Neurobiology

Dr. Nitin Gupta

Department of Biological Sciences and Bioengineering

IIT Kanpur

Week - 02

Lecture 2.6: Length constant

Welcome back to Neurobiology. In the last video, we saw what factors determine the rate of charging or discharging of a membrane. We saw it's the resistance and capacitance that affect the rate at which the membrane potential can change. In fact, it's the product of these two quantities, resistance times capacitance, which is also known as the time constant, that single parameter that determines the rate of charging. Now, in this video, we are going to look at a similar quantity known as the length constant of a branch of a neuron. So, let's see what this is about.

We have been talking about membrane potential in the last few videos. And we have made an assumption that the membrane potential is identical at different parts of the neuronal membrane. So, effectively, we have assumed that the neuron is a small, maybe a sphere-like body, and the membrane potential inside at every point is identical. And when we inject current into a neuron, we have assumed that the membrane potential will rise equally at different parts of the membrane.

But in reality, we know that neurons have these thin and long branches, the dendrites, the axons. So, the πcture, real πcture is very different from a spherical object. These dendrites and axons can be together called neuronal branches or neurites or neuronal processes. These terms include all types of branches, both axons and dendrites. Now, imagine if we are injecting current at a certain point in the branch.

So, the current flows in and this current will start charging the membrane and it will also start flowing out through the ion channels. And eventually, a saturating value of membrane potential will be reached here. Similarly, the current will also flow along the length of the axon. So, some current reaches here and this current will again charge the membrane here and it will flow out through the ion channels and some of it will pass along further. So, the saturating value of membrane potential that is reached here will be dependent on how much current is coming here, which is smaller than the current here.

So, the farther we go from the injection point, the smaller will be the change in the membrane potential that we will observe. So, how does this saturating membrane potential decrease as you go farther away from the point of injection? This is what we will try to understand in this video. Now, let us look at in slightly more detail what the current flow looks like when we inject current into a neurite. So, this is our stimulating electrode and we are injecting current here. And you can imagine that the current will flow along the length of the neurite.

It can flow to the right and to the left. And because this is a neuronal membrane, so the current can also flow out through the ion channels in the membrane. So, there is a membrane component of the current and there is this axial component of the current where the current is flowing along the length of the neurite. Now, since some current is flowing out here, a lesser amount of current reaches this point and again some current is flowing out flowing out here. So, lesser current reaches this point.

So, the farther we go from the point of injection, the lesser current is available to flow out through the membrane. And this current that is flowing through the membrane charges the membrane and changes the membrane potential. So, the final value of membrane potential that will be reached here closer to the point of injection would be higher compared to the final value that is reached farther away from the point of injection. And this decay in the final value is given by this relationship.

$$\Delta V\ (x) = \Delta V_0 e^{-\frac{x}{\lambda}}$$

So, this is also an exponential decay. The maximum value is reached when x = 0 that is at the point of injection and that value we are denoting by delta V_0, the change in membrane potential that is reached here. And then as we go farther away, we see an exponential decay. And the rate of this decay is determined by this parameter λ. So, when $x = \lambda$, then the value becomes $\Delta V_0\ e^{-1}$ or it becomes 37% of the maximum value.

So, that distance is equal to 1 λ. So, the membrane potential change at any distance x from the point of injection is given by $\Delta V_0 e^{-\frac{x}{\lambda}}$ and λ is the length constant. So, λ must have the same units as x for this to be a dimensionless quantity. So, it is in the unit of meters. Now, let us see if we can figure out what parameters would λ depend on.

So, the time constant τ dependent on the resistance and capacitance. Do you think λ would depend on capacitance? Well, since we are looking at the steady state membrane potential, so when we start injecting current, the membrane potential would rise gradually and then a steady state value will be reached. At different values of x, different steady state values will be reached and we are looking at the distribution of these steady state values. And since we are looking at these steady state values where the charging has been over and the values are not changing over time, so we can ignore the capacitance because capacitance only affects the initial charging period. But once the charging is done, then the final value that is reached does not depend on capacitance.

So, the length constant should actually depend on the resistance values. How much resistance the current encounters as it is flowing through the membrane and how much resistance it encounters as it is flowing along the length of the neurite. So, we define these two parameters R_m and R_a. R_m is the membrane resistance per unit length and R_a is the axial resistance per unit length. So, both these resistance values will depend on the length of the neurite that we consider.

And if we increase the length of the neurite, then we increase the surface area, so the resistance would decrease with length. So, if we consider length L, the resistance of that length will be R_m/L, if R_m is the resistance per unit length. So, R_m must have the units of ohm times meter and if we divide it by L, then the final resistance that we get will be in ohm. Now, let us see if we can predict how the λ will depend on R_m. If we have more R_m, will we have more λ or less λ? Pause your video and see if you can predict that before I show the answer.

So, when we increase R_m, we are making it more difficult for the current to flow through the membrane, which means that more current will be able to flow along the axial direction and it will be able to go a farther distance. So, the λ should increase. So, there is less leakage and current travels farther distance along the length of the axon or the dendrite and so we have a bigger λ. Now, R_a is the axial resistance per unit length and as we increase the length, the current encounters more hindrance. So, the resistance would increase as we increase the length.

Resistance of a length L will be R_a times L, where R_a is the resistance per unit length. So, the unit of R_a would be ohm divided by meter and when we multiply it by the length, then it becomes ohm. So, the final resistance of any πece of neurite will be in ohm. Now, again let us see if we can predict how λ would depend on R_a. So, if we have more R_a, will we have more λ or less λ? So, as we increase R_a, we make it more difficult for the current to flow along the axial direction.

So, it would go a shorter distance and therefore the λ would decrease. So, more R_a should result in a smaller λ and the exact relationship between λ and these two quantities is given in this manner. So,

$$\lambda = \sqrt{\frac{R_m}{R_a}}$$

And since R_m has the units of ohm-meter and R_a has the units of ohm/meter, this term has the units of meter^2 and after taking a square root, λ gets the unit of meter. Now, let us see how the length constant λ depends on the axon diameter. So, if we have an axon with a large diameter and another axon with a small diameter and everything else between them is same, which of these two axons would have a larger length constant? So, let us try to figure that out.

So, we know that the length constant depends on these two quantities, membrane resistance per unit length and axial resistance per unit length R_m and R_a and these two quantities will be affected by the diameter. So, let us see how the R_m would be affected. So, R_m is the membrane resistance, the current is flowing from inside to outside the axon and this is the surface area that the current is crossing. So, as this surface area increases, the membrane resistance would go down and the surface area here is $2 \pi R$, 2π radius times the length. So, R_m varies as 1 over 2π radius and R_a is the axial resistance.

So, current is flowing in this direction and this is the cross section area that it is encountering. So, the axial resistance will be inversely proportional to this cross sectional area and this area is πr^2. So, R_a would be inversely proportional to πr^2. So, we have R_m inversely proportional to r and R_a inversely proportional to r^2. So, when we look at R_m/R_a, it becomes $(1/r) / (1/r^2)$ or becomes proportional to r.

So,

$$\sqrt{\frac{R_m}{R_a}} \propto \sqrt{r}$$

So, it still increases with radius. If you have a larger radius, you would have a larger value of length constant and what that means is that thicker axons will have more length constant and they will be able to conduct signal to a longer distance with less attenuation. So, that is a good thing. We want the axons to carry the signals to longer distances with less attenuation with less leakage.

So, we would want neurons that have larger length constants and we can achieve that by having a larger diameter. In fact, that is the reason why the giant axon in the squid has a large diameter. So, why don't we have neurons with larger diameter? Well, it would be nice to have larger diameter, but how would we pack them in the same brain? Our brain is already filled to the capacity and there is no more room to have larger diameter axons. So, instead of increasing the diameter, the way we have achieved more λ is by increasing R_m and that can be done by increasing the insulation on the axons and that is where myelination comes in. So, by doing myelination, we can increase R_m and then increase length constant.

So, in summary, in the last video and the current video, we have looked at two constants. In the previous video, we looked at the membrane time constant, which tells us how slowly the membrane charges or discharges, how much time it takes before the charge dissipates to a certain level. In the current video, we have looked at the concept of length constant of a branch of a neuron, which can be an axon or a dendrite and that tells us how far the signal travels in the branch, how much distance it can cover before the signal dissipates to a certain level. Thank you.