

Computational Systems Biology
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Lecture – 19
Network Models

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Computational Systems Biology
Network Models


► Random (Erdős–Rényi) Networks

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In today's video, we move on to network models and motivate why we need network models and look at a very interesting, random network model which was proposed by Erdos and Renyi, right.

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NETWORK MODELS



The moment you have a network, you want to try and understand what are the properties of a network, right. We always want to classify things in science and engineering so that we can bin them into observations that we can study collectively.

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Why do we need network models?

- ▶ Create models that mimic real-world networks, based on theory
- ▶ Or rather, discover generative models for real-world networks, based on theoretical principles
- ▶ 'Bin' our observations on real-world networks into the *canonicals*
- ▶ Improve understanding of real-world networks
- ▶ And also the ability to analyse/simulate them, predict behaviours ...

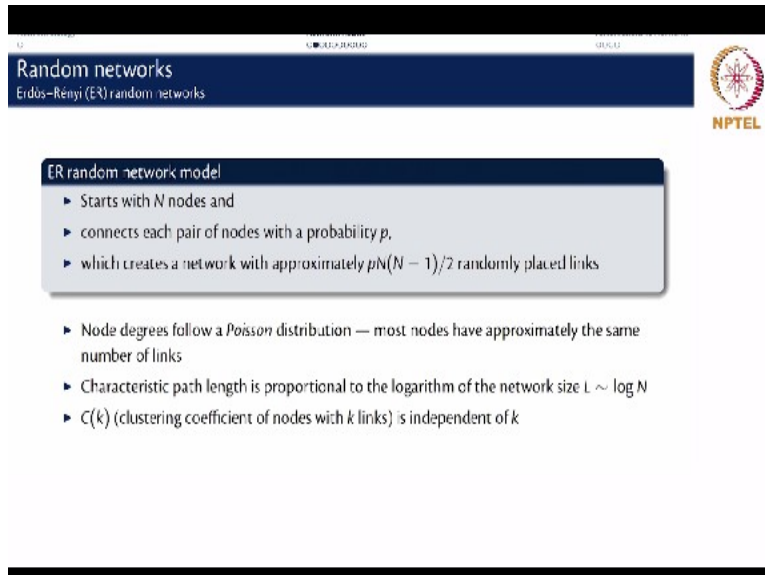
So, we want to create models that mimic real-world networks based on a theory. Maybe I make some assumptions. This is how people connect to one another and I use those assumptions to build a network and then see if it matches my social network then I can make some comment that oh maybe this is how people, networks get formed in real life as well. Essentially, as we have always mentioned, the highest goal would be to actually glean out some design principles, right.

Or what is the generative model for a particular type of real network, be it a gene regulatory network or a protein interaction network or a metabolic network, what is the underlying principle for generating that kind of a network. And we have always liked to bin our observations into some canonicals, right. Oh, this is one class of networks, this is another class of networks, this is another class of networks.

So, my network looks like this class A. The other network looks like this class B network and so on. And this will obviously help us to improve our understanding of real-world networks and also the ability to analyse, simulate them or predict how they perform in different scenarios. For example, what happens when you start removing nodes from different kinds of networks, how

does a perturbation cascade, how does a perturbation you know does it affect the network, does it destroy the network or does it have no effect on the network and so on.

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Random networks
Erdős-Rényi (ER) random networks

ER random network model

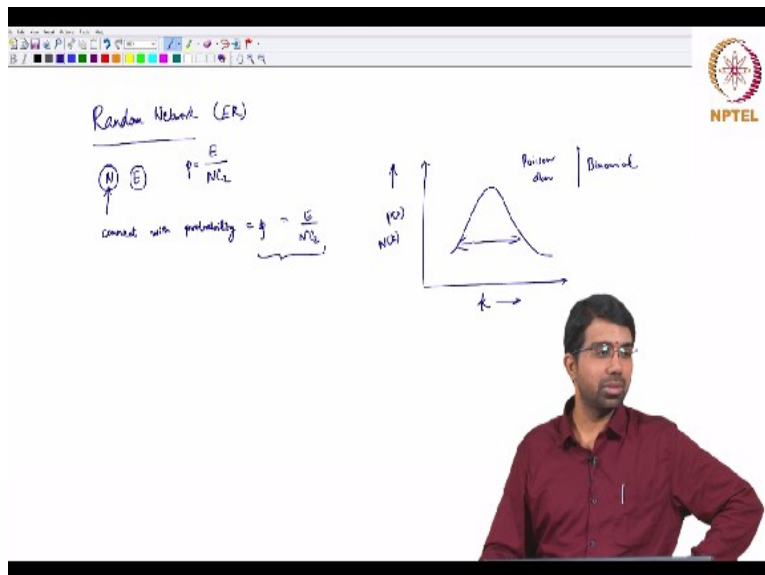
- ▶ Starts with N nodes and
- ▶ connects each pair of nodes with a probability p ,
- ▶ which creates a network with approximately $pN(N-1)/2$ randomly placed links

▶ Node degrees follow a *Poisson* distribution — most nodes have approximately the same number of links

▶ Characteristic path length is proportional to the logarithm of the network size $L \sim \log N$

▶ $C(k)$ (clustering coefficient of nodes with k links) is independent of k

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Random Network (ER)

N nodes, E edges

connect with probability $p = \frac{E}{N(N-1)}$

$P(k)$ vs k graph showing a Poisson distribution curve.

Poisson dist. | Binomial

So, the first type of a network is a random network or what we call an Erdos-Renyi random network. These were the 2 scientists who came up with this network way back in 1960, okay. So, let's say I want to create a network with N nodes and E edges, right. How do you create a network? What will be a random network, right? What do you think is a random network?

“Professor - student conversation” What does, what would random mean in the first place?

There is no pattern, right. There is no special, specific structure in the network, right. If you have no structure, how should it be? How should the network be connected? The connections are formed randomly and no node is special, right.

Which means I have E edges, I have to literally take those E edges and throw them such that they land randomly amongst the possible NC_2 edges. I have NC_2 possible edges out of which I want to fill E edges at random, which means I could basically just take a probability p equals E/NC_2 and with that probability, connect every pair of nodes, right. So now I will just keep picking node pairs at random from, from here and connect them with probability p , right, which is basically nothing but E/NC_2 .

So what would this give you? It will give you a network where no node is sort of special, right. What would the degree distribution look like? What is the degree distribution again? It is a count of the number of neighbours for every node, right. So degree distribution will essentially look like this. So $p(k)$ or $N(k)$, number of nodes is degree k versus k . So common mistake is to assume that the degree distribution might come out as uniform, right.

Because you are connecting every node with probability p but if you look at it closely, you will see that, it's like tossing a coin. You have a probability p of success and what are you counting? The number of successes, right. So, the number of successes for a given node, right. So what is that going to look like? A binomial or it will start looking like a Poisson in, yeah. It will look somewhat like this, okay.

So you can compare this with binomial as well, right. So this is the limiting case of a binomial distribution. So you will see that this is how it looks but you know this spread is not too much. This is not a normal curve where you have like a big spread, right. So the spread is not too much. You will find that no node has much higher degree than another node, Erdos-Renyi random networks.

So, you start with N nodes and connect each pair of nodes with a probability p which will give you approximately pNC_2 randomly placed edges. So, the node degree follows the Poisson

distribution. Most nodes have approximately the same number of links. You also have to figure out what is the characteristic path length look like and may be what does the average degree look like?

What is the diameter of the network and so on? So characteristic path length happens to be proportional to $\log N$ which is quite small, yes. So there are mathematical derivations for all of these. I am going to skip all of them in this class but there are mathematical derivations for all of these and $C(k)$ versus k is actually independent of k . The clustering coefficient for a node with degree k does not depend on k .

So it is essentially like flat lining. But it turned out that this does not really represent many real networks. Of course, the idea here wasn't to represent real networks but the idea was to get a sort of a base model to compare real networks with but then people started wondering how do I make network models that are more realistic, that look like real networks.

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The slide is titled "Recap" and is divided into two sections. The first section, "Topics covered", lists "Random (Erdős-Rényi) Networks". The second section, "In the next video ...", lists "Small-World (Watts-Strogatz) Networks" and "Power law Networks".

Section	Content
Recap	
Topics covered	▶ Random (Erdős-Rényi) Networks
In the next video ...	▶ Small-World (Watts-Strogatz) Networks ▶ Power law Networks

In today's video, I hope you had a good introduction to random networks but real networks are seldom random. In the next video, we will look at some important canonical models of real world networks namely the small-world networks and power law networks.