

Introduction to Biomedical Imaging Systems
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Lecture - 27
Nuclear Med_Phys



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Course Plan*
* Subject to change if need be

Topics

- Introduction, Signals Systems review, Image Quality metrics
- Projection Radiography
- X-ray CT
- Nuclear Medicine- Scintigraphy, PET/SPECT**
- MRI
- Ultrasound Imaging
- Other recent modalities

Textbook:
Medical Imaging Signals and Systems by J. L. Prince and J. M. Links,
Pearson Prentice Hall, 2006, ISBN 0130653535.

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The slide is titled "Topics covered" and features a list of topics on the left side. The topics are: Nuclide Imaging Overview, Physics of Radioactive Decay, Planar Scintigraphy (with sub-points: Scintillation camera, Imaging equation, Image Quality), Single Photon Emission Computed Tomography (SPECT), Positron Emission Tomography (PET), and Image Quality consideration (with sub-points: Resolution, noise, SNR, blurring). Red handwritten curly braces group the topics into three sections: the first two topics, the Planar Scintigraphy sub-points, and the last two topics. On the right side of the slide, there is an NPTEL logo and a video feed of a male presenter in a white shirt.

- Nuclide Imaging Overview
- Physics of Radioactive Decay
- Planar Scintigraphy
 - Scintillation camera
 - Imaging equation
 - Image Quality
- Single Photon Emission Computed Tomography (SPECT)
- Positron Emission Tomography (PET)
- Image Quality consideration
 - Resolution, noise, SNR, blurring

Welcome back. We are all set to start a new the new module on Nuclear Medicine right. So, as per our flow of contents right we will start nuclear medicine with first talking about the physics part of it. A quick overview and then physics and then the different you know sub modalities. The nuclear medicine is one big branch within that what are the different techniques that are used to acquire right. So, different images that you can get.

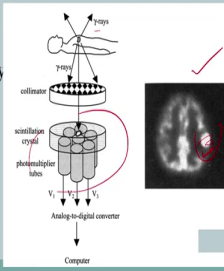
So, we will go in that order. So, what we will do first is start with a overview and then physics. Spend some time on planar scintigraphy it is like varies and then on your T's. Very similar to the template that we followed right X ray imaging. So, physics we covered then projection radiography. After projection radiography we started to talk about the tomography part of it right.


So, similar order we will cover here and then conclude with image quality considerations for PET and PET ok. So, let us get ourselves introduced to nuclide imaging right. In fact, we introduced this in some sense during our introduction. So, it is going to be a quick overview of what it is.


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Nuclear Medicine?

- Also known as nuclide imaging
- Introduce radioactive substance into body
- Allow for distribution and uptake/ metabolism of compound
⇒ *Functional Imaging!*
- Detect regional variations of radioactivity as indication of presence or absence of specific physiologic function
- Detection by “gamma camera” or detector array (Image reconstruction)







So, what is nuclear medicine? I mean nuclear medicine might seem like it is medical subspecialty, right that is how it is used nuclear medicine, we are engineer, so why is it nuclear medicine?

I mean, traditionally that is how it has been host, but our interest we want to look at it from a nuclide also known as nuclide imaging. So, what is nuclide imaging? The name is very suggestive right, imaging of the nucleus. So, what we did so far; we will again review that,

but we were talking about in X ray in the earlier modules about X ray we were more interested about the electrons right.

Now, we should be more interested in the nuclear aspect of it right of the atom. So, quickly this is the logic. So, you here what is happening is, we actually send some radio tracers, radioactivity, all these terms we will kind of describe in this module to begin with.

But essentially you are you are taking some you are sending some radioactivity into the body in the form of radio tracers. After that what happens is you give it some time, it will distribute itself right.

So, you introduce radioactivity into the body. That substance will distribute because you put it in the blood it will go through wherever the blood is taking it and get used right or get accumulated or go to location where it is used right. So, and then because this is radioactive tracer, it is going to give out radioactivity or it is going to give out gamma energy. So, it is going to give out gamma energy.

So, when we detect this gamma energy our interest here is not in the energy per se. Our interest is after you detect the gamma where is it coming from right that is our interest. So, you give some time right for uptake metabolism to happen, after that you detect. You detect the regional variation of this activity right and the idea is the presence of absence of any activity can be tracked by the variation in the uptake right.

So, detect regional variations of radioactivity as an indication of presence or of absence of a physiological function. So, the key message here is, even before we said X ray, X ray came out on the other side and we detected. And we said, depending on the attenuation we could tell what is at depending on the number of photons that are received right.

On the detector we used to tell what is the attenuation along the path. So, it was a distribution of μ in X ray energy. Here, on the other hand, we are not really interested in the attenuation

of the or what is the μ or anything. Here, the activity is coming. Our objective is to say where that activity came from, because that activity is indicative of some action.

So, it this is functional imaging. This is the key. So, this is very important; functional imaging. So, whatever we did in X ray it was through transmission, you sent through got the energy on the X ray energy on the other side.

Whereas here, as you can see in this diagram here the source is sent into the body right source is sent into the body, your objective is to detect where the source is. So, based on the detection you have to locate where the source is. So, it is slightly different way of looking at things, but it gives very powerful look.

For example, so you have some instrumentation which we will cover and then you transform this to an image. Notice here, right there is a region that is high activity say for a if you have a tumor right cancerous tumor. You know cancers tend to grow aggressively, so they are metabolically more active. So, when you send radioactive tracer. It will go accumulate more right in the region where there is more metabolic activity.

So, the radioactivity will come from more from this location, because the radio tracer that you sent is supposed to go there ok. So, you engineer it so that it goes to that location ok good. So, we will go into all these terms that I said right radioactivity, tracer, giving out energy, gamma rays, detector, all this we will cover ok, but this is the rough idea that we want to know.

So, once this gamma rays come we can either use gamma camera or a detector array. So, how we did for chest x projection radiography right. We got a X ray imaging, we got an image or in CT X ray CT we detected it and then did a reconstruction.

So, we will do similar things here, in one we call as scintigraphy the other is your PET aspect depending on what you are reconstructing ok. So, so much for the introduction of what we are it is a big picture overview of what we are going to cover.

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PET vs CT

- X-ray projection and tomography:
 - X-ray transmitted through a body from a outside source to a detector (transmission imaging)
 - Measuring anatomic structure ✓
- Nuclear medicine:
 - Gamma rays emitted from within a body (emission imaging) ✓
 - Imaging of functional or metabolic contrasts (not anatomic)
- Brain perfusion
- Myocardial perfusion
- Tumor detection (metastases) }

NPTEL

So, just to relate it to what we have covered so far or what you know from X ray CT right where are the difference. So, in projection radiography right what we covered? We covered projection radiography and tomography. So, that was a through transmission image, that is the key ok. And, most of the time what the images that we showed right. It was structural imaging or measure measuring the anatomical structure ok.

Whereas, in nuclear medicine as we just saw, this is about detecting the gamma rays, but this is from inside emitting from the source inside the body. So, this is a emission imaging ok, it is not through transmission this is emission. So, you send the radio tracer radioactive tracer into the body and it starts to emit gamma rays.

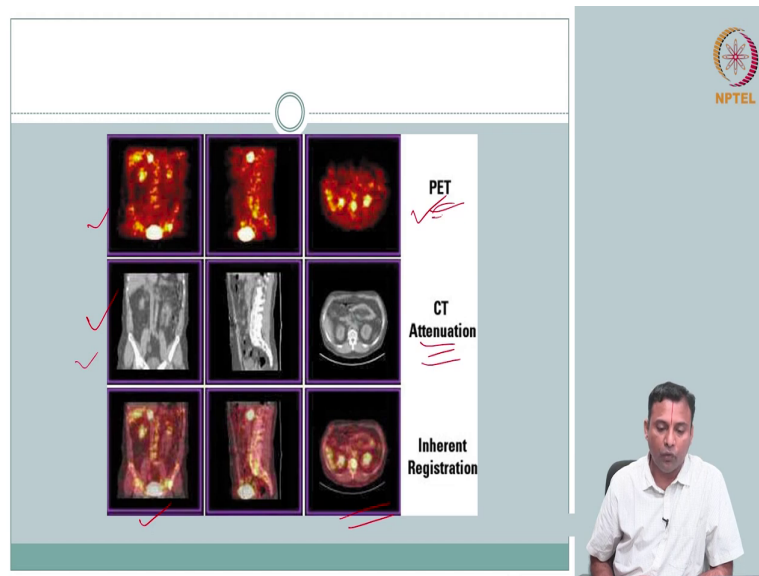
So, you are essentially catching the emissions. So, it is called as where is the emission coming from, how much of emission is coming from where, that is what you are capturing.

So, it is your emission imaging. Again, like we talked about it is a functional, so you are not really talking about anatomical contrast here. You are not actually trying to measure the tumor dimension right. You are trying to capture or you are trying to see the tumor based on the activity that is happening. So, you are not going to measure the tumor size based on this image, for that we can use CT.

We are going to capture the activity of the tumor that you can use PET. How metabolically active is the tumor, that you can catch from PET ok. So, it is actually, so there are several other applications brain perfusion, myocardial perfusion. This is another important.

So, you know whole body in one of the introduction slides we showed PET you can use it for whole body right. So, you can actually detect where all the activity is it will light up in the body. So, it is a very good modality to capture or see the metastases meaning a primary tumor is there and it goes branches out tumors come in different locations. So, you can do a whole body PET scan, you might find out possible locations where that cancer is active ok.

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So, by itself it is a functional image, anatomy is not the main interest. So usually what happens? So, this is an example of PET images and the color scheme that it is usually plotted. So, you can notice that of course bladder will show up anything that you take it is going to come to the bladder right, so you can always catch up in the bladder.

But the key difference here is, this is the corresponding attenuation image or from your CT. Notice you see so much details here which is actually not present here. So, typically you also call what we call as a PET CT right. So, they fuse these two images, it is a multi modal imaging here it is dual mode PET and CT scanner. So, PET CT scanner essentially it tries to register the PET on top of a CT, so that you get both structural image and functional aspect of it. So, this is very powerful.

Clearly, you can appreciate this image gives you lot more information than either of the two images in isolation ok. So, that is a very powerful you know very clinically significant applications. And so what we will try to do now is step in and understand what is the physics. That is the first part that we said right here. We understood attenuation that forms the basis for the image.

The pixel if you see, right where does it come from, what is the meaning of the pixel, what are the units of the pixel, all that we covered right. Hopefully you are able to appreciate that. So, we will try to now do the same thing. When you see a red color or a yellow color or bright yellow, what does it mean, what is the units of that, how did we get it, what all does it capture, how all it got, how can I increase it decrease it right what plays a role in that.

So, all these aspects both from physics fundamental physics and then the instrumentation and imaging and image quality we will cover for. Of course, this is PET. We will cover for nuclear medicine. This is only one of the mode you know one of the techniques in nuclear medicine, ok, fine.

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Basic Atomic Structure

- An atom={a nucleus,electrons}
- nucleons = {protons; neutrons}
- mass number $A = \# \text{ nucleons}$
- atomic number $Z = \# \text{ protons} = \# \text{ electrons}$
 - Define an element with a particular symbol: H, C, etc.
 - An element is denoted by it's A and Z
 - Ex: C-12 or $^{12}_6\text{C}$

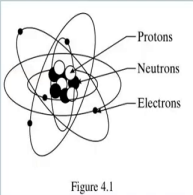





Figure 4.1



So, what we will do now is, get to the same slide that we started with right with the atomic structure. What you notice is this is the same slide, what we were focusing on in the previous module with X ray.


We talked about only the electrons and how the energy, x energy, X ray energy interacts with these electrons, but now the focus is going to shift to the center ok. So, we are going to now talk about the nucleus part. What is there? You have protons and neutrons, you have protons and neutrons.

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Stable and Unstable States

- Stable nuclides:
 - # neutrons \sim # protons ($A \sim 2Z$)
- Unstable nuclides (radionuclides, radioactive atoms)
 - Likely to undergo radioactive decay, which gives off energy and results in a more stable nucleus




So, now we need to address the similar question ok. We also talked about stable nuclides right. Of course, we will pay now attention to this approximately equal to, that is one take home message you will you will get across, but there is a relationship between neutrons and protons for the nuclei to be stable ok.

So, if under certain conditions you can have unstable nuclides. So, if a nuclide is unstable that atom is called as radionuclides radioactive atoms ok. So, what is the big deal about this? The big deal is something is stable. We talked about this also for the whole of electrons interactions right.

If something is unstable the nature has it that it will come back to stability by sending out the excess energy right. This is a big picture philosophy right. You have to give out the energy to come back to ground state which is usually the more stable state.

So, likely to undergo radioactive decay right, so the radioactive element, radioactive atom sends out the extra energy comes back to ground state ok. So, we will see what kind of atoms of course, we talked about unstable nuclides give rise to radioactive atoms. So, we will see the conditions under which this happens or some of the radioactive atoms and elements and how it can be engineered, we will look at it.

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Iso-

- Isotopes: atoms with the same Z but different A
 - E.g. C-12 and C-11
 - Chemically identical ✓

- Isobars: atoms with the same A but different Z
 - Different elements ✓
 - E.g. Carbon-11 and boron-11

- Isotones: atoms with the same number of neutrons but different A

- Isomers: atoms with the same Z and A but with different energy levels (produced after gamma decay)



So, before we jump in, there are several different terminologies small variations in the number game right. So, what is an isotope? You have the same mass number sorry, you have the same atomic number, but different mass number. So, here for example, right you have

carbon 12 or carbon 11, carbon is the same element so chemically identical ok. So, these are called isotopes.

But, you could also have what is called as isobars. It is here is the thing right. So, atoms with same mass number but different atomic number right. So, if it is a different atomic number we also have we said this redundant information. So, now, boron carbon we also named it. So, carbon, boron these are different elements, but they have same mass number. So, these are called isobars. There are few more isos that we will cover ok.

So, isotones, we were even talking about mass number sorry electrons and protons right electrons, protons or your nucleus right, but in the nucleus we start you have protons and neutrons isotones, what happens? Isotones have same number of neutrons, but have different A right mass number.

So, within the neutrons, right within the nucleus you have neutrons and protons. Here, is a case where isotones atoms with the same number of neutrons, but it has a different mass number.

Isomers; this is another interesting aspect. Isomers, it has same everything same Z and A , but it is at a different energy level ok. So, these are you know these are interesting because these are sometimes used in the process that we are going to look at right.

So, sometimes these come out because of the decay process ok. So, there are several iso's that you should be very familiar. It is slightly confusing, but then when you look at the context and see how it is exploited I think it will become you should be able to get over that confusion ok.



So, so much for the different types. So, now, what we need to talk about is, ok we have radioactivity. So, you have unstable nuclides it has to shed energy to become stable, so this process right you have some radioactivity.

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Mass Defect and Binding Energy

Sum of masses of the constituents of atoms is > than the atom's actual mass

- This difference is called the "mass defect"
- Consider C-12: $6xM_p + 6xM_e + 6xM_n = 12.098934 \text{ amu}$
- "mass defect" accounts for "missing" energy called the "binding energy"
- $1 \text{ u} = 931 \text{ MeV}$; therefore for C-12 = $0.098934 \times 931 = 92.1 \text{ MeV}$
- Binding energy per nucleon = $92.1/12 = 7.67 \text{ MeV/nucleon}$
- Nuclear binding energy \rightarrow energy required to separate p and n
- Radioactive decay is the process by which an atom rearranges its constituent protons and neutrons to end up with lower inherent energy



So, we need to define why this is happening or where all this energy right the excess energy to shed where is it coming from right. We talked about similar thing; binding energy when we talked about in electrons right electron binding energy. Now our interest is in the nucleus.

So, we need to talk about two concepts; one is mass defect, the next is it is equivalent, binding energy of the constituents of the nucleus. So, what is mass of an atom right? We know mass of all this right. Have you heard of atomic mass unit right? You would have heard about that right. So, it is one-twelfth of the carbon, 12 right, that is what we use as a units for talking about the mass.

But it turns out that you have a mass, but then I have the constituents right which are electrons protons and neutrons each of these particles have a mass. So, what happens if I sum the masses of the constituents of the atoms right. Atom consists of these components right,

these constituent elements particles. So, if I add them that should be the mass of the atom, but that is not the case, the sum of the masses of the constituents is actually greater than the atoms actual mass ok.

So, it turns out let us take the example right. So, this difference is called as the mass defect. So, there is a difference between the sum of mass of the constituent particles and the actual mass of the atom. These two are different and this is called as the mass defect.

So, let us take I mean because you know carbon 12. So, one twelfth of carbon mass is your atomic mass unit right, but actual like that is your atoms actual mass, but what is the mass if you calculate based on the constituents.

So you have 6 mass of protons plus 6 mass of electrons plus 6 mass of neutrons, if you add that you are actually getting 12.098934 amu's. This is clearly different from 12 amu right so there is a fraction there is a small difference this is called as your mass defect.

So, what does this indicate, where does this extra mass right? Actual mass is only 12, but yes when you add all this it comes to this much and we are talking about this excess mass ok and we loosely said there has to be something to do with energy states. So, how do we connect the mass and energy? We know that right E is equal to MC^2 .

So, we can connect mass and energy using the equation famous equation E equal to MC^2 square therefore, this mass defect right this mass extra mass accounts for missing energy which is called as which called as the binding energy. So, when it is binding right it has shed this extra energy, you can look at it that way right.

So, when it is intact right together, that is how we talked about even electrons. When the electron was part of the atom it was at low energy correct. You have to supply energy to bring free the electron. So when it came back in it shed the excess energy, similar concept right.

So, here there is a mass defect this mass defect has a equivalent energy, that is the one that is binding the constituent particles right. So, binding energy so the relationship between mass and binding energy is from your E is equal to MC^2 . So, if you take one unit right, we have always talked about energy in terms of electron volt. That is how we have been defining in this imaging system physics right.

So, when we do that, 1 units has about 931 million electron volt. So that means, your mass defect is going to correspond to so your 0.09893 whatever. That is a small mass defect right. That mass defect is going to correspond to 92.1 million. What is this? That excess energy, that is your binding energy ok that is your binding energy.

Of course, instead of talking about binding energy clearly the heavier the atom, right more number of constituent particles probably the defect is also going to be little more and therefore your energy could be more. So, instead of reporting just that in normalized binding energy per nucleon ok per nucleon. So what is the nucleon here? You have 12, 6 6 protons and 6 neutrons, put together you have 12 particles in the nucleus.

So, you have your 92.1 is the energy that is a defect whatever mass defect right, from the mass defect, that energy is your binding energy binding energy per nucleon per 12 constituent, neutrons plus protons is about 7.67. So, essentially you will notice that if right you will kind of notice a behavior like this with say this is your number of nucleons and this is your binding energy per nucleon right. So, you will have several elements, so this is a behavior that this depicts ok.

So, it does not really increase or anything it has an increase and then it kind of plateau. So you have a list. In fact, there is a table you can find this plot with exactly which element is lying where ok. But the idea here is, nuclear binding energy required to separate p and n the constituents of your nucleus.

So, the radioactivity right, the radioactive decay can be thought of the process in which you have the radio unstable right. So, it has to rearrange itself rearrange itself so that it comes to

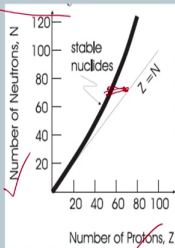
ground state inherent lower ground state ok. So, there is this rearrangement in the nucleus so that you get to a lower energy state. So, if it is unstable, it will become stable by shedding out the excess energy. How does it come down? It rearranges the constituents. So, this nuclear binding energy.

So, very equivalent to I mean analogous I should say, analogous to the concept of a electron binding energy that we saw right. So, and then it had to rearrange in the process it gave out the excess. Similar thing happens here, but this is now happening in the nucleus ok.



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Line of stability

- Two groups of Nuclides
 - Non-radioactive { stable atoms }
 - Radioactive { unstable atoms }
- The concept of “Line of stability”



The graph shows the relationship between the number of protons (Z) on the x-axis and the number of neutrons (N) on the y-axis. The x-axis ranges from 0 to 100, and the y-axis ranges from 0 to 120. A thick black curve, labeled 'stable nuclides', starts at the origin and curves upwards. A dashed line labeled 'Z=N' is also shown. A red arrow points to the curve with the text 'Line of stability'.



So, we can actually look at now make this, remember I told about this number of protons is approximately number of neutrons and I said, we will come look at that relationship between neutrons and protons right. So, we have two types of nuclides, two groups of nuclides; one is stable, the other is unstable.

So, the stable is fine. These are considered non radioactive. We are more interested in the case which is radioactive or rather these are unstable atoms ok. So now, the question is what kind of atoms. So, we know we talk about unstable atoms which are radioactive.

Can we talk about their stability? We just talked about their binding energy and number of nucleons right, binding energy per nucleon. Is there any clue on what could be unstable right.

So, that is where this concept of line of stability comes in line of stability comes into picture. So, here so before you jump or this line which seems this is not the line of stability. Line of stability is actually this curve that you see. So, what is this? This is a plot between number of protons and number of neutrons.

The two constituents of your nucleus. So, now you notice when it is a smaller atom, probably this is along the line, but as the size increases right, as the number of protons increases for the atom right it becomes a heavier element, your actually number of neutrons should be slightly more than your number of protons so as to be stable. So, these are stable nuclides.

So, it is not even though it is called as line of stability, line of stability is not a line right. Line of stability is basically telling, if how much more neutron should I have for a given proton so that this nuclide is stable. So, in some sense this is the line Z equal to N . Here you are talking about stable means; it is actually having more neutrons than protons as you as you increase the mass number ok.


So, this is a very important concept, because as you will see the whole idea about you can now start to think about radioactivity as nothing, but right the stability or instability that you can start to think about in terms of the atom wanting to rearrange the neutron to proton.

So, if I am here for example, right I am unstable. I may want to have right protons, so I may want to convert some on some way rearrange so that I want to get to this line to become stable right.

So, I have to increase the neutron, decrease the proton right to reach this line. So, how this rearrangement takes place? It we will talk about that, but the idea is this line of stability is important because that tells you when an atom is.


So, you can think about radioactivity as the process by which the atom tries to abide by this line of stability. It rearranges so that it forms on this line. So that is your line of stability. Now, we need to talk more about we talked about radioactivity decay. It is trying to rearrange itself very very loose terms.

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Radioactivity

- Radioactive decay: rearrangement of nuclei to lower energy states
{parent atom → daughter atom}
- Daughter atom has higher binding energy/nucleon than parent { aka, becomes more stable!}
- Energy is released during radioactive decay when disintegration of radioactive atom takes place



So, we will now use the next few minutes to organize our thoughts into terminologies that will be consistently used. So, radioactive decay. What is the radioactive decay? Is the

rearrangement of the nuclei to low energy state. So, we will have what is called as a parent atom. Parent atom is a radioactive unstable atom.

What will happen? It will shed the energy and come give out daughter atom right. The daughter atom is supposed to be more stable or in other words you can also view it as daughter atom has higher energy, higher binding energy per nucleon higher binding energy per nucleon. Remember, the other cons binding energy per nucleon we talked about right. So, that again you can also view it like that.

So, if you have more binding energy per nucleon, then it tends to be at a less energy state right. So, you can also view it in terms of binding energy per nucleon. So, daughter atom has higher binding energy per nucleon and therefore it is more stable ok.

So, becomes more stable. So, it so when the parent atom decays, you get daughter atom and the daughter atom is more stable. So meaning; you can think about it as having higher binding energy per nucleon ok.

So, this rearrangement takes place. So, naturally that means, in this rearrangement energy is released right. So, when we talk about this radioactive decay, we talk about some disintegration takes place right. So, you have this atom parent atom becoming daughter atom; that means, this atom is disintegrated to, while it is giving that energy out during the radioactive decay ok.

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Measurement of Radioactivity



- Radioactivity, A , # disintegrations per second

$$1 \text{ Bq} = 1 \text{ dps}$$
$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$$

- Bq=Bequerel Ci=Curie
- Naturally occurring radioisotopes discovered 1896 by Becquerel
- First artificial radioisotopes produced by the Curie 1934
- The intensity of radiation incident on a detector at range r from a radioactive source is

$$I = \frac{AE}{4\pi r^2}$$

A: radioactivity of the material; E: energy of each photon



So, we can think about it in terms of or radioactivity in terms of number of disintegrations per second ok. So, radioactivity when you say it happens what is happening? This parent atom is giving daughter atom so it is disintegrating. How many disintegrations per second happens that talks about your radioactivity, how active right; number of disintegrations per second. So, radioactivity, number of disintegrations per second. Is there any units?

The SI unit is 1 becquerel or equal to 1 disintegrations per second or more commonly what is used is curie right. So, naturally these two are related right. 1 curie is 3.7 into 10 power 10 becquerel. What is this curie and becquerel? these are all named after (Refer Time: 33:02) right, people who actually did contribute to these concepts. So, becquerel and curie are in honor of right, people who contributed naturally occurring radiotop radioisotope was discovered by Becquerel.

Artificial radioisotope was produced by Curie. So, the units of this disintegration, right SI unit is honored with the becquerel, but more commonly the dosage that we are using right, we tend to use millicurie, curie millicurie. Especially in bio imaging we are talking you know most commonly in terms of curie both are heavy heaters who contributed to this ok. So, that is your radioactivity ok fine.

So, now we slowly are getting into the nitty gritty. So, we got the radioactivity, we know this energy shedding, so we have some names right, we have some units. So, what do we need to, I mean before we jump in completely what is the idea here. So, you give some radioactivity right.

We talked about giving a unstable. So now, you know so you so you are going to give some radioactive tracer into the body and the radioactivity is happening; that means, disintegrations per second is happening; that means, while it is disintegrating it is giving excess energy out.

So, what is the intensity? So, this energy comes out and you want to detect it, so what is the intensity of radiation? Incident on the detector. So, let us say, if the detector is kept at some r distance from your radioactive source.

Same concept we use right, you have a source, you have a detector, how much will be right, what will happen is there any law that comes into your mind. Exposure right when we talked about, what did we say? stay away every distance you have safety of one by d square inverse square law.

So, here also the same thing holds good, but what is your intensity? intensity has to do with number in our previous X ray when we covered what did we say? Number of photons into number of energy per energy per photon. Here, activity gives rays right, so number of activity times energy per activity so number radioactivity right, number of disintegrations. So, each disintegration it is giving some $h\nu$ out that is your energy.

So, this is your inverse square law $4\pi r^2$ remember. So, this is the intensity that will fall on the detector when you have r distance due to radioactivity ok. Of course, A is your radioactivity of the material. So now, you see the problem. The problem is, I can give ok I have some radio chemist background, I am putting some radioactivity, I have, you saw all the laws how to make it radioactive, how to make it unstable so that it becomes it will give radioactivity.

So, you are a radio chemist, you know how to work with it, your engineer a compound then engineer an atom. You give it, then it gives out energy. So now the task is, how do I detect this and say where it came from ok. These are the important aspects. So, you know how to measure radioactivity that is the goal right. So, I know the intensity.



So, if I know my intensity, if I know my distance, if I know my energy, if I know I mean what, you see this is very important. What is it that we need to know from here, what is the known, what is the unknown. So, this you are going to measure radioactivity and energy per.

So we need to still go into details to see how what decay happens, what is the energy that is coming out this we need to understand. Of course, this r comes into business when we do the geometry right. When how where do we place the detector instrumentation, and then perhaps from there we can talk about where the source is located ok. So we will get to that clear.

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Radioactive Decay Law

- $N(t)$: the number of radioactive atoms at a given time
- $A(t)$: is proportional to $N(t)$
$$A = -\frac{dN}{dt} = \lambda N$$
$$\lambda: \text{decay constant}$$
- From above, we can derive
$$N(t) = N_0 e^{-\lambda t}$$
$$A(t) = \lambda N_0 e^{-\lambda t}$$
- The number of photons generated (=number of disintegrations) during time T is
$$\Delta N = \int_0^T A(t) dt = \int_0^T \lambda N_0 e^{-\lambda t} dt = N_0 (1 - e^{-\lambda T})$$



So, we need to measure this radioactivity. So, let us talk about when you measure radioactivity right when we did this for X rays, what did we do? We sent an X ray, we detected the X ray. What happened to the X ray? It got attenuated along the material and so we used that is our information.

How much of attenuation happened through the material or the material property was characterized in terms of its ability to attenuate the X ray energy. So the μ was the material property that we were going after. Here what is the problem? I am not actually going after the material property of the tissue. All I am interested is I have engineered this radioactive material I have put it in.

I want to know when you put it into the body where is this radioactivity coming from? So, when I put it into the body it will go through the body it will distribute itself and it will

perhaps go to locations where it is going to be involved and the radioactivity is going to come from there.

So, my objective here is to detect, the radioactivity is coming from where. How much of radioactivity is coming from which location. That is my I am not really interested in the attenuation property of my tissue clear.

So, here what you want? So, the there why I said that is you had your μ your attenuation, we came up with the fundamental attenuation law which was exploited. Now, here attenuation is not the big deal. Here, there is a fundamental, what is that we are interested, we are interested in the radioactive decay. That is what we are interested. So, there is a fundamental radioactive decay law that we need to you know put in put here, so that that can be exploited for imaging.

So, what does that state? That says, ok you have you start with says the num n number of radioactive atoms at any given time right. What is radioactivity? Disintegrations per time. So, it is your A of t is your radioactivity is proportional to number of radioactive atoms that are there at that time ok.

So, you can write A , your radioactivity I mean, just pretend that N is continuous ok. So, you have rate of change, so your radioactivity is number of disintegrations right. So, in time how much atom number of atoms change. So, A is equal to minus dN by dt right is equal to your λN . So, the λ is your decay constant ok.

So, if this is the case then we can quickly look at it and rewrite right. What do we want? We want, what are we looking at. We are looking at when we did X rays right, X ray energy. We were looking at how much we sent, how much came out after it crossed the material. Here, we have a radioactivity and we have some time, so after some time how much of radioactivity is left, is happening ok.

So, because you send the radioactivity in, you give it some time it goes redistributes itself, after that radioactive decay is happening and you are detecting it. So, you are interested in the decay, how you know the decay rate how do you capture it over time ok. So, from here we

can talk about N of t is N_0 . So, this is from your boundary condition whatever at t equal to 0, when you started whatever was there right number of activity is $e^{\lambda t}$ power minus.

So, here also you talk about say exponential decay that is what is happening. So, the radioactivity is happening so that you have a exponential decay model. So, your A of t is activity you start with at t equal to 0 right and then it decays with a constant λ , and therefore you can have A of t equal to whatever you start $\lambda N_0 e^{-\lambda t}$ clear.

So, this means you start with some activity, it starts to decay this is exponential so it is never going to go to 0. There is some radioactivity that is going to be always there right, it is never going to go to 0, but it will become insignificant after some time right and that depends on your λ . What is the time depends on your λ . That is the inherent property of that atom ok.

So, the number of photons generated during this time. What is the number of photons that are generated? It is decaying and it is sending out right. So number of photons generated is going to be number of disintegrations that has happened right.

So, how much is it going to be? Observe sometime, radioactivity is number of disintegrations per second, so I want to see number of disintegrations over T . Just integrate the activity over time right. So, you integrate the activity you integrate the activity over the given time which you call as the number of photons that are generated clear.

So, you have some photons N_0 at T equal to 0 radioactivity, this is radioactive atoms. $1 - e^{-\lambda T}$ So, $N_0 - N_0 e^{-\lambda T}$ is your leftover. So, the number of photons that are generated which is equal to your number of disintegrations clear. So, fine, so this is actually a good place to just quickly think about one concept.

So, when you talk about starting point and right after some time this much radioactivity has happened. When we talked about the analogous concept, when we did X ray imaging, when

we talked about number of photons that go in fundamental attenuation law; $e^{-\mu x}$ and what comes out.

We talked about an important metric as ok. How much is the material, how much the material. So we talked about μ as a material property. Here, we are talking about radioactivity as the property and we have this fundamental law.



So, natural quantity of interest is, when is it 50 percent. There we talked about half value layer thickness when the attenuation was 50 percent. Here, likewise you start with some N_0 some radioactivity, when does it become half right, that is a quantity of interest.

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Half-Life

- Half-life is the time it takes for the radioactivity to decrease by $1/2$.
- Half-life, $t_{1/2}$, is when $\frac{A_{t_{1/2}}}{A_0} = \frac{1}{2} = e^{-\lambda t_{1/2}}$

$$t_{1/2} = \frac{0.693}{\lambda}$$



So, half life is the time it takes for the radioactivity to decrease by half ok. So, same concept how would you do? We start with the previous equation right. We need $t_{1/2}$; $t_{1/2}$ is nothing but you start with some A_0 . When does it become half A_0 ?

So ratio is equal to $\frac{1}{2} = e^{-\lambda t_{1/2}}$. So essentially, it depends on the material property right. There we talked about half value layer thickness when we talked about we talked about inherent material property was the μ , here radioactivity ok.

So, this again has similar workings right. So, you have a half, you have a e , so we could do this learn and we can get the value. So, $t_{1/2}$ is $\frac{0.693}{\lambda}$. As a formulation as a formulation this looks exactly same, I should say similar, just because here instead of λ we had μ ok. Instead of λ we had a μ . Whereas, here it is λ , otherwise the quantity of interest is half life there it was half value layer.

So, this half value both are exponential models ok. So, what does this signify? So, this signifies again, this λ is the constant radioactive decay constant of that material radioactive atom. So, the atoms change, their radioactive decay, some can decay very fast, some can decay slow. So, now, you see the issue, so you can send some material which is radioactive.

That material, what is that material property of interest to us is the depending on it is decay rate right it is going to give so many it is going to decay at certain rate which is going to give you number of photons that are coming out at that rate at certain energy and you have to capture that.

And say, so you cannot want the so we will talk about the importance of this parameter right when we when we get slightly ahead. So, the ideal characteristics are what do you want of this parameter. See the idea is, μ we could not really change much because it was inherent distribution.

Of course, there also we did change, we sent a contrast agent that was finely tuned. Here, you are actually engineering it and sending it, so you have little more leverage ok. So, as long as we understand what this is, we can also ask discuss about the desirable properties of a atom. Say, what should be a good value for λ and that maybe we can send in ok.

So, we will stop right here. What we will do next is, we will understand we said decay, but we did not really go into the details. How is it, what are the modes of decay? What are the different types of decay that are possible? What are the energy levels that are coming right?

These are we just said, decay energy level comes out, but we need to be little more specific on that. So we need to talk about modes of decay, what are the energy that comes out, and then similar to previous X ray photon side you have to talk about energies that are coming out, energy packets that are coming out.

So, when this decay happens, what energy comes out? Is there any probability randomness in that, ok. And, and after that you really think about the photon energy the interaction with the body, is not much we have already covered that when we talked about X ray photon. It is just that it is going to be of higher energy as you will see. So, that will be for the physics. So, we will continue on the modes of decay right after this.

Thank you.