

**Aspects of Biochemical Engineering**  
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**Lecture – 47**  
**Transport Phenomenon in Bioprocess I**

Welcome back to my course Aspects of Biochemical Engineering. Now up till now I discuss mostly on this, we I we started with this reaction kinetics, then we started and we discussed the reactor and this is, then we surely discuss the enzymatic reaction kinetics, then we also discuss the mobilized enzyme system, then microbial system cell growth subset degradation and product formation and to a last two lectures I concentrate on scale up of bioreactors.

Now, this present lecture is this is a very important aspects as per bioprocess is concerned, this is the transport phenomena in bio processor. Now transport phenomena the term itself tell kind of transportation many kind of transfer that you know how it takes it place.

Now, there are different type of transfer that take place in the bioprocesses. So, that kind be decided in the three different types, one is called momentum transfer, another is called heat transfer, another we call the mass transfer.

The though we now there are different type of transfer processes we have and we try to discuss how these process play a the important role in these in these biochemical process. As for example, that mass transfer process a particularly I can tell you plays important role as per the anaerobic fermentation process is concerned, because I told you that major limiting factors for the anaerobic fermentation process is the dissolve box un concentration.

So, so that since oxygen is sparingly soluble in water, so you shall have to increase the solubility that you know that dissolve oxygen concentration that directly related with the mass transfer, that you know how you can you can do that.

Another important aspect is the heat transferred, because as we now that that any kind of fermentation process, that this is a microorganism there are very sensitive to the environment. And since they are very sensitive to the environment that, what you what

you have that that a any kind of temperature if we shall have to maintain a particular temperature.

So, that your organism can give the maximum performance in the in the reactor, because if you are if organism walk well, then in only then you can expect their desired amount of product formation coordination to the extend you are expected.

So, to maintain the temperature, which will be the heat transfer plays very important role and as appear as you know that in the biochemical processes that most of the processes they are they are exothermic in nature. Now, exothermic means the heat liberating am I right.

So, heat liberating; that means, there is every possibility that temperature rises, but, but you know that in case of summer, what is what is happening; since we are in true tropical country our ambient temperature increases as high as 40-45 degree centigrade and may be our fermented temperature required may be 33 or 30 35 degree centigrade. So, it is quite less.

So, you know the; so question come that, the cooling effect that requirement will be more. So, we shall have to pass some kind of chilled water, there to control the or you know we shall have to pass more water and; so that temperature we can maintain.

Now, in case of winter ambient temperature may be may fall as low as 15 degree centigrade then also you have to increase the temperature to some extent because, there will be always some kind of heat loss that takes place in the in the fermented.

So, heat transfer plays very important role and momentum transfer also plays very important role, because the because you know that we have to when you when you when you study any kind of any kind of fermentation process you have to keep the cells in suspension and for the suspension of the cell we have to use kind of starters. So, there you know the sales remain suspended and freely interact with the sub state and give the product.

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**Introduction**

- ✓ Subject which **deals with the movement of different physical quantities** in any chemical or biochemical process
- ✓ Describes the **basic principles and laws of transport**
- ✓ Also describes the **relations and similarities among different types of transport that may occur in any system**

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Now, let me start with this that this the subject deals with the movement of different physical quantities in any chemical and biochemical processes. So, that is the movement, I told you three type of movement is there as a momentum heat transfer and also you have mass transfer. Then describe the basic principle and law of transport and also describe the relation and similarity among the different type of transport that may occur in any system.

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**Classification of transport phenomena**

- ✓ **Momentum transport** deals with the transport of momentum in fluids and is also known as **fluid dynamics**. Examples- **blood circulation in body, mixing phenomena in bioreactor**
- ✓ **Energy transport** deals with the transport of different forms of energy in a system and is also known as **heat transfer**. Examples- **sterilization of reactor, temperature control in bioreactor**
- ✓ **Mass transport** deals with the transport of various chemical species themselves and is also known as **mass transfer**. Examples- **oxygen transport from bubbles to aerobic microorganism**

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Let us say let us see the classification of the transport phenomena. This is the classification of that; we have three different classification: momentum transport, energy transport and the mass transport. What is momentum transport?.

It deals with the transport of momentum of fluid and is also known as fluid dynamics. For example, blood circulation in the body, mixing phenomena in the bioreactor, both that comes under the momentum transport, because this is a, this is very easy to understand the mixing phenomena you just explained.

The energy transport deals with the transport of different form of energy in a system and with also know that heat transferred. As per examples sterilization, because I forget to mention the sterility of the reactor is very important, because we want to grow our desired organism in a particular environment, so that we can get the desired product. And also temperature controlling the bioreactor just I pointed out that organisms are very specific with respect to temperature that we particular temperature they give the maximum product.

Mass transferred deals with the transport of various chemical species themselves and also known as mass transfer as for example, oxygen transport from the bubble to the aerobic microorganism. So, you know that the; from these we can easily understand, how crucial is the transport phenomena as per biochemical processes are concerned.

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The slide is titled "Physical quantities used in transport phenomena" and lists three categories of physical quantities:

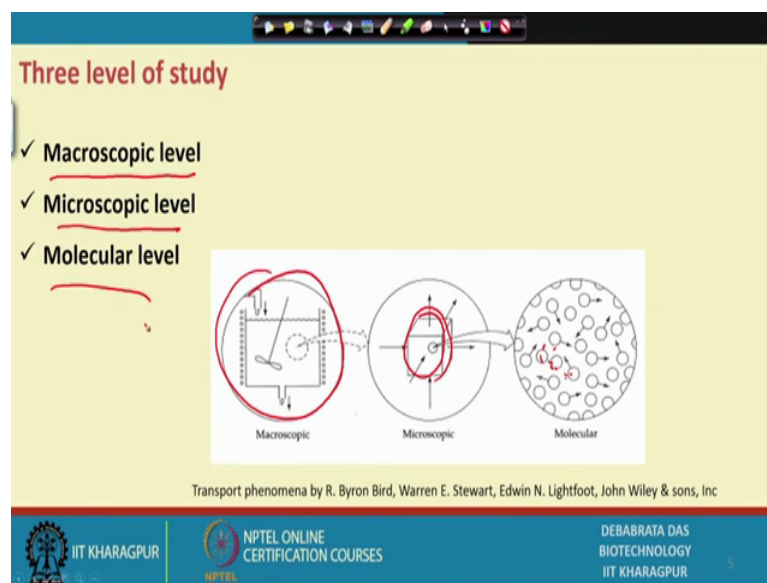
- ✓ **Scalars** (temperature, pressure and concentration)
- ✓ **Vectors** (velocity, momentum and force)
- ✓ **Second order tensor** (stress or momentum flux and velocity gradient)

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Now, when you when you try to solve it we come across different physical quantity am I right; and what is the one scalar; scalar means it has some values, you know I mean as per example temperature, pressure and concentration. But vectors they have some direction velocity and momentum and force. And second order tensor is the stress or momentum flux and velocity gradients.

So, these are the different quantities will be using during describing this transport phenomena.

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Now, if you look at that three level study: one is microscopic level, another microscopic level, and the molecular level. Now, macroscopic level that the inside the fermenter, what is happening; if you study something like this and we; now here inside that the you might be using some kind of a cell or in a particular portion of their liquid, you want to study in details.

Then we have we have these microscopic analysis. And then molecular level means, how the molecules inside this cell; they are they are they are interacting with each other and give the products. So, you know that three different level of studies are there.

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**Three axioms**

- ✓ Mass is conserved, which leads to the equation of continuity.
- ✓ Momentum is conserved, which leads to the equation of motion (Newton's 2<sup>nd</sup> law)
- ✓ Energy is conserved, which leads to equation of thermal energy.

These equations are collectively called the governing equations.

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Now three axioms are there mass is conserved, which will lead to equation of continuity. Momentum is conserved, which leads to equation of motion and energy is conserved, which lead to equation of thermal energy.

So, these three equations collectively called governing equation. So, these are all these based on the basis of mass conservation on the basis of momentum conservation, on the basis of energy conservation; that we can try to solve this.

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**General statement for any transport process**

$$\text{Rate of transport process} = \frac{\text{driving force}}{\text{resistance}}$$

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Now, general statement of transport processes. Now how you can do that? Rate of transport process is the driving force blindly defined by resistance. So, you know driving force means, suppose you know you want to heat some liquid; now what is the driving force? Driving force is the temperature difference and resistance is the media that you know they hinders that you know that that um that flow of the, that heat. So, you know that.

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**Driving force**

- ✓ Factor that will make the transfer occur.
- ✓ Driving force for momentum transport → velocity gradient  
(microscopic/molecular)/velocity difference (macroscopic)
- ✓ Driving force for energy transport → temperature gradient  
(microscopic/molecular)/ temperature difference (macroscopic)
- ✓ Driving force for mass transport → concentration gradient  
(microscopic/molecular)/ concentration difference (macroscopic)

**Resistance**

- ✓ Factor that will slow down the transport process.

Handwritten notes on the right side of the slide:

- $m(c_2 - c_1)$
- $m s (t_2 - t_1)$
- $k L a (c^* - c)$

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Now, that you know this driving force I can, we can explain like this, factors make again that the driving force is the momentum, transport is the velocity gradient, and microscopic it will be microscopic molecular and velocity difference. This is the momentum as I understand that mass into velocity am I right; also it is the mass is the velocity gradient that plays very important role.

Energy transport is the temperature gradient; since we know  $m s t$ ;  $m s t$  is what that you know mass is specific heat and temperature. The temperature gradient plays very important role and driving force is a mass transport is the concentration gradient like we can we can  $k L a$  into  $c^* - c$  that this is the gradient. And here also here, what is the gradient;  $t_2 - t_1$  this is the gradient this here  $v_2 - v_1$  that is this is the gradient at forces we have.

The resistance factor which is slow done the transport the purpose of resistance means they slow down the kind of transport processes.

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**Mechanisms of transportation**

Momentum, heat and mass transfer phenomena usually take place by two means:

1. **Molecular transfer**
2. **Convective transfer**

In case of heat transfer there is another mode of transport. The mechanism is called **radiation**. It is not much important for momentum and mass transfer phenomena.

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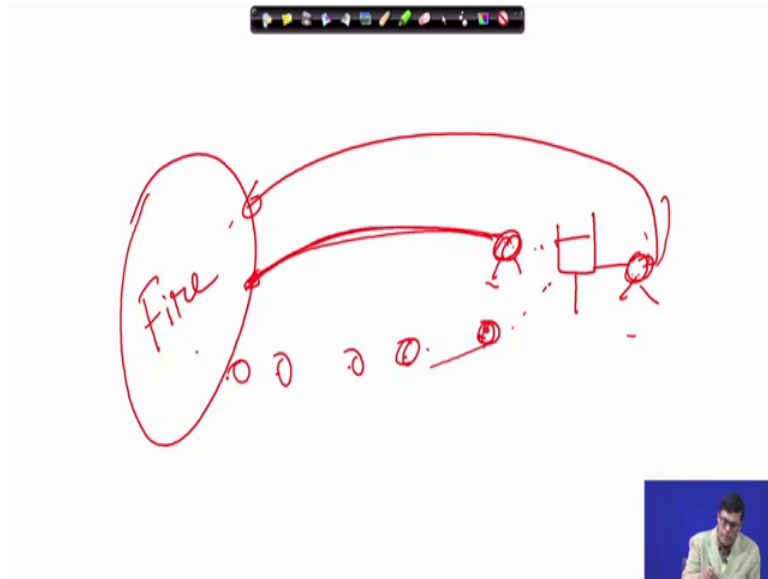
As I point out the interesting that mechanism of transportation that momentum, heat and mass transfer phenomena usually take place by two different means: molecular transfer, and what you call convective transfer.

And in case of heat transfer another mode of transport is there what you call radiation, which is not the applicable in case of in case of this mass and in case of momentum transfer. The radiations transfer is applicable only in case of heat transfer. And this is not and it is not much important for momentum and mass transport phenomena.

Now, let me explain more elaborately that what do you mean by that; then let us I can give a small example that I can give a small explain, suppose there is a there is a if a fire take place here this is fire. So, we want to extinguish the fire.



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So, it can be fire can be extremely such that three different phenomena: one is suppose we know water is the best media through which we can extinguish the fire. So, we have bucket; this bucket of water I can transfer in three different means, how we can we one person can person can carry this one directly to this field. Another the different person they can stand here and they can hand over this bucket to one after another and transfer here. And another we can use a pump here we can use a pump and with the help of pump we can press this water. So, three different means we can we can do that.

Now, if you look at look at three different means at three different purposes. Now here that that you know that person is physically moving with the material am I right; this is if the material there are moving, about here the person is not moving, but materials is moving that one day, two day, three they area like this.

And here then your with the help of pump your you are doing this. Now this is this particular pumping this can be explained with the help of what you call radiation and this is this particular things this may be consider the conduction and this consider as convection.

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**Molecular momentum transfer**

- ✓ Responsible for fluid flow
- ✓ Can be explain by **Newton's law of viscosity**

$$\tau_{yx} = \mu \frac{dv_x}{dy}$$

Final velocity distribution in steady flow

Transport phenomena by R. Byron Bird, Warren E. Stewart, Edwin N. Lightfoot, John Wiley & sons, Inc

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Now, molecular momentum, if you transfer; how you can calculate. Suppose, there is a two plates this is the two plate, this is one plates and one plates the water is there stagnant water is there. Always suddenly you move this bottom plates, then what will happen; the velocity of the lower fluid will be like this, it will be the profile will be like this can explained that with the help of Newton law viscosity this is the shear stress and I right and this is a shear rate.

So, this is equal to mu into this, and what is mu; mu is considered the viscosity of the liquid. So, viscosity is basically nothing, but shear stress by shear rate.

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### Viscosity

- ✓ The proportionality constant in Newton's law of viscosity
- ✓ Measure of its resistance to gradual deformation by shear stress
- ✓ For example, honey has a much higher viscosity than water
- ✓ Also called dynamic viscosity



<https://en.wikipedia.org/wiki/Viscosity#/media/File:Viscosities.gif>

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The viscosity is the proportionally constant of the Newton law of viscosity. Measurement of resistance is gradually deformation of the shear stress. For example, honey has much higher viscosity then water and also as dynamic viscosity.

So, he if you if you look at that, now pouring of water this is water is pouring this is water and this is honey am I right the honey has higher viscosity. So, it weight is pouring it is a different way you know that and this is weighting very easily. So, this is how we can explain that.

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### Units

the unit of momentum is given by

$$\text{momentum} = \text{mass} \times \text{velocity} = \text{kg} \cdot \text{m/s}$$

Whereas, that momentum flux is given by

$$\text{momentum flux} = \text{rate of momentum per unit area}$$
$$= \frac{\text{kg} \cdot \text{m/s}}{\text{m}^2} = \frac{\text{N}}{\text{m}^2} = \text{Pa}$$

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Now momentum as I pointed out this is equal to mass into velocity, what is the unit of mass kg. And what is the unit of velocity that is mass per unit time.

So, this (Refer Time: 15:11) mass flux flux is always with respect to per unit surface area. So, in the kg, kg meter per second than the kg per meter second is the neutron, neutron per second this is usually expressed as Pascal. Now, shear stress that you know this is I told you this is shear stress is nothing, but neutron per unit surface area this is Pascal.

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**Units**

SI units of viscosity  $\mu$  can be arrived at as follows. We have

$$\tau_{yx} = \frac{N}{m^2} = Pa$$

$$v_x = [m/s]$$

$$y = [m]$$

$$\therefore \mu = \frac{Pa}{(m/s)(m^{-1})} = [Pa \cdot s]$$

The slide also features a 3D coordinate system with x, y, and z axes drawn in red. At the bottom, there are logos for IIT KHARAGPUR, NPTEL ONLINE CERTIFICATION COURSES, and DEBABR, BIOTECH, IIT KHARAGPUR, along with a small video inset of a person.

And this is a the velocity  $v_x$  is the direction, because you know that if you if you look at this direction of flow may be of this is x, this way this might be y or you know this might be z and this might be y that you know that they are different direction it can it can it can move.

So, this is the x direction y is that that y is mean meter and viscosity equal to Pascal per unit meter per second and meters (Refer Time: 16:11) is the Pascal second.

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**Dimension of viscosity**

$$\mu = \frac{\text{Force/Area}}{\frac{dv_x}{dy}} = \frac{MLT^{-2}L^{-2}}{LT^{-1}L^{-1}} = ML^{-1}T^{-1}$$

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Now, similarly we can we can we can find out the, what should be the unit for viscosity that way this is I told you this is your shear for shear stress by shear rate, if you if you give you will get this equation.

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**Newtonian fluid**

- ✓ Fluids which obey Newton's law of viscosity
- ✓ All gases and most liquids which have simpler molecular formula and low molecular weight such as water, benzene, ethyl alcohol, CCl<sub>4</sub>, hexane and most solutions of simple molecules are Newtonian fluids.

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Now, Newtonian fluid, what do you mean by Newtonian fluid, which obeys the Newton law of viscosity. That all gases most liquid which have simpler molecular formula and lower molecular weight such as water, benzene, ethyl, alcohol, carbon tetra chloride, hexane and most solutions of simple molecules are Newtonian fluids.

So, this is this is how Newtonian fluid can be defined.

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**Non-Newtonian fluid**

- ✓ Fluids which do not obey the Newton's law of viscosity are called as non-Newtonian fluids.
- ✓ Generally non-Newtonian fluids are complex mixtures: slurries, pastes, gels, polymer solutions etc.,

<http://www.mssubbu.in/in/fm/Unit-I/NonNewtonian.htm>

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Now this is very important, but you know if you if you if you if you plot; this is shear stress versus shear rate, if you if you plot. And this is for this will be for Newtonian liquid and this is for non Newtonian liquid this all are non Newtonian liquid.

So, what is non Newtonian liquid or fluid? The fluid which do not obey the Newtonian law of viscosity called the non Newtonian liquid. Generally non Newtonian fluids are complex mixtures: slurry, pastes, gels, polymer solution all considered, while with here we have given the example of Bingham plastics. This is actually happen in case of fermentation broth fermentation broth. Some fermentation broth they behave like Bingham plastics. So, this is also a non Newtonian fluid, this is not Newtonian if a Newtonian it will follow this one.

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**Convective momentum transfer**

- ✓ Transportation can be done by bulk flow the fluid
- ✓ Let the fluid enter with a velocity  $v_x$ , then the momentum flux per unit volume is  $v_x \rho v$ .
- ✓ Similarly for the velocity in  $y$ - and  $z$ - directions, we get momentum fluxes  $v_y \rho v$  and  $v_z \rho v$ .
- ✓ Once we know these fluxes, then multiplying by the area perpendicular, we can calculate the forces.

*Handwritten notes:  $\rho v$  (circled), mass/vol*

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Now, convective momentum transfer, that I already explained you, what is convective? The transportation can be done from the bulk flow of the fluid. And that you know that fluid center has the velocity  $v_x$ , then the momentum will be what; this is  $v_x$  into  $\rho$  there  $v_x$  into  $\rho$  into  $v$ . What is  $\rho$  into  $v$ ?  $\rho$  into  $v$  is the volume here  $\rho$ ,  $\rho$  volume a  $\rho$  is the mass per unit volume am I right; mass per unit volume and this is volume.

So, volume will cancel, this will be mass the mass into velocity is the is the is the what you call momentum flux per unit volume, that can be explained. Similarly the velocity  $y$  and  $z$  direction we can we can have the momentum flux like this  $v_y \rho v$  and  $v_z \rho v$ . This we can; we know that the flux when multiply this the, where the area perpendicular we can calculate the force this is I have already shown you before.

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**Boundary condition**

- ❑ Solid-liquid interface
  - ✓ No slip condition
- ❑ Liquid-liquid interface
  - ✓ Free slip condition
- ❑ Gas-liquid interface
  - ✓ The shear stress at the gas-liquid interface will be zero

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There are different boundary conditions that we have in this; what you call solids-liquid interface, there will be no slip flow. And in case of liquid-liquid interface that the free slip flow is there and gas-liquid interface the shear stress of the gas liquid interface, where will be 0 that is the, that is how over these over the that is the boundary condition that we have.

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**Flow through a circular tube (gravity flow)**

Let a fluid of constant density  $\rho$  and constant viscosity  $\mu$  be flowing in a circular tube of radius  $R$  and length  $L$ . Let us consider that the fluid is flowing in the  $z$ -direction. Hence only  $v_z$  is considered.

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The now let us calculate with the flow of the fluid through the circular tubes that is the gravity flow. Now let the fluid constant have the fluid constant density  $\rho$  and the



viscosity  $\mu$  be flowing in the circular tube of radius  $R$  capital  $R$  and length  $L$ . Let us consider the fluid is moving in the  $z$ -direction. And hence  $v_z$  is considered;  $v_z$  is the, that velocity in the direction of  $z$ .

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**Flow through a circular tube (gravity flow)**

**The assumptions**

- ✓ Steady-state conditions.
- ✓ Laminar flow, i.e.  $N_{Re} \leq 2100$ . - 2100 - 4000
- ✓ Incompressible fluid, i.e. density  $\rho$  is constant and viscosity  $\mu$  is constant.
- ✓ Unidirectional flow, i.e. liquid flow is only in the  $z$ -direction. Hence,  $v(\theta) = 0$  and  $v(r) = 0$ .
- ✓ Newtonian fluid.
- ✓ No slip between the liquid and the solid surface of the wall.
- ✓ No end effects. The disturbances of the flow of liquid at the edges (i.e. at  $z = 0, z = L$ ) are neglected.
- ✓ The flow of liquid is under gravity only.

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Now, the flow of the circular tube the assumption following assumptions remain: one is steady state condition, laminar flow we know that laminar flow occurs, when the Reynolds number is less than 2100, if a; so I can tell you that 200 10 to 4000 it is transient, you know the neither laminar no node you know non laminar flow is that non Newtonian flow ah, but below that it is Newtonian flow or 4000 we have turbulent flow. This is laminar flow is less than 2100.

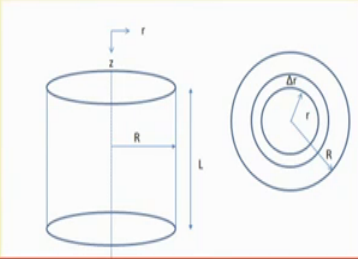
The incompressible compressible fluid that density is constant and viscosity also constant so, on unidirectional flow liquid flow only in the  $z$ -direction, the  $v_\theta$  equal to 0 and  $v_r$  equal to 0; Newtonian fluid no the no slip between the liquid and solid surface of the wall, no end effect the disturbances of the flow of the liquid at the edge and the  $z$  equal to 0  $z$  equal to  $L$  are neglected. And flow of the liquid in under gravity only. Though this are the couple of assumption we remain to analyze this flow.

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**Flow through a circular tube (gravity flow)**

Under these assumptions, the z-momentum considered will be as follows:  
Molecular momentum flux =  $\tau_{rz}$ .

i.e.  $\tau_{\theta z} = 0$ ;  $\tau_{zz} = 0$



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Now, let me explain that flow through the circular tube now, under this assumption that z momentum is considered as follows; that momentum molecular momentum plus this tau r z and if theta z equal to 0 and z z equal to 0 that we assume.

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**Flow through a circular tube (gravity flow)**

□ **The governing equation**

Rate of molecular "momentum in" across the cylindrical surface at  $r$  – rate of molecular "momentum out" across the cylindrical surface at  $(r + \Delta r)$  + gravity force acting on the fluid = 0.

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Then here we also considered a segment you can see that segment; this is the  $\Delta r$  we consider see just to integrate from  $r$  to  $R$  right.

So, following; the flow through the circular tube can be explained at the rate of molecular "momentum in" across the cylindrical surface  $r$  rate of molecular "momentum

out” across the cylinder  $r$  plus  $r \text{ del } r$  and plus gravity force acting on the fluid that should be equal to 0.

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**Flow through a circular tube (gravity flow)**

- ✓ Rate of molecular “momentum in” across the cylindrical surface at  $r$ 

$$= (2\pi r L) \tau_{rz} |_r$$
- ✓ Rate of molecular “momentum out” across the cylindrical surface at  $(r + \Delta r)$ 

$$= (2\pi r L) \tau_{rz} |_{r+\Delta r}$$
- ✓ Gravity force acting in the  $z$ -direction on the fluid  $= (2\pi r \Delta r L) \rho g$

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Now, rate of molecular momentum in can be explained like this. This is the surface area,  $2\pi r L$  right; this is per unit surface area. So,  $\tau_{rz}$  this is  $r$ , then this is  $r$  plus  $\text{del } r$  and this is the gravitational, this  $\rho g$  you multiplied you will get the gravitational force acting on the  $z$  direction of the fluid.

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**Flow through a circular tube (gravity flow)**

- ✓ Combining the above equations
$$(2\pi r L) \tau_{rz} |_r - (2\pi r L) \tau_{rz} |_{r+\Delta r} + (2\pi r \Delta r L) \rho g = 0$$
- ✓ Dividing the equation by  $2\pi \Delta r L$  and taking the limit  $\Delta r \rightarrow 0$ , we get the differential equation
$$\frac{d(r\tau_{rz})}{dr} = r\rho g$$

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Now, combining these whole equation, we will get this whole equation we will get to the this is equal to 0. Now we divide by 2 pi del r into L and taking del r tends to 0, we get this differential equation that we that will simplified the r rho into g. Then we by through integration, that if you if you look at here this is like this, now if you do the integration here, then what will get this is equal to C 1 is the constant am I right this is constant.

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**Flow through a circular tube (gravity flow)**

✓ Integration both sides, we obtain

$$r\tau_{rz} = \frac{r^2}{2}\rho g + C_1 \text{ Constant}$$

$$\tau_{rz} = \frac{r}{2}\rho g + \frac{C_1}{r}$$

Where  $C_1$  is a integration constant

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Then this is equal to; this is if you divide by r, you will get this equation, where C 1 is the integration constant.

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**Flow through a circular tube (gravity flow)**

✓ At  $r = 0$ , centre of the tube, the fluid velocity will be maximum and at the wall of the tube, the fluid velocity will be zero.

✓ So physically, the molecular momentum flux,  $\tau_{rz}$  will be finite at the centre of the tube.

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So, flow through the size circular tube, what is happening; if  $r$  equal to 0, if  $r$  equal to 0, center of the tube, the fluid velocity will be maximum. Suppose, this is the tube and you know that; if the velocity is the maximum here, why it is velocity; where you have less friction as you come to the close to the wall there will be friction between wall and the liquid.

So, you know that the velocity will be will be effected. So, physically the molecular momentum that flux that is the  $\tau_{rz}$  will be finite at the center of the tubes.

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**Flow through a circular tube (gravity flow)**

**Boundary condition 1:** at  $r = 0$ ,  $\tau_{rz}$  = finite

So,  $C_1$  must be zero, otherwise the momentum flux would become infinity at the centre of the tube. Therefore

$$\tau_{rz} = \frac{r}{2} \rho g$$

Applying the Newton's law of viscosity, we get

$$\tau_{rz} = -\mu \frac{dv_z}{dr}$$

The slide also features logos for IIT KHARAGPUR, NPTEL ONLINE CERTIFICATION COURSES, and DEBABR. BIOTECH IIT KHARAGPUR, along with a small video inset of a presenter.

Now flow through the; again we can find out the boundary conditions like this  $r$  equal to 0,  $\tau_{rz}$  finite I told you, and  $C_1$  is must equal to 0, otherwise the momentum plus will be infinity at the center of the tube.

Therefore, we can write this is equal to  $r \tau_{rz}$  equal to  $r$  by 2 rho into  $g$ . Applying the Newton's law of viscosity, we can find out this is equal to that that, we know this is now; this is we already we have seen. Now, we can write this equal to this we can write this that is exactly what we have shown here.

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Flow through a circular tube (gravity flow)

Therefore,

$$-\mu \frac{dv_z}{dr} = \frac{r}{2} \rho g$$

Integrating the above equation

$$v_z = -\frac{\rho g}{4\mu} r^2 + C_2$$

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And this is like this, then we can derive this equation that  $v_z$  equal to then again this  $C_2$  is a integration constant.

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Flow through a circular tube (gravity flow)

Boundary condition 2: at  $r = R, v_z = 0$

$$0 = -\frac{\rho g}{4\mu} R^2 + C_2$$
$$C_2 = \frac{\rho g}{4\mu} R^2$$

We obtain velocity distribution as

$$v_z = -\frac{\rho g}{4\mu} r^2 + \frac{\rho g}{4\mu} R^2$$
$$v_z = \frac{\rho g}{4\mu} R^2 \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

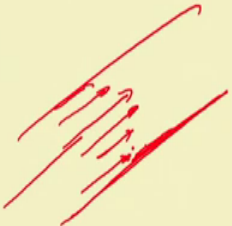
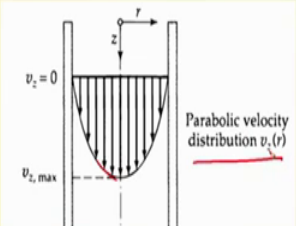
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Then again we apply the boundary conditions here,  $r$  equal to small  $r$  equal to capital  $R$ , I showed you the small  $r$  here and capital  $R$  here, then  $v_z$  equal to 0, then the equation will be this and then that is it would be this, then combine this we will get this equation and final equation will be; if you take this common where final equation will be this.

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**Flow through a circular tube (gravity flow)**

- ✓ This velocity profile is for laminar, incompressible flow of a Newtonian fluid.
- ✓ The velocity profile is parabolic in nature



Transport phenomena by R. Byron Bird, Warren E. Stewart, Edwin N. Lightfoot, John Wiley & sons, Inc

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Now, flow through the circular tube the velocity profile in the laminar, of a incompressible liquid Newtonian flow, this is the this is the velocity profile with a what I want to show, this is suppose the liquid is flowing through this pipeline. So, it will be having like this. Now, why because you know this is exactly 45 as the liquid comes close wall of the tube, there will be friction between liquid and the and the and the and the wall.

So, your velocity will be retarded. So this is the parabolic velocity distribution that we will obtain.

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**Flow characteristics (Laminar and turbulent)**

✓ A parameter used to characterise fluid flow is the Reynolds number. For flow in pipes with circular cross section, **Reynolds number (Re)** is defined as

$$Re = \frac{D u \rho}{\mu} = \frac{D u}{\mu / \rho} = \frac{\text{inertia force}}{\text{viscous force}}$$

✓ Where,  $\rho$  is fluid density,  $u$  is fluid velocity,  $D$  is pipe diameter and  $\mu$  is fluid viscosity.

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The one important parameter, that is use to characterize the flow characteristics of the fluid that is the Reynolds number. And Reynolds number can be explained as  $D u \rho$  by  $\mu$  am I right. So, this is  $D u$  you consider as a inertial; inertia force and  $\mu$  by  $\rho$  we consider the viscous force. So, if the Reynolds number is nothing, but that if the ratio between the inertia force and the viscous force.

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**Flow characteristics (Laminar and turbulent)**

✓ Laminar flow  $Re < 2100$

✓ Turbulent flow  $Re > 4000$

✓ Transition flow  $2100 < Re < 4000$

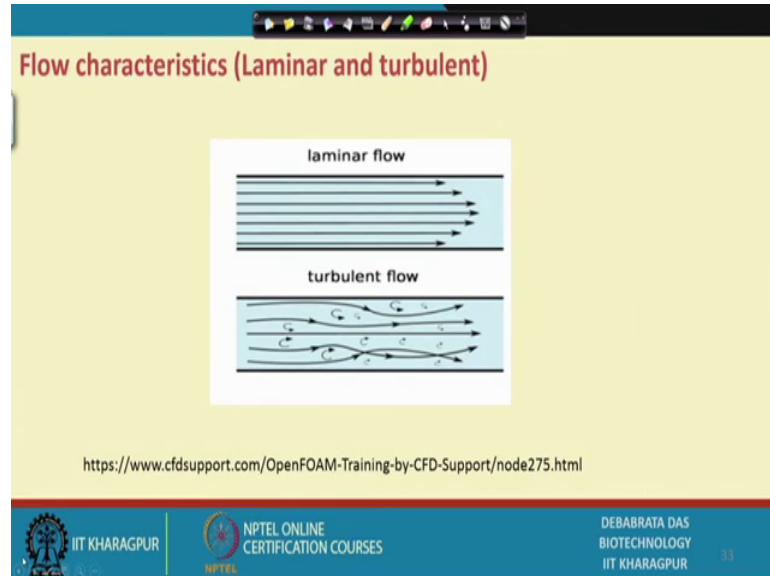
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Now, I was telling like this that you know, if the Reynolds number less than 2100 is laminar flow, it is more than 4000 we call it turbulent flow. And transition flow is if it



lies in between it is neither laminar flow nor turbulent flow this will be between 2100 to 4000.

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This is the pattern that we have that is in case of laminar flow clearly we can see the velocity gradient and this velocity gradient can be minimized, when we have the turbulent flow. When we have; if the velocity of the fluid is maximum, then we can see here the velocity gradient is reduced to a great extent as compared to this laminar flow.

In this particular lecture we try to cover a new topic it is transport phenomena in the bioprocess, and I told you transport phenomena plays a very important role as a in the biological process, because it has three different approaches: one is momentum transport and the heat transfer and the mass transfer.

And basically that that momentum transfer is applicable for the mixing of the fluid, because which is very much required as per biochemical industries is concerned, because how the cells are remains suspension how you add this some kind of media that you know in radiance to the media, how quickly is dispersed, then we have heat transferred, because organisms are very sensitive to temperature.

So, temperature is to be maintained that right there heat transfer plays a very important role and mass transfer I told you the anaerobic fermentation process we require to have

proper dissolved oxygen concentration for the growth of the microorganism, because the oxygen's are sparingly soluble in the in the water.

So, all these transport phenomena can be explained. By the first in this particular lecture I try to discuss the momentum transferred, how different parameters involve with this particular things. And we try to correlate take out we try to find out what is viscosity? Viscosity of the fluid is nothing, but ratio of shear stress by shear rate.

Then we we try to analyze the system, when which passes through the cylindrical tubes and finally, I try to try to find out that that the; how the Reynolds number influence the flow characteristics of the fluid with the Reynolds number is less than 2100 we call it laminar flow, if it is more than 4000 we call it turbulent flow, and if it is lies in between 2100 and 4000 you call it the transition flow.

Thank you very much.