Introduction to Biomicrofluidics
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Lecture – 05
Pressure Driven Flows

We will start our discussion on Theoretical Biomicrofluidics with Pressure Driven Llows. So, what do we mean by pressure driven flow? We mean that there is a fluid in a micro channel and there is a pressure gradient or difference in pressure that is created across the fluid, so that by virtue of the pressure gradient the fluid starts flowing. This method has it is own advantages and disadvantages. But one has to understand that this is one of the very classical methods of driving the flow. Even in your household application you see that there is a pump that is driving the fluid from the lower level to the higher level.

Similarly, you will see that in micro fluidics there are different other types of pumps not really the type of pump that you use in the household application, but a pump where you may have a piston driving the flow by getting displaced inside a cylinder. So, that type of pump looks like a big injection syringe and that is often called as a syringe pump. Syringe pump is very important for medical applications and it is used for many applications including metering of flow and drug delivery and so many other issues.

So, it is therefore quite important that we learn that what are the fundamental aspects of a pressure gradient driving of fluid flow? To understand that one classical way is you know to start with the basic equations in fluid dynamics which for certain special types of flows like Newtonian fluid and so on are governed by well known may be a Stokes equations.

But here what we will do is that we will not go through the root of the Navier Stokes equation and this I am doing deliberately because I can understand that this being an interdisciplinary course there are many participants in this course who have who may not have done a very rich basic course in fluid mechanics.

So, if you have not done that then it will be difficult to follow if we start as such with the Navier Stokes equation. So what I will do is that I will start with a basic force balance in the fluid to explain how the fluid flow takes place using pressure gradient.

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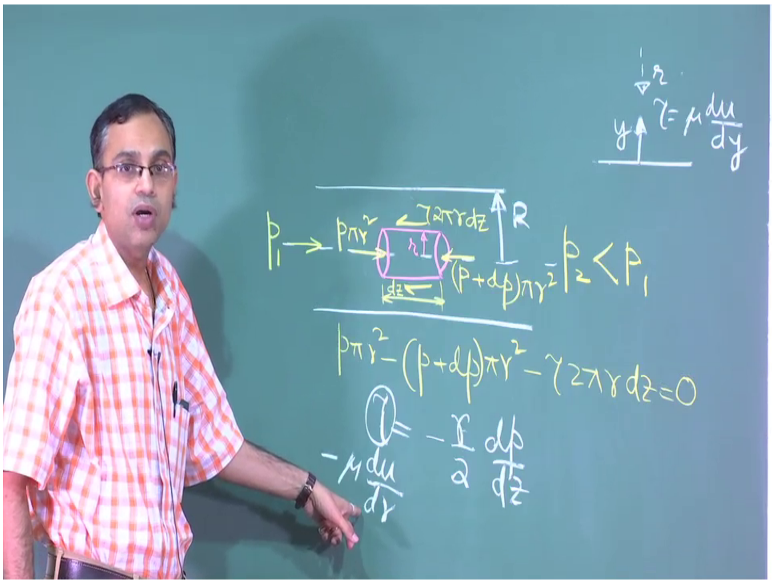

So, to understand that let us consider a circular channel which in the large scale would be a pipe in a small scale it is a circular micro channel or a micro tube. So, how you are driving the flow is you are having a pressure here p 1 here you are having a pressure p 2 which is less than p 1 and by virtue of this pressure difference the fluid is flowing. Of course, you have to understand that sometimes this pressure includes the fluid pressure plus the effective pressure due to the height, which is the gravity effect like effective pressure due to height is h rho g so that might add to this.

But in case of a horizontal channel that effect is nullified so what drives the flow is essentially not just p, but p plus rho g h where h is the height g is a acceleration due to gravity and rho is the density. Under these situations let us take a small fluid element with radius r ok, radius of the pipe or the channel is capital R. So, now let us identify various forces, so the first force that we can identify is the shear force. Why is this shear force coming? You have various layers of fluid and one layer of fluid is trying to slide over the other at the wall. Because of the fact that wall molecules are immobile they are trying to hold the fluid molecules therefore the velocity is 0.

But as you go away from the wall the velocity becomes more because the effect of the wall is not directly filled, but it propagates through the fluid by virtue of the fluid property called as viscosity. So, viscosity make sure that although this is not directly in contact with the wall there is a shear here which is transmitted from the wall to this and that transmission of shear across various layers is by virtue of the fluid property viscosity. So, now if you call this as tau which is the shear stress then and let us say that this is dz then this force is tau into the surface area of this imaginary cylinder.

So, 2 pi r into dz then there will be a force due to pressure gradient, here the force is p into pi r square here it is p plus dp into pi r square. Now the question is are all those forces balanced there is no necessity that these forces will be balanced, but under the condition that these forces are balanced and these forces are balanced in most circumstances in micro channel flows, I do not have enough time to get into the reason why all these forces will be balanced.

But when all these forces will be balanced the situation resembles to something which is called as a fully developed flow. So, to give you a qualitative understanding of what is a fully developed flow I will try to go through this slide.

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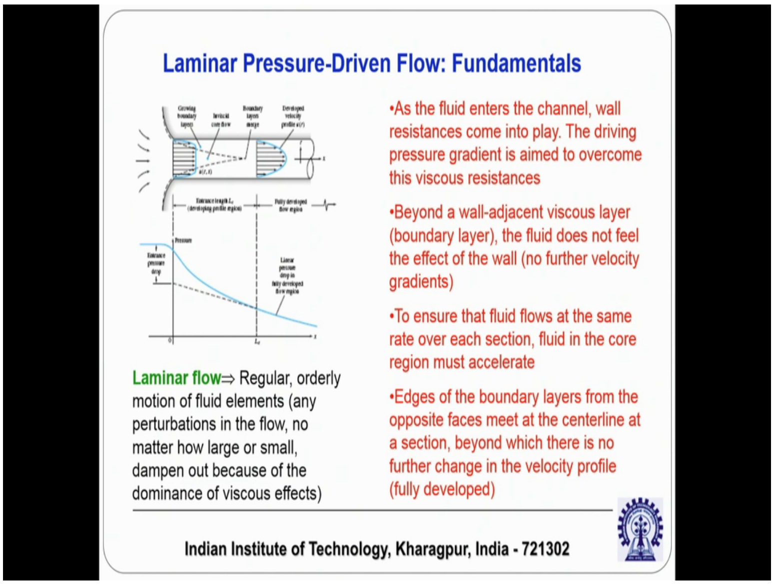

So, you see here that the fluid enters the pipe with a velocity profile such that there is a layer close to the solid boundary where the viscous effects are important, but as you go away from the solid boundary the viscous effects become less and less. So, this layer within which the viscous effects are important and outside which the viscous effects are not important this layer is called as the boundary layer.

So, the dotted line here is basically the edge of the boundary layer, so when the boundary layers from the all the walls they merge then after that you will have a velocity profile which is called as fully developed velocity profile. So, this fully developed velocity profile is the consideration that we are assuming here, when the flow is fully developed the velocity profile does not change further as you move along this direction.

So, if you see here in the slide if you if you see in the slide I request them to show the slide please low please show the slide yes. So, if you see this velocity profile this velocity profile will not change further with the axial position and that means that all these forces are balanced. Because if the velocity profile changes with the axial direction there will be an acceleration and acceleration means there is an unbalanced force. So, here there is no unbalanced force that means you can say that p into pi r square minus p plus dp into pi r square minus tau into 2 pi r dz that is equal to 0.

So, if you simplify this you will get tau is equal to minus r by 2 dp dz ok. So, then by Newtons law of viscosity if this fluid is Newtonian, now this is where you can make sure that you apply this equation to different types of fluids. So, if it is a Newtonian fluid that is the Newtonian fluid is a fluid which will obey the Newtons law of viscosity, then this tau will be minus mu du dr why minus mu du dr see if you have a solid boundary, then for a flow unidirectional flow tau will be mu du dy this is Newtons law of viscosity that all of you have learnt through some of your basic courses in physics.

Now, here so this is the wall the r direction is towards the wall, so mu du dy will be minus mu du dr because r and y are in opposite direction. So, for a Newtonian fluid which will be minus mu du dr, if it is not a Newtonian fluid let us say it is a fluid layer fluid like blood.

So, then what we will do instead of this whatever is the constitutive behavior of that fluid that we need to substitute instead of this expression, so this is where we will model the bio fluid in a particular way. So, here the fluid is modeled as a Newtonian fluid as an example, but if it is modeled as a bio fluid modeled as a non Newtonian fluid then the corresponding expression has to come out here. So, now so this u is nothing but the velocity in the z direction.

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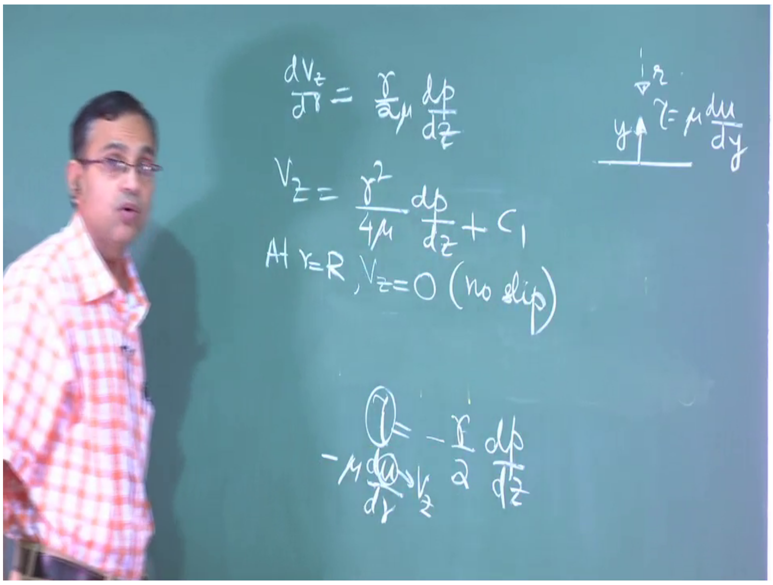

So, we will call it is as a V z z is the axial direction of the tube. So, dVz dr r by 2 mu dp dz. So, if you integrate this once more Vz is equal to r square by 4 mu dp dz plus a constant of a integration C 1. So, how do you get C1? You get C1 by noting that at small r is equal to capital R, which is the solid boundary you have Vz equal to 0, this is called no slip boundary condition. We will see briefly, what is slip in case you have slip instead of no slip.

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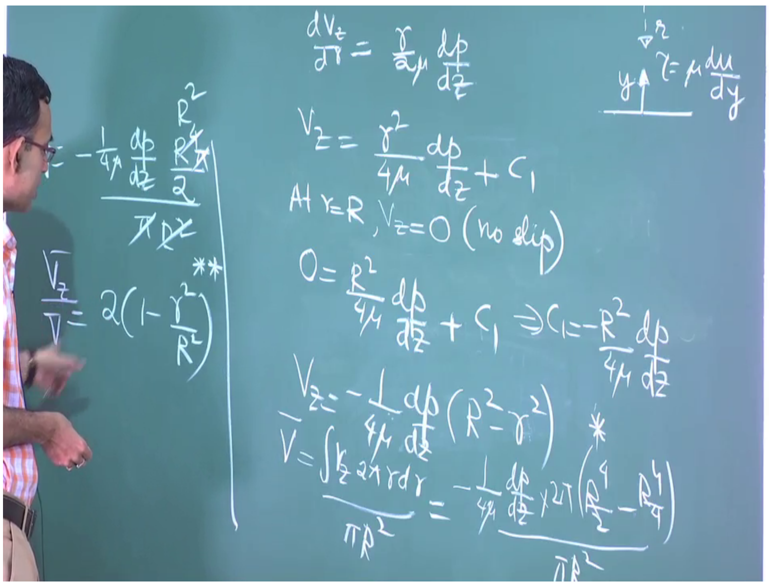

So, you will have zero is equal to r square by 4 mu dp dz plus C1. So, C1 is sometimes it is we will keep this equation in mind because, we will be using this for certain purpose a But you can also express it in terms of the average velocity why average velocity is important because, in experiments you can measure flow rate the total rate of flow at a given section in the pipe and if you divide the flow rate by the cross sectional area you will get the average velocity.

So, average velocity is an experimentally obtainable parameter and that is why instead of this you can express the same in terms of the average velocity average velocity is the flow rate divided by the area of cross section ok. So, this becomes so this r square into rdr is. So, r cube so this becomes rdr that is r square by 2, so this becomes r 4 by 2 minus this is r cube dr that is r 4 by 4.

So, let us work it out here so V average, so this becomes r 4 by 2 minus r 4 by 4 this is r 4 by 4. So, r 4 by 4 into 2 pi so that becomes minus 1 by 4 mu dp dz into so 5 get is cancel this becomes r square. So, by considering equation star and equation double star you can write this flow is classically known as Hagen Poiseuille flow and although it appears to be a very fundamental aspect of fluid mechanics, this was actually derived and developed by a person called as Poiseuille who by profession was a physician.

So, he was a medical doctor and his interest was actually to see how blood flows through the capillaries in human bodies and this is one of the very simplified old models that tried to mimic that physical picture. So, this is the velocity profile and it shows that the velocity profile is parabolic, which is the velocity profile that I showed in the slide. So, the velocity profile is something like this a parabolic velocity profile.

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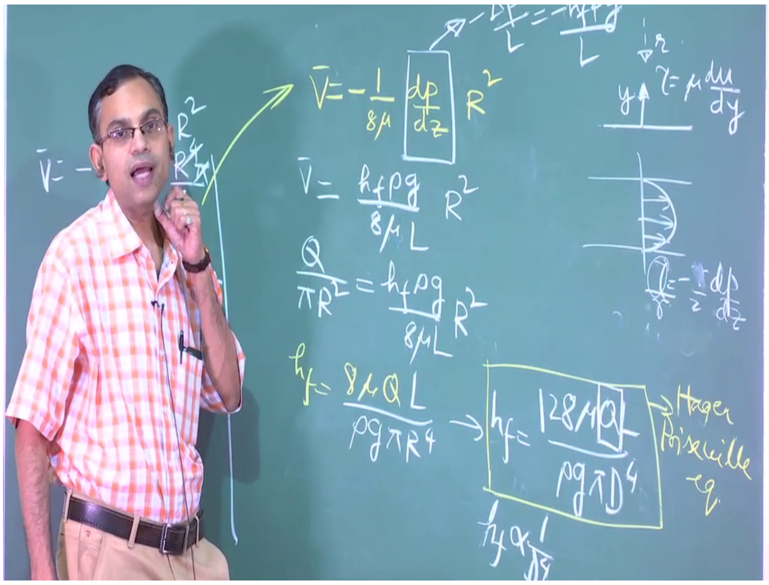

Now, the big question is see as a bio microfluidics engineer, you may be interested in the velocity profile of course are the fundamental information, but you may also be interested you know how much power will I require to drive the flow. Because that is where your pumping cost effort and all those things will be related. So, the primary things are how much pumping power I will require to achieve a given flow rate that is the you know output of my device. So, that in hydraulics is often measured by a quantity. So, we will look into this equation and build it up from here.

So, V average is minus 1 by 4 mu minus 1 by 8 mu actually 1 by 4 and a half is there dp dz into r square. Now this dp dz you can write because pressure versus z is linear; why pressure versus z is linear you can see from this equation I mean that equation is erased from this board, but if you look the look at the first equation tau is equal to minus r by 2 dp dz I am writing this again. So, tau is a function of r only.

So, tau by r is a function of r only. So, in this equation the left hand side if you bring r here. So, you may bring r here tau by r is a function of r only dp dz is a function of z only a function of r is equal to a function of z only if it is a constant so; that means, dp dz is a constant. So, p versus z is linear. So, you can write dp dz as minus delta p over L; where L is the length of the tube. So, then this delta p you can write the pressure drop, you can write in terms of a length unit which we call as head loss hf.

So, why head loss? Head loss is essentially an energy loss to overcome viscous effects in the flow. So, physically why do you require a pumping power? You require a pumping power because there is viscosity in the fluid and viscosity in the fluid resists the relative motion between the fluid layers. So, you have to maintain, to maintain the fluid motion you have to give certain energy to overcome the viscous resistance and that energy per unit weight is called as head h, f for f subscript for friction; so, minus hf rho g by L.

So, you will get V is equal to these 2 minus signs get cancelled out. So, hf rho g by 8 mu R square, so V average you can write as a Q by pi R square where Q is the flow rate in the unit of Q is meter cube per second so Q by pi R square is the average velocity, so this is hf rho g by 8 mu r square. So, hf is 8 mu Q there is there was a L right there was a L 8 mu QL by rho g pi R to the power 4.

Sometimes engineers prefer to use diameter instead of radius so R is d by 2. So, it becomes hf is equal to 128 mu Q L by rho g phi d 2 the power 4. This is classically known as the Hagen Poiseuille equation, the corresponding flow is p let me write it properly the corresponding flow is Hagen Poiseuille flow and this equation is Hagen Poiseuille equation. So, what do we get from here, so if we want to maintain a particular flow rate in a pipe then the head loss why head loss is important because, to overcome this loss you have to give pumping power as a input, so more the loss more will be the pumping power.

So, the head loss is proportional to fourth power of the hydraulic diameter or the diameter of the pipe in this case. Hydraulic diameter is an effective diameter, but for a pipe it is the diameter itself. So, what does it indicate it indicates that it indicates a very important scaling law for microfluidics applications, that if you reduce your pipe diameter from 1 meter to one micron ok, so 1 meter to one micron is 10 to the power 6 times the reduction.

So, 10 to the power 6 to the power 4 that is 10 to the power 24 times you will have the increase in the head loss. So, to maintain the same flow tremendously large pressure you have to pumping power you have to give. So, it is not normally advised that you use microfluidic devices for achieving large flow rates.

So, typically you use microfluidic devices only when small flow rate, but in precise volume is needed to be delivered and that is the hallmark of microfluidic applications as compared to other applications. I will talk little bit about another aspect which is called as the slip at the wall, in microfluidics instead of the no slip boundary condition sometimes there happens a slip at the wall. So, I will explain it through a small slide.

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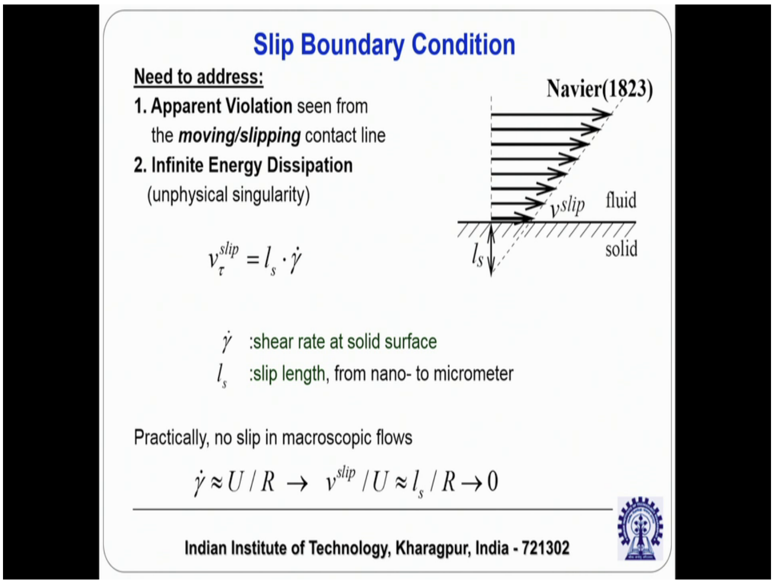

So, the explanation is something like this so if you look into the velocity profile at the wall, there are some cases when the velocity profile is does not show 0 velocity at the wall. So, to quantify the velocity at the wall what is done is the tangent to the velocity profile which is the dotted line here is extrapolated and it shows 0 not at the wall, but at the, but at a distance of ls from the wall, this ls is called as slip length. So, let me draw it with a figure to understand to make you understand better.

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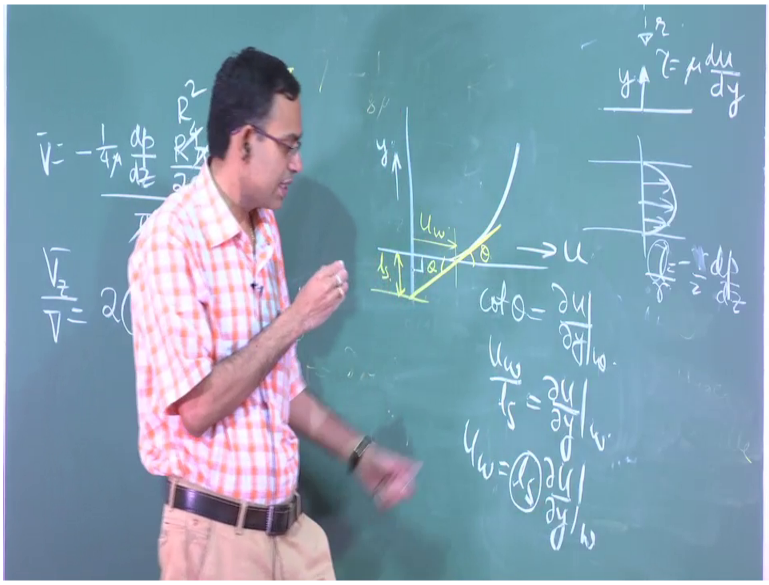

So, you have a velocity profile like this, velocity profile means here you are plotting velocity here you are plotting the distance from the wall. So, you draw a tangent this tangent meets here at ls, this is the velocity of fluid at the wall and let us say this angle is theta. So, if this angle is theta you will get cot theta because tan theta is dy du so cot theta is this one.

So, cot theta from this right hand right angle triangle is u wall by ls. So, you can write u wall is equal to ls del u del y at the wall. So, this ls is called as Navier slip length because, in Navier in 1823 he was a very famous mathematician in the domain of fluid mechanics and mathematics applied mathematics and he first postulated this boundary condition.

So, if ls is 0 then velocity of the fluid at the wall is 0 that is the no special no slip boundary condition. So, now the question is why will the fluid slip at the wall we are arguing that there could be a slip it is quantified by this slip length and this can be obtained either through experiments or through molecular simulations, but the question is why it will slip.

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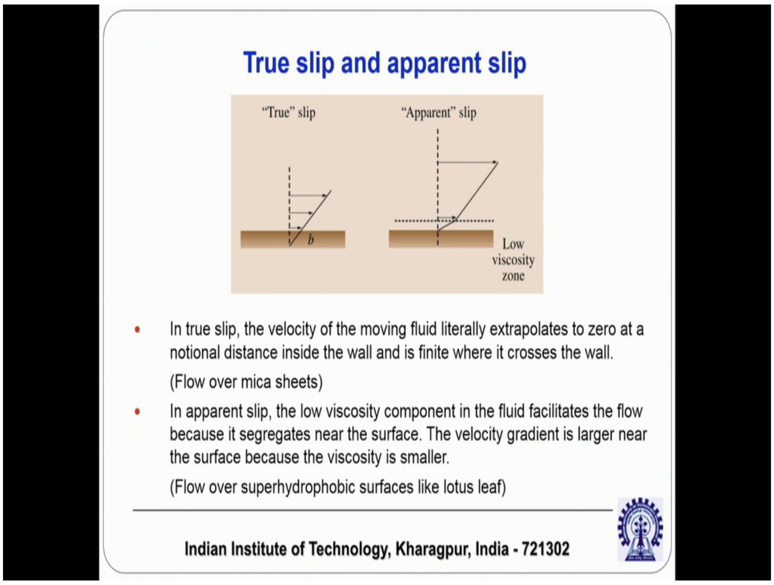

So, why it will slip is described in this very simple model, where you will see that at the wall there can be a small layer a thin layer of nano bubbles or a thin layer of rarefied gas. So, when the liquid is flowing on the top of this wall the liquid is not feeling the effect of roughness of the wall directly, the nano bubble layer which is shown up to this dotted line this will act like a cushion and the liquid will slide or glide over this cushion that will make it as if it is slipping over the solid boundary.

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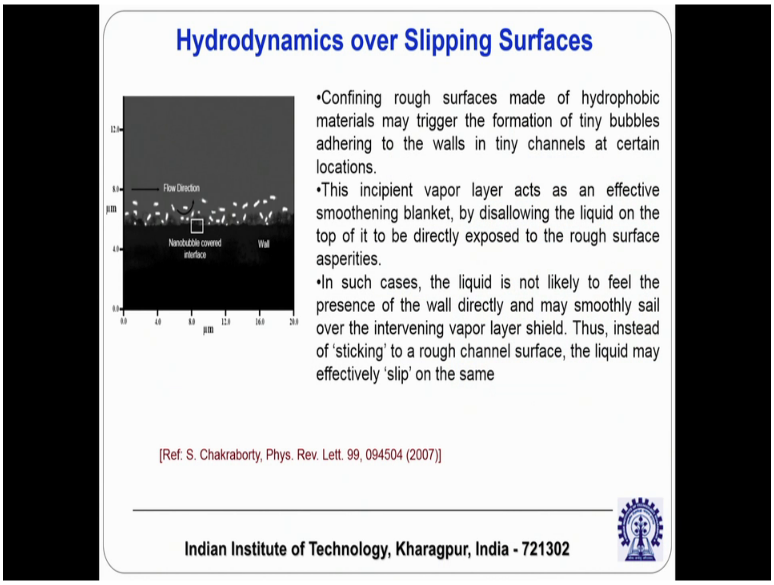

So, that is one of the very important considerations and there has been lot of research work actually in this domain to understand hydrodynamics over slipping surfaces. So, it is important because if you try to understand how you can control the slipping of fluid over a very rough surface, rough and hydrophobic surface can promote slip in a confinement research has shown this.

Then it may be possible that overcoming the huge resistance that you can otherwise get in a pressure driven flow, you can pump fluid at an unprecedented rate and this kind of large level of pumping by utilizing slip has been a hallmark of the fluid flow in the nano channels, where in carbon nano tubes or fluid flow over graphene type of materials have shown that you can have a high level of slip and if you can have a high level of slip.

So, in the nano scale it is because of the behavior of the material fluid interaction in the nano scale, in the micro scale it could be that you can use roughness as a blessing in disguise. The roughness of the wall might trigger the formation of small gas pockets and liquid can flow on the top of that without creating much of a frictional hindrance.

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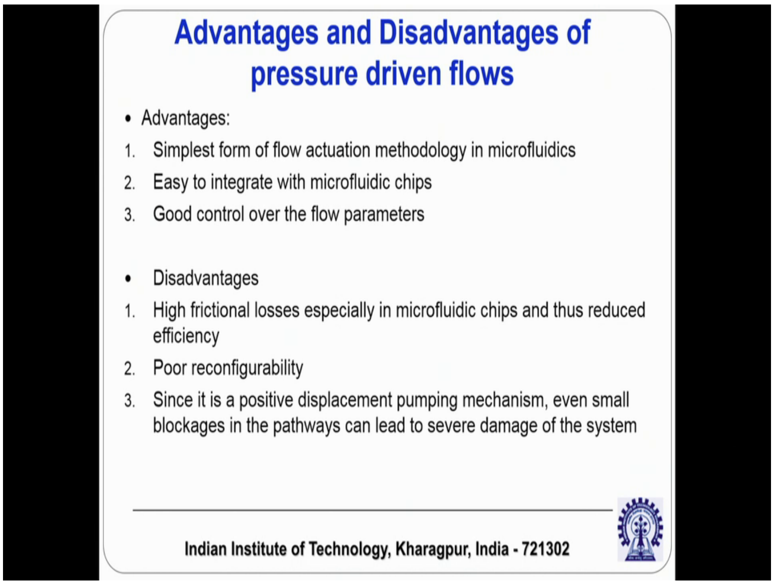

So, we will summarize the discussion or conclude the discussion in this lecture by pointing out advantages and disadvantages of pressure driven flow. So, for that I will refer to the slide, the advantage of pressure driven flow is that it is the simplest flow actuation right you we just need a need a syringe pump to drive the flow.

It is very easy to integrate with microfluidic chips anybody who has started working in a microfluidics lab we will be knowing how to operate a syringe pump and it can have a good control over the flow parameters. But this has disadvantages also for example, the primary disadvantage is high frictional loss especially in a microfluidic conditions were you know the head loss being inversely proportional to the fourth power of the diameter, if the diameter becomes very small the head loss becomes very large.

It is not easily reconfigurable since you essentially depend on a displacement of a plunger. So, it is a positive displacement type of device even small blockages in the pathways can lead to severe damage in the system. So, mechanical friction wear tear damage these are some of the bottlenecks of implementing this. Despite that being one of the very traditional methods the pressure driven flow is commonly considered to be one of the very prominent methods of flow actuation in a micro scale.

But in micro scale the advantage also remains, what is the advantage that instead of creating a positive displacement pressure gradient you can use surface tension to generate the equivalent pressure gradient, because surface tension is a beautiful force in a small scales.

So, in our next lecture we will see what is surface tension and how surface tension can be used to manipulate a flow, which is fundamentally pressure driven but the pressure gradient is created by surface tension.

Thank you very much.