Neurobiology

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Week - 02

Lecture 2.5: Time constant of the membrane

Hi everyone. In this series of videos, we have been looking at various electrical properties of neurons. In the last video, we saw how the membrane potential can be measured and how it can be manipulated by injecting current. If you inject positive current into a neuron, you can make the membrane potential more positive. And if you inject negative current, you can make it more negative or hyperpolarized. One very interesting thing we observed is that even if you give a very sharp current pulse, the change in membrane potential is not as sharp as shown in this picture again.

So even if you have a square current pulse, the membrane potential changes slowly. So if you give a positive current, it will rise slowly or if you give negative current, then it will become more negative slowly. So what determines the speed of this change in membrane potential? That is what we will try to understand in this video. So let us understand what is going on.

When we apply a current, the current changes the membrane potential because it accumulates charge on the membrane. Positive charge or negative charge is accumulated on the inside or the outside of the membrane. That tells us that the capacitance is at play here. So let us look at our membrane again. This is the inner terminal, outer terminal.

We have the lipid bilayer that acts as a capacitor and in parallel we have ion channels that are embedded in the membrane and these act as resistors or conductors. So the equivalent circuit of the piece of membrane is this resistor and capacitor in parallel which have the same terminals, the inside of the membrane and the outside of the membrane. Now when we apply a constant current source, so let us say we have two stimulating electrodes, one inside, one outside and we start passing some current from inside to outside. So this current I, constant current I that we have applied through the stimulating electrode will flow from this terminal towards this terminal and it can take these two possible paths. It can go through the resistor or it can go through the capacitor.

So in the beginning, will the current split equally? No. The current will flow in a way that it can satisfy one very important requirement and that requirement is that the voltage across this resistor which is the voltage difference between these two points and the voltage across the capacitor which is also the difference between these two points. So these two voltages must be identical. The voltage inside relative to outside whether we look at it through the resistor or whether we look at it through the capacitor, this voltage difference must be identical at every time point. And since we assume that the capacitor had no charge to start with, then the voltage across the capacitor would be zero because voltage of a capacitor is charge divided by the capacitance.

So that voltage is zero across the capacitor in the beginning. So voltage across the resistor will also be zero in the beginning which means that no current can be flowing through the resistor. And in the very few initial nanoseconds or picoseconds, because the current through the resistor is zero, all of this current is actually passing through the capacitor. And all of this current I is passing through the capacitor, so the capacitor will start getting charged and then it will start developing a voltage which is proportional to the charge stored on the capacitor. And as this voltage across the capacitor develops, the same voltage will also be available across the resistor.

A current also starts developing in the resistor and this current will gradually increase as the voltage across the capacitor gradually increases. But the rate of the voltage increase across the capacitor is proportional to how much current is flowing through the capacitor. So in the very beginning when all of the current was flowing through the capacitor, the voltage across the capacitor would have been rising faster. But as more and more current starts flowing through the resistor, lesser and lesser current passes through the capacitor. And so the rate of increase of voltage across the capacitor will also reduce.

So voltage was rising faster and then it will rise slowly. And at some point a steady state will be reached where the current across the capacitor becomes zero and all of the current flows through the resistor. And at that time point, the capacitor is fully charged with the maximum voltage that it can reach and the voltage across the resistor at that time point will be equal to IR because all of this I current is flowing through the resistor. So the voltage across the capacitor will also be equal to I times R at that time point. So with a constant current source, we have seen that the membrane gets charged gradually and it reaches a saturating voltage.

Now if we turn off the current source at that time after the saturating voltage has been reached, then let us see what would happen. So then we would have this capacitor and the resistor connected at these ends. And because there is a voltage across the capacitor, this voltage will drive current through the resistor. So a current equal to whatever voltage is present across the capacitor divided by the resistance V over R will flow through the resistor. And as that happens, the capacitor will get discharged gradually because as current is flowing in this direction, positive charge is coming out.

And so the voltage across the capacitor reduces over time and the current across the resistor will also reduce over time. And eventually all of this when all of this charge has flown out, the capacitor will become fully discharged. So there will be no voltage left across the capacitor. And then the current across the resistor will also become zero. So in the steady state, the system will return to the baseline.

So we have seen that if we apply a current source, we can charge the membrane. And if the current source is removed, then the membrane gets discharged. Can we figure out what will be the rate of charging or discharging of the membrane? Or can we come up with a formula that tells us exactly how the charge across the membrane or the voltage across the membrane would change over time. So let's see if we can do that in next two, three minutes. So say the membrane was fully charged to a voltage V_0.

And at time t equals to zero, the external stimulations were removed and the membrane was allowed to discharge. So let's see how the voltage across the membrane would change over time. So let's denote this voltage by this factor V. V will be a function of time at $t = 0$, it is equal to V_0. And then gradually it will reduce over time.

And the current that is flowing, let's denote that by symbol I. Again, this I is a function of time. The resistance and the capacitance are of course constant, so they are not going to change. And at any point of time, the current I will be equal to V over R. So whatever is the voltage, that will determine how much is the current flowing through the resistor.

And the charge across the capacitor, we can denote that by symbol Q. So,

$$Q = CV$$

So both Q and V are functions of time. Capacitance is of course constant. In a small time dt, we can say that if charge changes by amount dQ, that will also cause a change in the voltage dV.

So we can write,

$$dQ = CdV$$

If we divide both sides by dt, we get

$$\frac{dQ}{dt} = C\ \frac{dV}{dt}$$

Basically saying that the rate of change of charge is proportional to rate of change of voltage. Now rate of change of charge is dependent on the current. So if more current is flowing, then the charge will be decreasing faster.

So,

$$\frac{dQ}{dt} = -I$$

Minus factor is there because we have discharging here. If current was flowing and charging the capacitor, then I would be equal to dQ by dt. But since we have discharging, I is equal to minus dQ by dt.

And,

$$I = \frac{V}{R}$$

So we can write,

$$\frac{dQ}{dt} = -\frac{V}{R}$$

Now combining these two equations,

$$C\,\frac{dV}{dt} = -\frac{V}{R}$$

And further rearranging these terms, we get

$$\frac{dV}{V} = -\frac{dt}{RC}$$

Now integrate this equation and apply the boundary condition that at t = 0, V = V_0.

That gives us

$$V = V_0 e^{-\frac{t}{RC}}$$

And since R and C are both constants, we can replace them or we can denote them by a single constant τ. So finally we have,

$$V = V_0 e^{-\frac{t}{\tau}}$$

So it is an exponential decay. At t = 0, this term will become one.

So $V = V_0$. And as t becomes larger, this fraction will become smaller than one. So we will reduce. And at $t = \infty$, V will become 0. At this terms becomes 0. So we have an exponential decay like this.

So this is how the voltage across the capacitor would change over time. Now this factor τ is very interesting. So this is the factor that determines how fast or slow the capacitor would charge. And this τ is equal to RC and it is known as the time constant of the membrane. One very interesting feature of this time constant is that it does not depend on the area of the membrane.

So if you have a larger membrane, your capacitance would be larger in proportion to the area and the resistance would be smaller in proportion to the area. So the resistance times capacitance, this product, still remains the same, whether you have a larger membrane or a smaller membrane. So it is dependent only on the properties of the membrane. What is the density of ion channels? What is the thickness of the membrane? But it does not depend on the surface area of the membrane. Let us try to understand this term membrane time constant in slightly more detail.

So we have,

$$V = V_0 e^{-\frac{t}{\tau}}$$

And one thing that is clear from this equation is that the τ would have the units of time because this term t over τ must be a dimensionless quantity. So τ has a units of time, seconds or milliseconds. We can actually predict what the range of τ would be in most neuronal membranes.

So $\tau = RC$. R is on the order of mega ohms, say 10^6 and C is on the order of nanofarads, say 10^{-9}. So R times C becomes 10^6 times 10^{-9}, that is 10^{-3}. So τ will be on the order of milliseconds. Now at t = 0, this term e^{-0} becomes 1, so $V = V_0$. As time increases and as t becomes τ, this factor becomes $e^{-\frac{\tau}{\tau}}$.

So it becomes e^{-1}. So when $t = \tau$, voltage becomes V_0e^{-1}. e is approximately 2.7, so e^{-1} is approximately 0.37.

So V has become approximately 0.37 of its original value. Or in other words, τ is the time in which V reduces by 63%. And if τ is larger, it would take more time for the membrane to discharge. If τ is smaller, it will take less time for the membrane to discharge. So τ basically indicates how much time it takes for the membrane to charge or discharge.

Although we have not derived the equation for charging, charging also follows a similar behavior. It's just the mirror image of discharging. So discharging is like this, charging is like this, just mirror image across the horizontal axis. And here also τ indicates how much time it takes for the membrane to reach 63% of its original charge. Let us again consider the fact that $\tau = RC$.

So it is proportional to both R and C. And that is easy to understand if we go back to our analogy with the water flow. So we have a tube in which water can flow. And the narrowness of the tube indicates the resistance. And we have placed a membrane in the middle of the tube that can stretch.

So if water flows, the membrane can stretch. And the stretchiness of the membrane indicates the capacitance. Now if we apply a constant flow of water, the membrane can stretch. And if we remove that flow, the membrane can come back to its natural shape. And this extension of the membrane or the flattening of the membrane will be slow.

It will take more time. If the membrane is more stretchy, so it has collected more water. Or if the tube is narrow, so it will take more time for the water to flow. So the time is proportional to both the capacitance, how much water can be stored, and the resistance, how narrow the tube is. And similarly in the electrical circuits, how much charge can be stored and how slowly the current flows through the resistance. Those two factors both affect the time constant.