

**Bioreactors**  
**Prof G. K. Suraishkumar**  
**Department of Biotechnology**  
**Indian Institute of Technology, Madras**

**Lecture – 13**  
**Solution to pp 3.2**

Welcome to lecture 13, the NPTEL online certification course on bioreactors. We are towards the end of module 3. We had assigned a problem in the last class, we will work that out and with that we can complete module 3. The problem, statement reads as.

(Refer Slide Time: 00:37)

**Practice problem 3.2.**

Processed fungi are good biosorbents for toxic trace metals, and thus they can be used to remove chromium, mercury, cobalt and others from industry effluents. A suitable fungus needs to be produced at  $500 \text{ g h}^{-1}$  for the above purpose. The growth limiting substrate concentration at the inlet of a chemostat to produce fungus is  $50 \text{ g l}^{-1}$ . The fungus follows Monod kinetics with the maximum specific growth rate =  $0.5 \text{ h}^{-1}$ . The substrate concentration that corresponds to half maximal growth rate =  $1 \text{ g l}^{-1}$ . It is aerobic growth with a cell yield coefficient from substrate of 0.5.

Find the minimum size of a chemostat needed for the above for a given inlet volumetric flow rate.

Processed fungi are good biosorbents for toxic trace metals, and thus they can be used to remove chromium, mercury, cobalt and others from industry effluents. A suitable fungus needs to be produced at 500 grams per hour for the above purpose, that is a production rate. The growth limiting substrate concentration at the inlet of the chemostat to produce fungus is 50 grams per liter. The fungus follows Monod kinetics with the maximum specific growth rate being equal to 0.5 hour<sup>-1</sup>. The substrate concentration that corresponds to the half maximal growth rate is 1 gram per liter. It is aerobic growth with a cell yield coefficient from substrate of 0.5, that is  $Y_{x/S}$  is 0.5. Find the minimum size of a chemostat needed for the above for a given inlet flow rate.

(Refer Slide Time: 01:47)

**What is needed?**

(a) the minimum size of a chemostat needed for the above for a given inlet volumetric flow rate

**What is known/given?**

Production rate,  $R = 500 \text{ g h}^{-1}$   
 $\mu_m = 0.5 \text{ h}^{-1}$   
 $K_S = 1 \text{ g l}^{-1}$   
 $Y_{x/S} = 0.5$   
 $S_i = 50 \text{ g l}^{-1}$

Monod kinetics for growth

**How to connect what is needed to what is given?**

If volume needs to be minimized, we need to be operating at maximum productivity,  $R_m$

The solution is as follows; we will ask our same questions and try to answer them. What is needed? The minimum size of a chemostat needed for the above given condition for a given inlet volumetric flow rate. What is known or given?

The production rate is  $500 \text{ g/h}$ ,

The maximum specific growth rate is  $0.5 \text{ h}^{-1}$

$K_S$  is  $1 \text{ g/l}$ ,

$Y_{x/S}$  is  $0.5$

The inlet substrate concentration  $S_i = 50 \text{ g/l}$ .

Monod kinetic for growth is given.

Now, how do we connect what is needed to what is given? If the volume needs to be minimized, then we need to be operating at the maximum productivity,  $R_m$ .

(Refer Slide Time: 03:01)

Our inlet flow rate,  $F$ , is fixed. And  $D \equiv \frac{F}{V}$  or  $V = \frac{F}{D}$

So, to minimize  $V$ , we need to maximize  $D$ . We need to find  $D_m$  which will also maximize productivity ( $R_m$ )

We have already derived  $D_m = \mu_m \left( 1 - \left( \frac{K_S}{K_S + S_i} \right)^{0.5} \right)$

If we substitute the values, we get  $D_m = 0.5 \left( 1 - \left( \frac{1}{1 + 50} \right)^{0.5} \right) = 0.43 \text{ h}^{-1}$

Now, if we know  $F$ , we can find  $V_m$ , the minimum size of the chemostat

We don't know  $F$ , but we know that  $R_m = 500 \text{ g h}^{-1}$  and  $R_m = F x_m$

$x_m$  is the outlet cell concentration at maximum productivity

Our inlet flow rate,  $F$ , is fixed.

Dilution rate  $D = F/V$

Therefore  $V = F/D$

So, to minimize  $V$ , we need to maximize  $D$  if  $F$  is a constant.

Therefore, we need to find essentially  $D_m$  which will also maximize the productivity  $R_m$

$$D_m = \mu_m \left( 1 - \left( \frac{K_S}{K_S + S_i} \right)^{0.5} \right)$$

Substitute the values

$$D_m = 0.5 \left( 1 - \left( \frac{1}{1 + 50} \right)^{0.5} \right) = 0.43 \text{ h}^{-1}$$

If we know  $F$ , we can find  $V_m$ , the minimum size of the chemostat. We do not know  $F$ , but we do know that the productivity  $R_m$  is 500 grams per hour.

$$R_m = F x_m$$

where  $x_m$  is the outlet cell concentration at maximum productivity

(Refer Slide Time: 05:12)

And we know  $Y_{x/s}$

Also, we know that  $x = Y_{x/s}(S_i - S)$  So  $x_m = Y_{x/s}(S_i - S_m)$

$S_m$  is the outlet substrate concentration at maximum productivity

To find  $S_m$ , let us use the relationship between  $D$  and  $S$

$$\frac{\mu_m S_m}{K_S + S_m} = D_m \quad \mu_m S_m = D_m(K_S + S_m) \quad S_m = \frac{D_m K_S}{(\mu_m - D_m)}$$
$$S_m (\mu_m - D_m) = D_m K_S$$

Now, let us do the above, step by step through substitution of the relevant values

$$S_m = \frac{D_m K_S}{(\mu_m - D_m)} = \frac{0.43 \times 1}{(0.5 - 0.43)} = 6.14 \text{ g l}^{-1}$$
$$x_m = Y_{x/s}(S_i - S_m) = 0.5(50 - 6.14) = 21.93 \text{ g l}^{-1}$$

We also know the yield coefficient  $Y_{x/s}$ .

Also the cell concentration  $x$  is

$$x = Y_{x/s}(S_i - S)$$

$$\text{Therefore, } x_m = Y_{x/s}(S_i - S_m)$$

$S_m$  is the substrate concentration at the outlet which corresponds to the maximum cell concentration  $x_m$ . How to find  $S_m$  ?

We know that

$$\frac{\mu_m S_m}{K_S + S_m} = D_m$$

Thus,

$$\mu_m S_m = D_m(K_S + S_m)$$

$$(\mu_m - D_m)S_m = K_S D_m$$

$$S_m = \frac{K_S D_m}{(\mu_m - D_m)}$$

Substituting,

$$S_m = \frac{1 \times 0.43}{(0.5 - 0.43)} = 6.14 \text{ } gl^{-1}$$

Now calculate  $x_m$

$$x_m = Y_{x/s}(S_i - S_m) = 0.5 (50 - 6.14) = 21.93 \text{ } gl^{-1}$$

(Refer Slide Time: 07:28)

$$R_m = F x_m \quad 500 = F \times 21.93$$
$$F = \frac{500}{21.93} = 22.8 \text{ } l \text{ } h^{-1}$$

So the corresponding minimum volume is

$$V_{min} = \frac{F}{D_m} = \frac{22.8}{0.43} = 53.02 \text{ } l$$

Since  $R_m = F x_m$

$$F = \frac{R_m}{x_m} = \frac{500}{21.93} = 22.8 \text{ } l \text{ } h^{-1}$$

So the corresponding minimum volume is

$$V_{min} = \frac{F}{D_m} = \frac{22.8}{0.43} = 53.02 \text{ } l$$

So, this is the minimum volume of the chemostat for the given set of conditions in the problem.