for the time thereby your final units are kilometers per day and you end up with the value of 8.64 times 10 power -3 kilometers per day.

If you looked at how I did this problem I have written down each number with the associated and the conversion factor where also written along with the units this ensures that I can appropriately use the conversion factor either for multiplication or division this will eliminate any confusion and ensure that I do not randomly multiply or divide and make unnecessary conversion errors.

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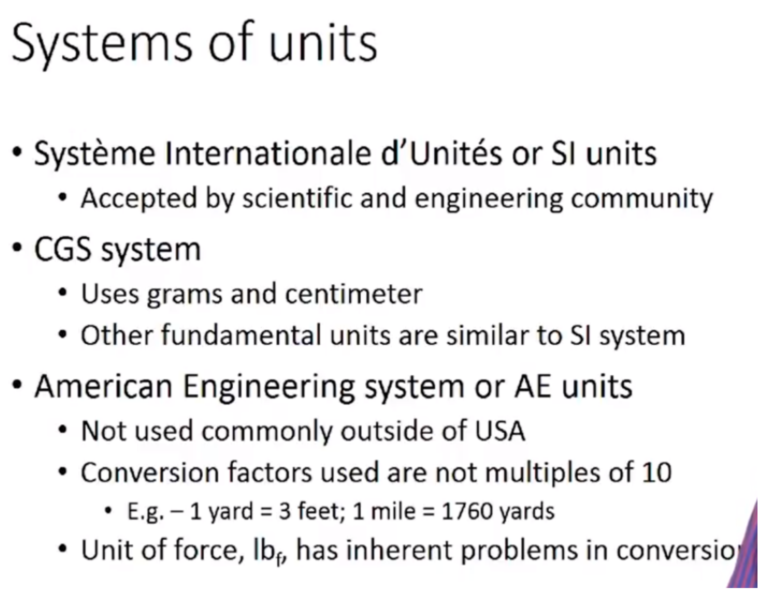

The unit which we use have different systems there are three different systems which are commonly used the first system is the SI units which is the most acceptance system of units amongst engineering the scientific community. You also have CGS system which is very similar to SI units except for the fact that length is measured in centimeters and mass is measured in grams instead of meters and kilograms which should be used in SI units.

Other fundamental units are similar to the SI units American engineering units is another popular system which is commonly used in US. Outside of US it is not common to use American engineering system because there are some inherent problems with the conversion factors which are used in American engineering system are not multiples of 10 this makes it very tidies to remember the conversion factors and remember the conversions.

For example 1 yard actually is 3 feet and 1 mile is 1760 yards it would be very difficult for most people to remember these numbers and apply them correctly. Another problem associated with the American engineering system is the unit for force the unit for force in American engineering system is pound force these has inherent problems in conversion we would look at what that is in the later slides of this lecture for now let us take it by phase value that pound force causes problems with conversions.

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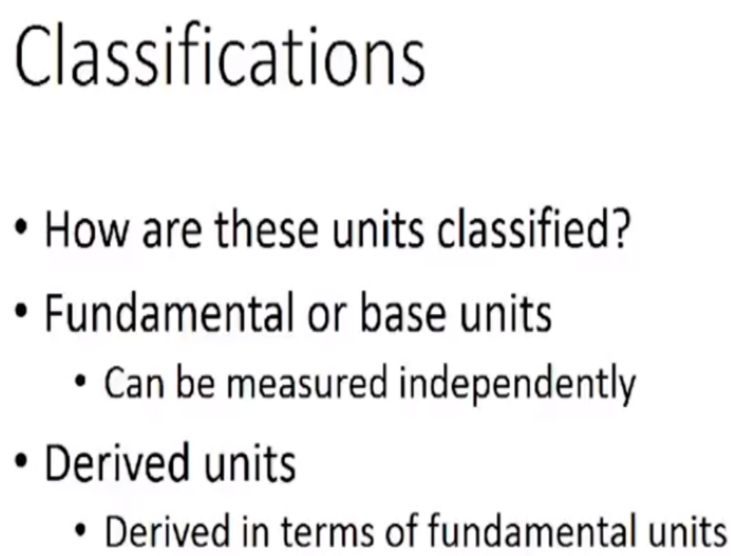

So the units can be classified into different types how are they classified the first type is the fundamental or base units these are units that can be measured independently so example would be a length which you can measure using a scale and so on. You also have derived units which are terms derived from the fundamental units either through multiplication or division.

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Base dimensions/units

- How many fundamental dimensions/units are there?
  - Seven
- What are the fundamental dimensions and their units?

Dimension	SI unit (symbol)	CGS unit (symbol)	AE unit (symbol)
Length (L)	meter (m)	centimeter (cm)	foot (ft)
Mass (M)	kilogram (kg)	gram (g)	pound (lb_m)
Moles (N)	gram-mole (mol)	gram-mole (mol)	pound mole (lb mol)
Time (T)	second (s)	second (s)	second (s)
Temperature (Θ)	kelvin (K)	kelvin (K)	degree Rankine (°R)
Electric current (I)	ampere (A)		
Luminous intensity (J)	candela (Cd)		

So we told there are base of fundamental units so how many base or fundamental dimensions in units are there you would have studied this in your high school and you should be familiar with this already. There are 7 fundamental dimensions or units okay what are they what are these fundamental dimensions and what are their units? The first fundamental dimension is length the SI unit for the length would be meters CGS units would be centimeters and American engineering unit would be foot.

Mass is measured as kilogram, grams and pound mass in the three system moles which is the amount of substance is measured as gram moles in SI units and CGS units and it is measured as pound moles in American engineering units. Time is measured as second in all the three units system temperature is measured as kelvin in SI and CGS units and as degree Rankine in American engineering units.

These are the five fundamental dimensions and systems which we keep using throughout these materials and energy balance course. In addition to these five fundamental dimensions and systems which we keep using throughout these material and energy balance course in addition to these five fundamental dimensions you also have electric current and luminous intensity which ear measured and ampere and candela in SI unit.

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Derived dimensions/units

- Derived by multiplying or dividing base dimensions/units
  - E.g. units of speed is m/s
- Some units are defined as equivalents for compound units

Quantity	Dimension	Unit	Symbol	Equivalent
Volume	L^3	liter	L	0.001 m^3
Force	MLT^{-2}	newton	N	1 $kg.m/s^2$
Pressure	$ML^{-1}T^{-2}$	pascal	Pa	1 $kg/(m.s^2)$ 1 N/m^2
Energy	ML^2T^{-2}	joule	J	1 $kg.m^2/s^2$ 1 N.m
Power	ML^2T^{-3}	watt	W	1 $kg.m^2/s^3$ 1 J/s

We will not be using these two dimensions for these course moving to the derived dimensions and units these are obtained by multiplying or dividing base units or dimensions one example could be units for speed is given as meters per second. So you could also have units which are defined as equivalent of these compounds units so the example are given here.

Volume is measured in liters which is basically 0.001 meter cube so similarly there are different derived for force, pressure, energy and power. Force in Si units is given as newton pressure is given as Pascal energy is given as Joule and power is given as watts. So this table gives you the dimension of these terms and the units which are given and the symbol which are used for the SI units and also they are equivalence is for the compound units.

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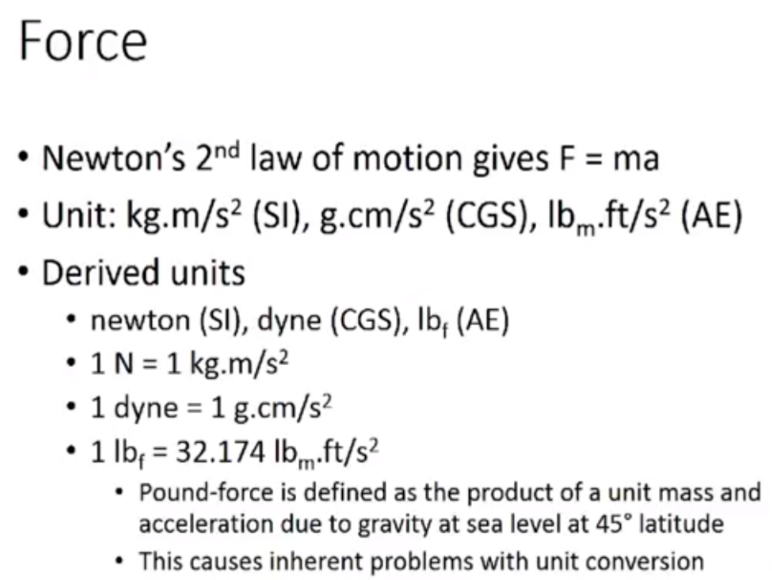

Now let us take one of the derived units which is force so this derived dimension force is define by second law of motion by newton which is force equal mass time acceleration. Units for these are kilo gram meters per second square, gram centimeter per second square and pound mass feet per second square in the three system. So it is simple mass is kilograms and acceleration is mass sorry accelerating is meter per second squared.

So this gives you kilo gram meters per second square so these derived compound units can also be written as derived units as newton in SI units dine in CGS and pound force in American engineering units. 1 newton is defined as 1 kilo gram meter per second squared 1 dyne is defined as 1 gram centimeter per second squared however in American engineering system is defined as 32.174 pound mass feet per second square.

This is because a pound force is defined as product of unit mass and acceleration due to gravity at sea level at 45 degree latitude. Because of this definition for pound force there is an inherent problem with conversion whenever we use pound force term. To avoid this and overcome this we use something called GC which is the conversion factor.

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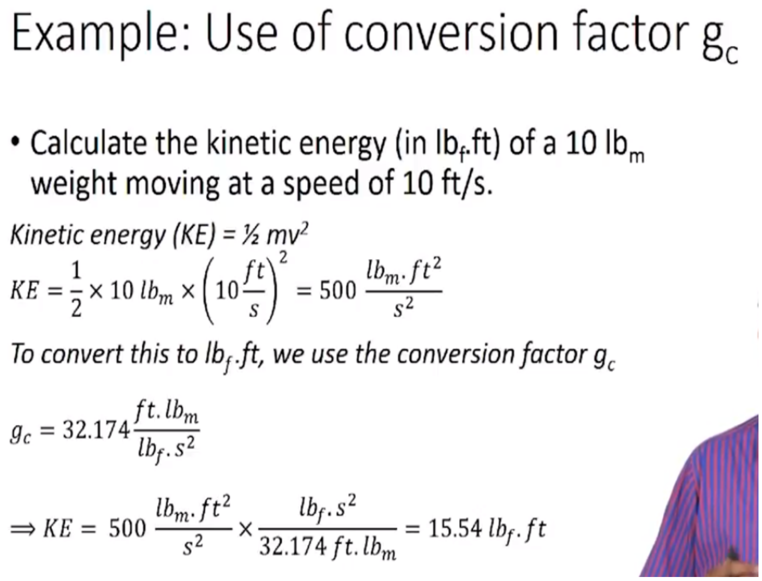

So let us look at an example problem which will hopefully help us understand how to use this conversion factor GC. The problem as you to calculate the kinetic energy in terms of pounds force time feet for a 10 pound mass weight moving a speed of 10 feet per second we all know that kinetic energy is given as 1 / 2 MV squared where M is the mass and V is the velocity.

So if you were to apply the values which have been given to us so we would directly get 1/ 2 times pound by mass times 10 feet by second whole square this gives you a value of 500 pound mass feet squared per second squared. So this is the value you for kinetic energy but the problem ask you to calculate kinetic energy in terms of pound force feet which means the pound mass feet squared per second squared term in this value you have calculated needs to be converted to pound force.

For this we would use the conversion factor GC so GC is 32.174 feet mass divided by pound force second square. So we apply this GC to the equation which was given earlier there by we get 500 pound mass feet second per second squared times pound force second squared divided by 32.174 feet pound. So basically what we done here is divided the kinetic energy which was calculated in terms of pound mass feed squared per second squared by GC giving you a final value for the kinetic energy as 15.54 for pound force times feet.

Hopefully with this you understand how to apply this conversion factor if you can remember that GC is the factor which is basically numerically equal to the acceleration due to gravity at sea level at 45 degree latitude with the units of feet pound mass per pound for second squared you would be able to apply this appropriately to make all the conversion.

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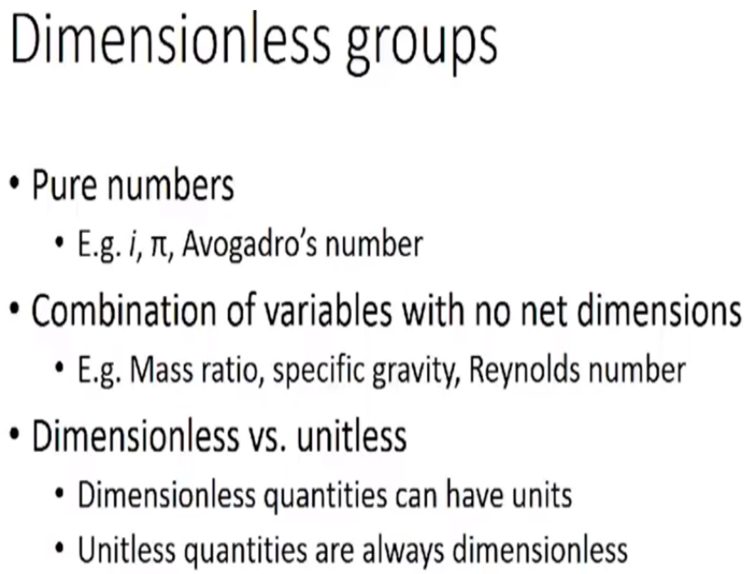

Other than these dimension and units there are also terms which are dimensionless there are pure number like I, pie, Avogadro's number which do not have any dimension you also can have combination of variables with no net dimension. Basically there could be ratios of different there the dimension get cancelled off examples could be mass ratio specific gravity and Reynolds number.

When we talk about dimensionless number we need to understand that there can dimensionless number and unit less numbers when I say dimensionless it does not mean that the quantity is unit less. Some of the dimensionless quantities can have units whereas all the units unit less quantities are dimensionless. Example would be mass ratio technically as units of grams per gram where it would be grams of one component divided by the grams of mixture or the grams of other components.

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Example: Dimensional analysis

- van der Waals equation is based on plausible reasons that real gases do not follow the ideal gas law

$$\left(p + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

where, p – pressure; V – volume; n – amount of substance; T – temperature. Based on dimensional homogeneity, can you find out the dimensions of a, b, and R?

Dimensional homogeneity is something which needs to be verified and valid for every equation. All valid equations would have uniformed dimension by that what I mean is the left hand side of the equation and right hand side of equation should have the same dimension another important parameter is all additive terms in an equation need have the same dimensions.

Here is an example problem you can check the first statement which I have made I have said that valid equations which would have uniform dimension now we know that potential energy is given as mass times acceleration due to gravity times height is which is MGH can you verify if this equation is dimensionally correct let us go to the first principles and try to identify what would be the dimension of the left hand side.

The left hand side term is energy is defined as force, time distance force is defined as mass time acceleration so energy is basically mass time acceleration times distance. So we know that the dimension of mass is M dimension of acceleration is LT – power 2 and distance is 12 and there it gives us total dimension of left hand as ML square T – 2 now let us calculate the dimension for the right hand side it is mass times acceleration due to gravity times height.

Dimensions of mass is M dimensions of acceleration is gravity is LT power -2 and height is L there by again giving M L squared T -2. So based on the dimensional homogeneity we can say that the dimension of the left hand side are equal to dimensions of right hand side this proves that this equation is dimensionally homogenous and correct.

So let us try another example problem where we try to identify the dimension of constants which we do not know van der Waals equation is based on plausible reason that real gases do not follow the ideal gas law. So we all know that ideal gas law is PV = NRT in real cases gases do not follow this and van der Waals equation is one of the model equation which describes such a real gas so the equation is given as P + N squared A divided by V squared times V – NB = NRT.

Here P is pressure, V is volume, N is the amount of substance, T is temperature and based on dimensional homogeneity you have been asked calculate the dimensions of the constant A, B and R.

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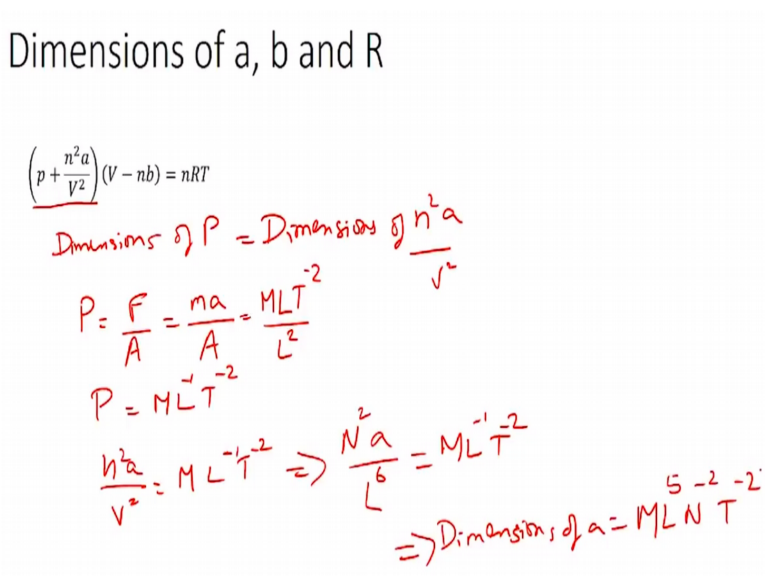

So to do this we first can take the first term alone so the first term which we have here as two aspect one is the pressure and other is the terms which contains A these two are added the based on our rules per dimensional homogeneity any additive term has to have the same dimensions which means dimension of P should be the same as the dimension of the second term which is N squared A divided by V squared. So let us try and identify what are the dimensions of P which is pressure.

Pressure is defined as force divided by area and force is defined as mass times acceleration divided by area. So now using this the dimension for mass is M dimensions for acceleration is LT power -2 and dimensions of area is L squared there by the dimensions of term pressure is ML

– 1 T -2. So this means the dimensions of the term N squared A divided by V squared would also be equal to M L – 1 T -2.

V being volume we know that the dimensions of denominator here would be L power 6 which is LQ squared. N square is basically the square of amount of substance. So if we were to half that as N as the dimension of amount of substance there will be N squared times A equals ML – 1 T -2. So this implies dimensions of A would be ML power 5N -2 T-2.

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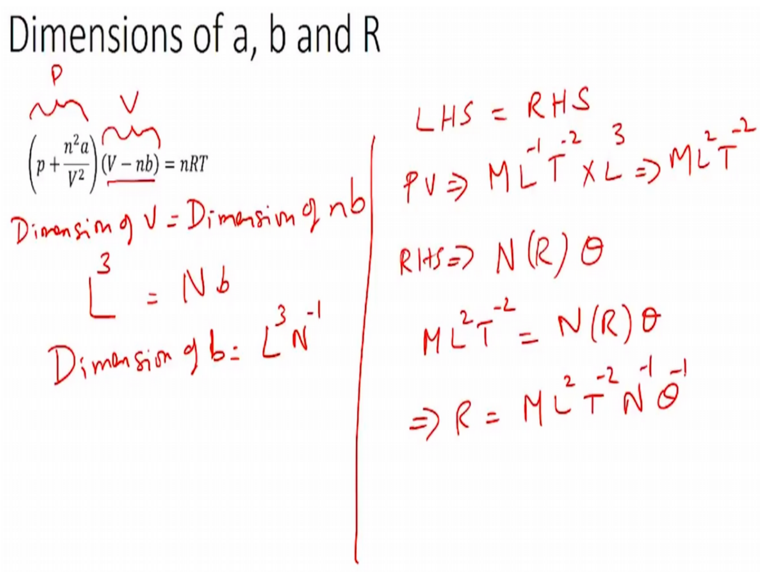

Moving on to the next section we will have to calculate the dimension of the second term B so let us look at these equation based on this the dimension of V which is volume should be the same as the dimension of the term NB. So the dimension of volume is L cube and the dimensions for N which is amount of substance is N which means the dimensions of B would be L cube N -1.

So can actually perform these calculations more systematically if we want to so what we can do here is we can actually assume the dimensions of each of these terms as M power X, L power Y T power A and N power A and so on and solve this equation to get all the powers which are there. So I am just doing it in a simplified fashion assuming that most of you would be familiar with how to these calculations if you have any queries feel free to contact me.

So now we can calculate the dimensions of R basically using the rule that dimensions off the left hand side should be equal to the dimensions of right hand side. So the dimensions of left hand side is basically the dimension for the pressure is the first term and the dimension for volume which is the second term. So now the left side is basically P times V which is pressure is M L – 1 T -2 times volume being LQ giving you a dimensions ML square T – 2.

So the right hand side would also have the should also have the same dimensions so we know that the right hand side basically as N which is amount of substance and whatever is the dimensions for R times theta which is the dimensions for temperature . So we know that M L squared P – 2 should be equal to N dimensions of times theta. So this implies that dimensions for R is M L square T – 2 N -1 theta – 1 based on these we have performed dimensional homogeneity calculations to identify the dimension of constants A, B and R.

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Significant figures

- You measure the mass three times using a balance with readability of 0.001 g and get the following values

Trials	Mass (g)
Trial #1	47.476
Trial #2	47.453
Trial #3	47.498

- A calculator shows the average mass as 47.47566667 g
- But, the balance had a readability of 0.001 g only
- The final answer can't be more accurate than your readings

Moving on we will move to the next important concepts when it comes to engineering calculations it is called significant figures before we learn how to identify the number of significant figures and why we use them how we use them let us look at that example problem.

When you are measuring mass of sum substance 3 times using a balance with the readability of 0.001 grams you will probably get values like this so the trial once as given you the value of 47.476 trial 2 as given you a value of 47.453 and trial 3 as given you a value of 47.498. Now if you were to calculate the average of these three which you can assume to be the closest to the accurate mass of what of the substance you measure you will end up with the value something like 47.47566667 grams which is what your calculator or excel would give.

However the value which you have written down here more accurate than all the values that you have measured because the readability of your weighing balance is only 0.001 grams and here you have written a final value which is much more accurate than that this cannot be true the final answer which you write down cannot be more accurate than your readings if you have a value of that sort then you do not have physical meaning for the numbers that you have written.

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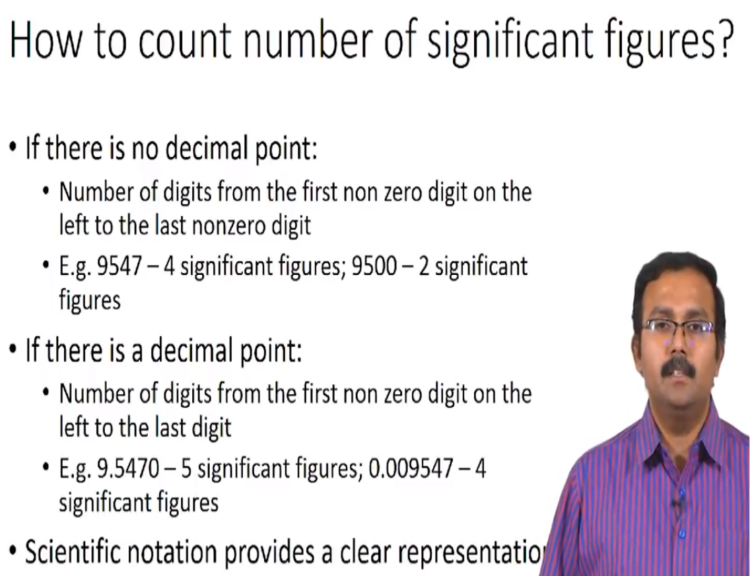

So to know and understand the significant figure to use them we first need to know how to count the significant figures for any given number if the number does not have any decimal points then what do you do is you start with the first non-zero digit on the left and start counting the number of digits till the last non zero digit. So if I were to have their example of 9547 this number would have four significant figures where the first non-zero digit is 9 and the last non zero digit is 7.

However if I were to use a number like 9500 then I have only two significant figure because the first non-zero digit is 9 and the last non zero digit is 5 giving me only 9 and 5 as the significant digits. If there is a decimal point then what you do is then count the number of digits form the first non-zero digit on the left to the last digit it does not matter if your last digit is 0 or non-zero example given here is 9.5470 this would have five significant figures because there is a decimal point.

I will start form the first non-zero digit on the left which is 9 and go all the way to the last digit which is 0. So that gives me five significant figures it I were to use the second examples which is 0.0009547 I still start with the non-zero digit on the left which is still a 9 and I go all the way till the last digit which is a 7 thereby I am giving you a 47 significant figures. If you write these number in scientific notation there is a clear representation of this significant figures.

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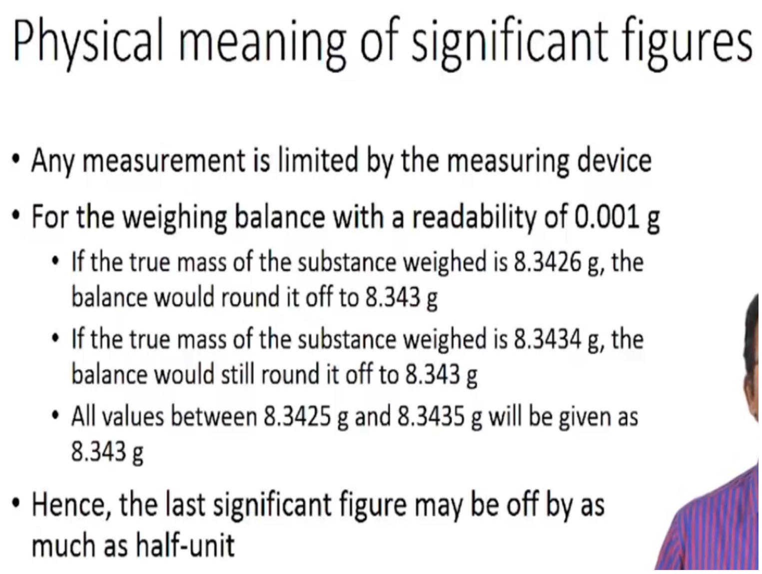

Why should we bother about these significant figures any measurement which we report is limited by the accuracy of the measuring device itself whatever number we use cannot be more accurate than whatever we measured that was the example which we look that consider the same example the weighing balance as a readability of 0.001 grams what does it mean?

If my substance as a true mass of 8.3426 gram balance would if it as 8.343 grams because the readability of this balance is only till 0.001 grams. Similarly if the mass of the weighed substance is actually 8.3434 grams the balance would still read it as 8.343 grams it cannot account for the last point 0.0004 grams what this means is any value between 8.3425 and 8.2435 will be represented as 8.343 by the abalone which has readability of 0.001 grams.

So as the last significant digit which you have can actually be off by as much as half a unit this has a lot of significance and that is why you need to ensure you follow the number of significant figure which are present.

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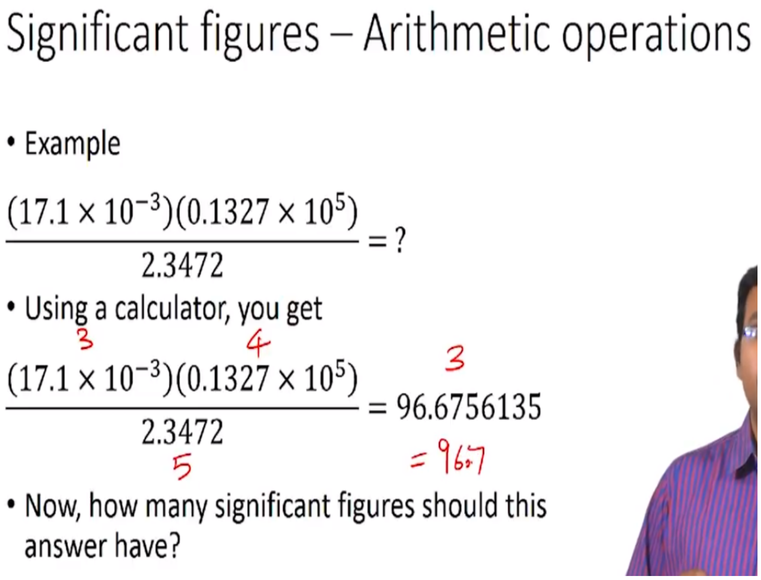

When we perform arithmetic operations we need to use significant figure carefully we need to know how to write the final answer with only the significant figures if you were to perform multiplication and division you would have to count the number of significant figures of the multiplicand and divisors and beside that the number which as the lowest numbers significant figures is the maximum at which you can go.

So the example we have given here will illustrate this you have 4.027 times 309 giving you a value of 1244.343. So this is what would your calculator would give you however the first term which is 4.027 has four significant figures and the second 309 has three significant figures. So the most number of significant figures your final answer which is the product have is three significant figures.

So you would write down your answer as 1242 match the significant figures this will accurately represent the physical validity of the numbers which you have used here is the another example problem. 17.1 times 10 power -3 into 0.1327 times 10 power 5 divided by 2.3472 if where to plug these values into a calculator you would end up with 96.6756135 this is mathematically accurate but physically it is not.

So what you do instead is you need to identify what would be the appropriate number of significant figures which you can use. Now can you identify how many significant figures your final answer should have let us see this the first term 17.1 has three significant figure and the second term 0.1327 has four significant figures starting from 1 to 7 the denominator has 5 significant figures 2 to 2.

So the lowest number of significant figures in your multiplicand divisors is for the first term which is 17.1 times 10 power -3. So your final answer can also have only 3 significant figures and this means you would write your final answer as 96.7 so this gives you the answer with the proper significant figures.

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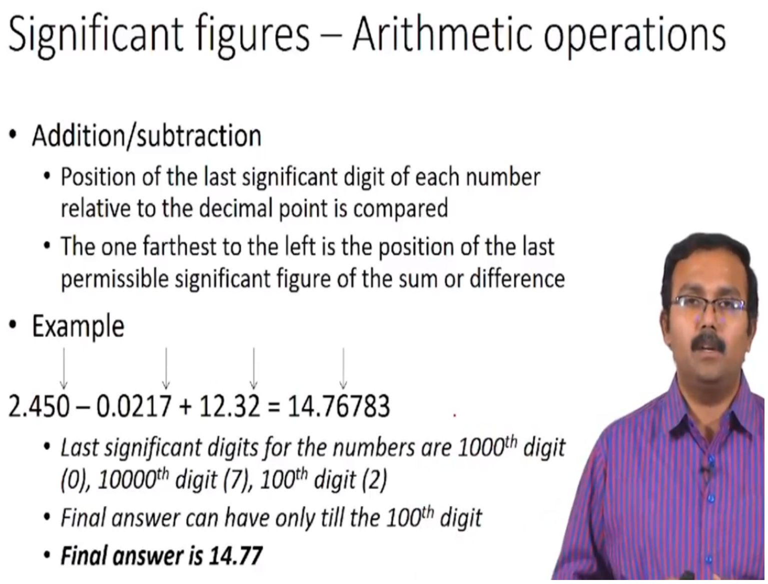

If you were to perform addition and subtraction then you need to use a different strategy to identify the correct number of significant figures for the final answer. The position of the last significant digit of the each number relative to the decimal point bas to be compared the one which is farthest to the left is the position of the last permissible significant figure for the some of difference an example problem will illustrate this condition clearly.

The example problem given here is 2.450 – 0.0217 + 12.32 giving this in a calculator you would end up with 14.76783 although this numerically correct it is again physically not accurate. So what you do is identify the position of the last significant digit in each of these numbers relative to the decimal point for the first number 2.450 the last significant digit is the 1000 digit which is 0 for the second number the last significant digit is the 1 10000 digit which is 7 and the third number which is the last significant digit the 100 digit which is 2.

So your final answer can actually be written only till the 100th digit there by you will write down the final answer as 14.77. Please understand that using these significant figures ensures that physical meaning of the measurement you have done is conveyed through the final answer at the same time I would strongly recommend that you do not round off every step to step the significant figures because which will it will add up to the errors which you finally calculate.

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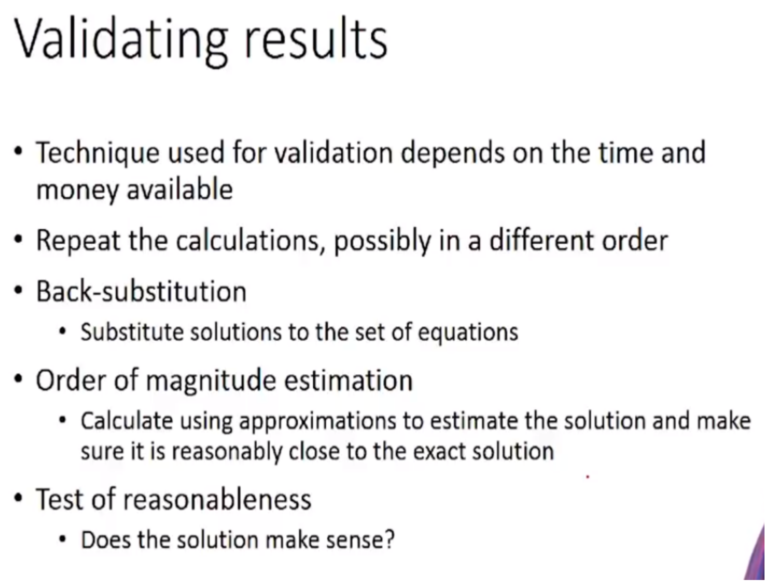

So rounding off can be done at the end after performing all the calculation and after using significant figures to represent the physical meaning ensuring that the units and dimensions are accurate and everything you can still make mistakes to ensure that you have not made there are different ways to validate the calculations which you have performed the technique used for validation will depend on the time and money available to you.

So if you need high powered computers to perform the validation the cost is going to be a problem if you are going to in exam hall and you have 10 minute left before you submit the answer sheet time becomes the problem considering what is available to you can use one of the following methods you can repeat the calculation possibly in a different order this will ensure that you perform all the calculation again and because it is coming in a different order it will ensure that you do not make the same mistake again.

You can also do back substitution where you take the solution you obtained and substituted back to the equations which you wrote down and ensure that these equations are still valid you can do something which is a little simpler called order of magnitude estimation. So basically whatever calculations you are performing you would have used different number these numbers would have their own orders of magnitude based on this you can roughly estimate what would be the order of magnitude of the final answer you are expecting.

If the final answer you have gotten is in the same order of magnitude changes are you have not made a gross mistake. If it is in a different order of magnitude then there is probably some crucial mistake which you have committed quite possibly due to unit conversion or multiplying something instead of dividing it and so on.

And the last and the crudest form of confirming the answer should be test of reasonableness that is the solution you obtain make sense this seems so simple and trivial but many times people do not think about it very carefully especially in an exam setting people do not look at whether the numbers which they have written down are actually physically meaningful for example many a times students write down the final answer for a flow rate which could be the mass flow rate or volumetric flow rate as a negative term.

Flow rate for mass cannot be negative it has to be a positive number this makes no sense however people do not think about it in the speed of the exam and try to hurry and submit something there by making an unnecessary silly questions. So these techniques are actually useful for you to ensure such errors do not crop up in your calculations please use this techniques ensures that all the calculations you perform or accurate.

I hope that whatever we have covered today with respect to the units, dimensions, dimensional homogeneity, significant figures and the techniques for validation would give you all the ammunition you need to perform calculation accurately and unsure that you excel in this course throughout this program and thank you.