

Introduction to Biomedical Imaging Systems
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Lecture - 52
MRI_RECON_S70_S82

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NPTEL

Content- outline

- Review of MR Physics
- MRI hardware
 - Magnet
 - Gradient coils
 - RF coils
- Data acquisition
- Image reconstruction
- Image Quality




Ok. So, I guess you had a chance to view and appreciate the beauty of MRI recording right the data that we got and why is it that the data that you are recording unlike other modalities. Here what you are recording is actually the Fourier transform even though signal is represented as s of t .

The data that you are recording actually turns out to be the samples of your Fourier spectra. So, you know it is very intriguing. So, now, what we will do is as you will see here this was


supposed to be the course content and we have covered until data acquisition. We are still left with image reconstruction and image quality.

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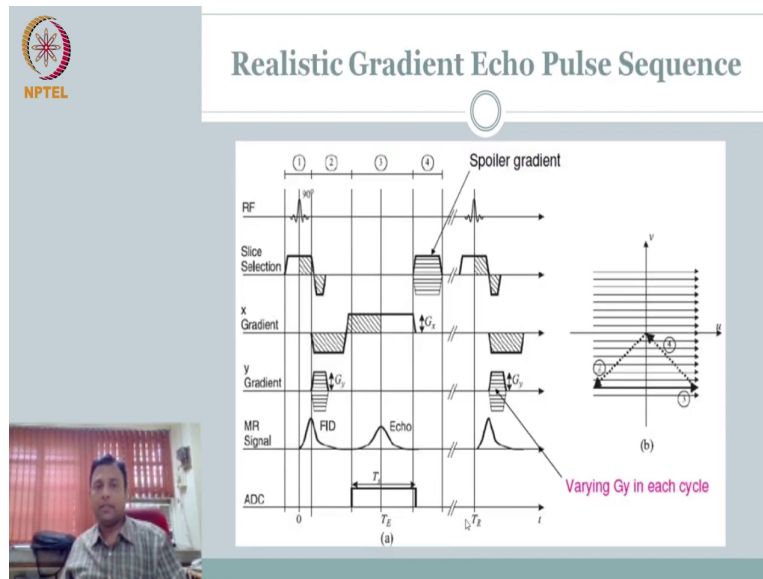
MRI Scan Review

- How to measure the signal at one particular location?
 - Using Z-gradient to vary the static field at different slices
 - Using RF pulses with a certain freq. to excite one slice at a time
 - Using X-gradient and Y-gradient to differentiate voxels in a slice
- Polar scan
- Apply X- and Y-gradient simultaneously with a given ratio, to scan one polar line
- Rectilinear scan
- Apply Y-gradient first to select one horizontal line in Freq. space
- Apply X-gradient to scan the line
- Received signal is samples of the 2D Fourier transform over a slice
- How to obtain the original signal?

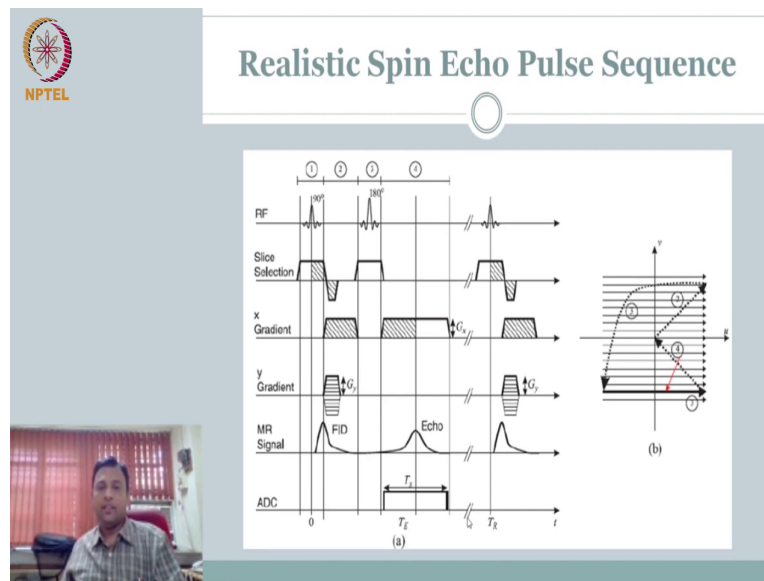


We covered what we will be covering today you know it is going to be very straightforward 10-15 minutes we will finish image recon because there is nothing new ok. So, how do you obtain original signal take, inverse Fourier transform.

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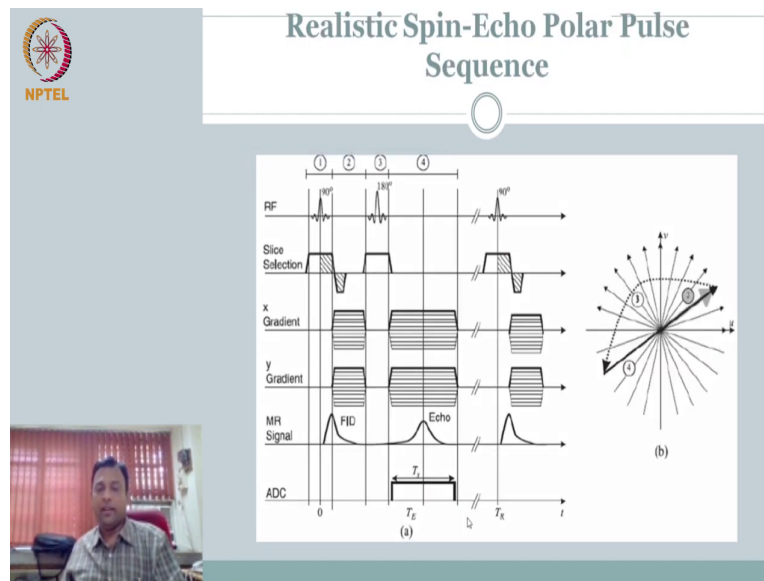


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So, I will just spend some time so, always revision of how we do data acquisition to get rectilinear polar.

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I will not spend time because that is not needed now because of the video lecture.

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Image Reconstruction

- Rectilinear scan
 - Acquired signal is the samples of $F(u,v)$ on a rectangular grid
 - Use inverse 2D FT
- Polar scan
 - Acquired signal is the samples of $F(u,v)$ on the polar grid
 - Use inverse 2D FT after interpolation to rectangular grid
 - Or apply backprojection approach




If you can review the previous video. So, where we will go is we will jump into image reconstruction. So, we have two kinds of data one is rectilinear scan the other is polar scan. So, what we have seen is acquired signal actually is we have samples right, we have samples of F of u comma v on a rectangular grid if it is rectilinear scan.

All you have to do is just take inverse Fourier transform 2D F of 2D inverse Fourier transform you will get your image. Likewise in polar scan acquired data already you have F of u comma v , but in polar grid. So, you can either do that you can convert the 2D to a rectangular grid and do it or apply it back projection approach ok.

So all of these reconstruction from Fourier to image we also did for CT. So, several of those algorithms can quickly guide you as to how you can do the inversion ok. So, we tried convolution remember. So, same things can be applied here nothing secret about it ok.

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Acquired Rectilinear Data


- Acquire data for all phase encode areas

$$A_y = G_y T_p$$
- Baseband signal

$$s_0(t, A_y) = \iint f(x, y) e^{-j\gamma G_x x t} e^{-j\gamma A_y y} dx dy$$
- Identify Fourier frequencies

$$u = \gamma G_x t$$

$$v = \gamma A_y$$




Only thing that I will do is just refresh within our equations what we have and how we start to look at the data or organize the data in a standard format. So, you acquire the data right in the rectilinear scan, you are having phase encoding; phase encoding comes with gradient y, you have the area A y right.

So, you are applying this gradient for a time period time for phase encoding. So, this is the phase that is accumulated. So, you have for each phase we have a baseband signal that you are getting with frequency encoding. So, your s of s 0 of t this t is what, x direction time over which you are acquiring essentially it translates through frequency correct gamma G x t is

going to give you the highest frequency in the read out direction, Δy is the in the v direction how you are moving in surfaces right.

So, what you have is s_0 of t comma frequency and phase essentially right. You have this is the signal f of x comma y e power minus j G_x x t e power minus Δy d y d x y . So, this is the same equation we saw before and we can readily identify like we said what is your u direction frequency, what is your v direction frequency.

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Reconstruction from Rectilinear Scan


- Fourier transform is built over repetitions

$$F(u, v) = s_0 \left(\frac{u}{\Delta G_x}, \frac{v}{\Delta y} \right) \quad 0 \leq u \leq \Delta G_x T_s$$

- Inverse Fourier transform

$$f(x, y) = \iint s_0 \left(\frac{u}{\Delta G_x}, \frac{v}{\Delta y} \right) e^{+j2\pi(ux+vy)} dx dy$$


- This is a fundamental equation in MRI



So, once you have this what you essentially have is F of u comma v is nothing but s_0 of u by ΔG_x . So, you are reorganizing the data right with this. So, that you recognize what you have in this form is nothing but F of u comma v . So, once you have that take the inverse Fourier transform this is the fundamental equation in MRI or rectilinear scanning. Similarly, what you

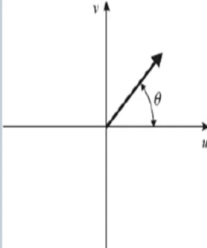
can do is for your polar coordinates; polar coordinates how do you organize the data you have to have r comma θ or r we will not call r we have been using ρ right.

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Acquired Polar Data

- In each RF pulse cycle, a different G_x, G_y is used to form a different θ
- Within each cycle, during the ADC read out time, a range of ρ is achieved



$$u = \gamma G_x t, v = \gamma G_y t$$

$$\rho = \gamma \sqrt{G_x^2 + G_y^2}$$

$$\theta = \tan^{-1} \frac{G_y}{G_x}$$

$$s_y(t; G_x, G_y) = F(\rho \cos \theta, \rho \sin \theta) = G(\rho, \theta)$$

$$G(\rho, \theta) \stackrel{b}{=} s_y\left(\frac{\rho}{\gamma \sqrt{G_x^2 + G_y^2}}; G_x, G_y\right)$$

Because so, in the acquired data in the polar coordinate you acquired it with G_x G_y . So, that you can get different θ . So, range direction we will call it as a ρ the angle is θ correct. So, essentially what we are doing here is we are identifying u as $\gamma G_x t$ and v as $\gamma G_y t$ we are applying simultaneously G_x and G_y and the readout is for the same duration.

So, here you notice v also has the t , if you understand the meaning you do not have to pay attention in remembering all this. This will just this will come straight forward by thinking about the physics using the mathematical notations you will arrive at the equations.

So, u is $\gamma G_x t$ because it is a frequency γG_y is the frequency you are acquiring it over a period t right t is the acquisition. So, you are having this readout gradient while you are acquiring. So, that is why this t is there. So, this was there from before. The phase direction also in polar means you are applying gradient G_y also simultaneously therefore, the t is here also.

So, your ρ is magnitude right square root of G_x^2 plus G_y^2 θ is as before \tan^{-1} of these two. So, you have reorganized you recognize what you are measuring it is nothing but it is in the polar coordinate F of $\rho \cos \theta$ $\rho \sin \theta$ for different ρ and θ right.

So, once you have this you can go to the time spatial domain which is your image clear. So, there is nothing more to add to image reconstruction with respect to MRI. If you understood the basic signal we are already having Fourier domain of the data 2D Fourier transform of the data take inversely. So, you organize it like this.

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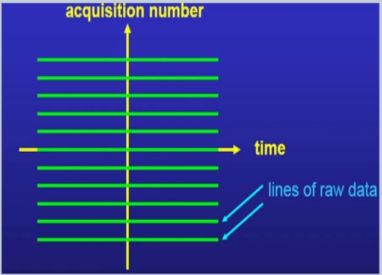



Image reconstruction in 2D

- It is useful to think of the raw signal from each successive acquisition as being recorded as lines in a two dimensional space, which we call **k-space**



- A 2D inverse Fourier transform of this data then provides the spatial distribution of the signal. In general the signal distribution is complex-valued, and the magnitude is taken to produce the image



So, how do we generally organized is. So, it is useful to think whatever you are recording the raw signal raw signal from each successively acquisition. So, you have one setting you acquire ones you have another setting you have acquire again right. So, you can label the readout direction is t . So, each acquisition has a time.

So, you can think about your use field different acquisition means you are moving in space in the v direction. So, you can start to think about raw signal from successive acquisition as being recorded as lines in 2D space, that is in case of you know rectilinear for example, it is easy to think about it as parallel lines right. So, we call this as k space I remember that we mentioned. So, Fourier space.

But, in the domain usually they instead of talking about frequency right when you have cycles per second in time is frequency correct or number of cycles you know in 2π radians. But,

when you talk about spatial variations not time variations usually it is advantageous to say number of cycles over a length rather than time right.

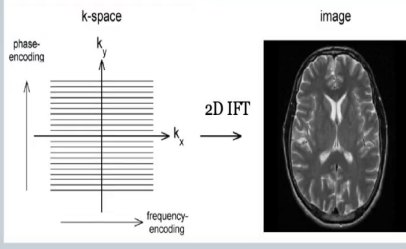

So, number of cycles one cycle is a λ right in your time domain you know frequency. So, number of λ s in the length 2π length is usually a wave number. So, it is convenient when you talk about spatial frequencies people talk about frequency in terms of k space, k is referring to the wave number ok.

So, that jargon is still there. So, people, but the way we have covered your u is k_x , v is k_y ok. So, you just have to be careful whatever we are covering is correct you just have to if you are reading different literature from different background they may call this as k space essentially k space it is similar to what you have.

Just denoting it as k space because of the jargon of wave number rather than the frequency. So, you align your data time is your u direction, acquisition number is your y , each time you acquire you align the data like this in your variable. So, this is your Fourier 2D Fourier transform of the data, once you have this you grid it take the inverse Fourier transform ok.

So, you can get your of course, the 2D inverse Fourier transform data provides spatial distribution rate. Generally, it is a complex value and therefore, magnitude is taken to provide the image clear. So, that is it for image reconstruction very simple right you know intuitively what it is by now.

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The relation between k_x and the time and between k_y and the acquisition number will be derived shortly

So, just to show another example right what we recorded. So, this is called as k space nothing different from u and v that we covered. One is your frequency encoding direction, the other is phase encoding direction. Whatever you are recording you align it like this in the variable space right, when you align it like this in your variable space and then take 2D inverse Fourier; inverse 2D Fourier transform that should be a inverse.

If you do that you get the image clearly you can see this is fantastic image, but it is really complex to visualize that it was coming from this data acquisition and this data alignment. But, such is the you know beauty of MRI that you can do several things. But, to get this basic feel for what is this gray, what is this black, what does this mean it is not straight forward I hope you repeat the physics lectures and then repeat the data acquisition lectures and get a feel for.

Because reconstruction lecture is straight forward if you understand this taking inverse Fourier transform coming here should not be that difficult to understand ok. So finally, last slide what we will do is put every signal right we have measured all the signal we will put it in the equation align it up. So, that we see the signal equation.

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Imaging Equations

Recall $f(x, y, t) = AM(x, y; 0^+)e^{-t/T_2(x, y)}$

If T_R is sufficiently long ($\gg T_1$), $M(x, y; 0^+) = M_0(x, y) \sin \alpha$

As long as $T_R \gg T_2$, after many pulse cycles, each spin reaches a steady state $M_z \neq M_0$.

Let this steady state value be $M_z^m(x, y)$, then $M(x, y; 0^+) = M_z^m(x, y) \sin \alpha$

We can show $M_z^m = M_0(x, y) \frac{1 - e^{-T_R/T_1}}{1 - \cos \alpha e^{-T_R/T_1}}$

$f(x, y, t) = AM_0(x, y) \sin \alpha e^{-t/T_2(x, y)} \frac{1 - e^{-T_R/T_1}}{1 - \cos \alpha e^{-T_R/T_1}}$

$\propto P_D(x, y) \sin \alpha e^{-t/T_2(x, y)} \frac{1 - e^{-T_R/T_1}}{1 - \cos \alpha e^{-T_R/T_1}}$

When $\alpha = \pi/2$

 $f(x, y, t) \propto P_D(x, y) e^{-t/T_2(x, y)} (1 - e^{-T_R/T_1})$

This is the imaging equation for MRI.

In spin - echo sequence, we measure at $T_E \ll T_2$

So, imaging equations we know f of x comma y of t is this guy some scaling constants is essentially a magnetization vector from x comma y at 0 plus right at the start, which is going to decay. So, this is your signal that you are recording right in the transverse plane right so, minus t and with time constant here.

But, if T_R is sufficiently large which we saw before then no issues this essentially becomes your M_0 that is your equilibrium value time $\sin \alpha$. But, notice here this is good

whatever we have covered so far is good when you do it once. There is an important concept that we will reveal now ok.

What we want to do is repeat it again and again and again right that is this is your magnetization vector and pushing it to the transverse plane again it comes back, again I am pushing it repeating the experiment with the different settings right. So, when I do it like this what happens is we were used to the term called allow the signal to come back to what; to equilibrium position your M_z right that is what we were doing so far.

So, if you allow it to come to equilibrium we will call that value as M_z . But, it so happens that if you do this multiple times well that magnetization vector never comes to equilibrium right you have to wait for a long time for it to come to equilibrium, but you repeat it. So, what happens is it does not reach the equilibrium, but it reaches a steady state.

So, each time you do right this experiment the value that it comes ready before you do the next one we can say that after several repetitions of this repeats of this you will call it as steady state M_z not M_z . M_z is equilibrium, which happens when you wait for long time without multiple push it comes to equilibrium.

The moment you are going to do this pushing experiment several times repeating the sequence several times you are pushing this vector and as you know every time with any displacements right when you do this dynamics it has to have some steady state you do this multiple times right static dynamics. So, when you try to push this several times you will have reach a steady state that M_z is not equilibrium, but it is a steady state.

So now, we will just rewrite our equations to correct for this if you are going to repeat the experiments right repeat the acquisitions. So, let the steady state value we will denote it as M_z at infinity comma x comma y . So, we are using this jargon instead of 0 we are putting infinity.

Then what happens is your M_x y at 0 plus is not M_0 , right, this is what we said before, but now it is M_z wherever you are starting M_z the steady state value not the equilibrium value.

So, your M_0 gets replaced by M_z at infinity $x, y, \sin \alpha$. So, once we can show that this essentially what this M_z is your equilibrium value with some you know deviation.

The deviation has to do with number of times you are repeating and the properties of how long you wait which will dictate your T_1 right. So, T_1 constant T_1 property of the tissue and how far you wait that your T_R ; this relative T_R to T_1 is going to have some deration of your equilibrium value.

So, we are going to end up using M_z the steady state value from the imaging equations instead of equilibrium value. And therefore, your $f(x, y, t)$ is nothing but you substitute right you substitute the values you get $M_0 \sin \alpha e^{-\text{power whatever times this guy right}}$.

So, which we can directly say this M_0 was proportional right. So, it can say it is proportional to proton density $\sin \alpha$ this guy. So, when α is also $\pi/2$ which is your maximum signal then you get $f(x, y)$ is proportional to this guy ok. So, this is your imaging equations this is modified; this is your $f(x, y)$.

Meaning what is your $f(x, y, t)$? This is the image right at every pixel x, y this is your image at every pixel x, y your basic signal strength is dependent on the proton density at that location. But, it is also dependent on the T_2 's and T_1 's and the time to repeat all that is going to.

So, we saw in contrast mechanism right how do you play with this to get a differential weighted images right, that is our $f(x, y, t)$ this is a governing equation depending on how you combine T_2, T_1, T_R you get different weighted images ok. So, of course, in spin echo we measure TE is far far less than T_2 right you repeat the experiment, clear? So, this completes in some sense the image formation.

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Image Quality

- Sampling parameters in Fourier space
 - Sampling spacing vs. field of view
 - Coverage area vs. blurring
- SNR



We need to next study about image quality.