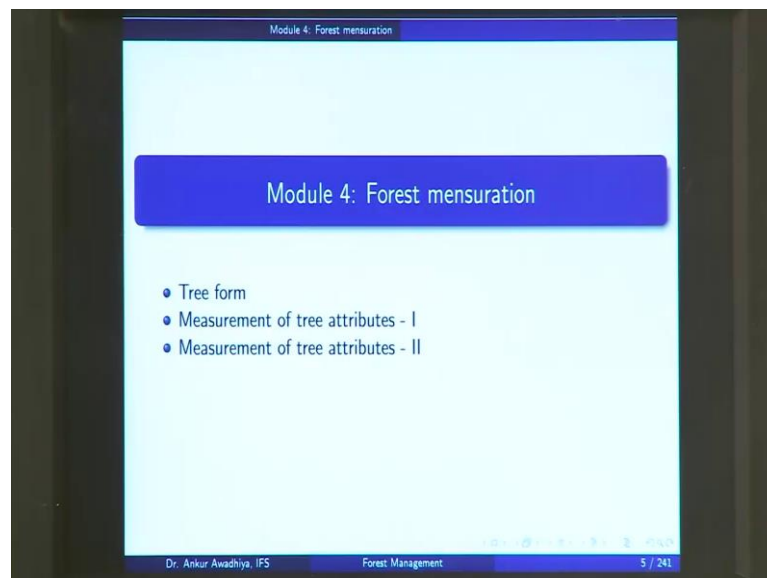


**Forests and Their Management**  
**Dr. Ankur Awadhiya**  
**Department of Biotechnology**  
**Indian Institute of Technology, Kanpur**

**Module – 04**  
**Forest Mensuration**  
**Lecture – 10**  
**Tree Form**

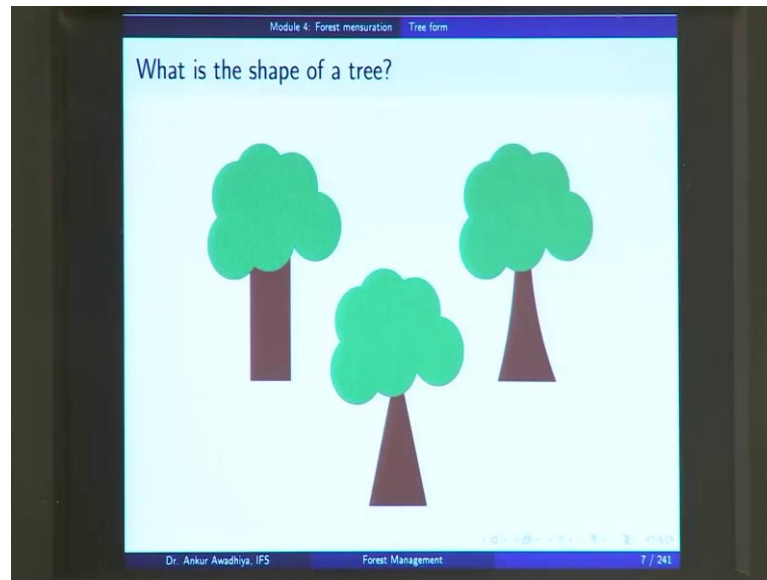
[FL]. Today we begin a new module which is Forest Mensuration. Now, ‘mensuration’ the word means ‘to measure.’ So, in this module, we are going to see how we are going to measure a forest.

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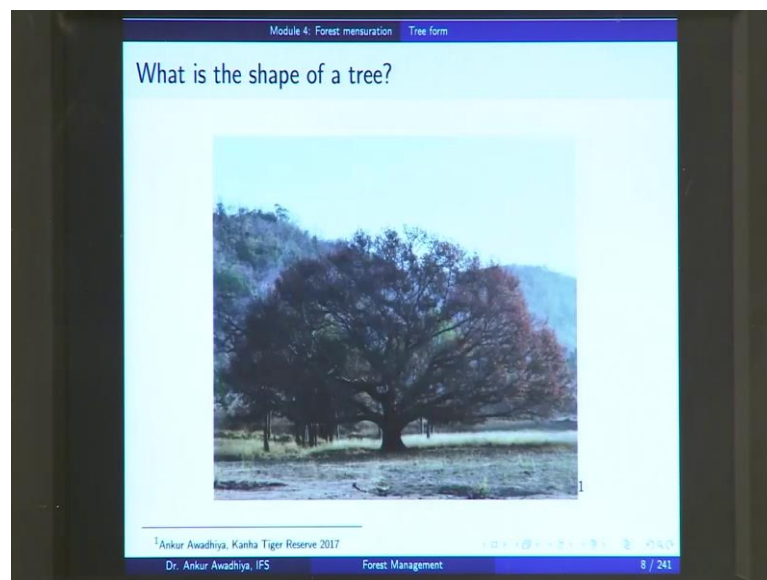
So, this module is going to have 3 lectures - Tree form, Measurement of tree attributes - Part I, and Measurement of tree attributes - Part II. So, we begin with tree form. Now, the question here is, what is the shape of a tree? You have seen a number of trees in your life, but then, if I ask you what is the shape of a tree, how are you going to describe the shape.

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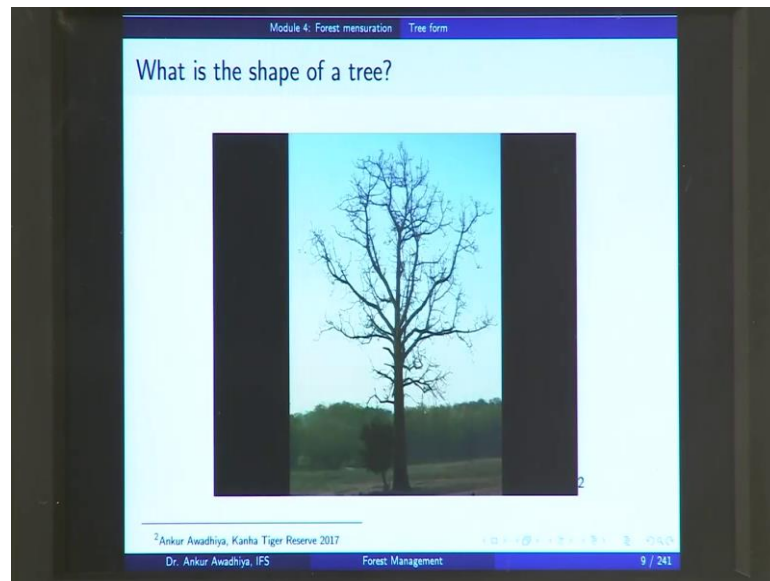
So, if you look at a tree, you will we will find a stem, and we will find the crown or the canopy. Now the question is, does the system look like a cylinder? Does it have a conical shape or does it have some other shape?

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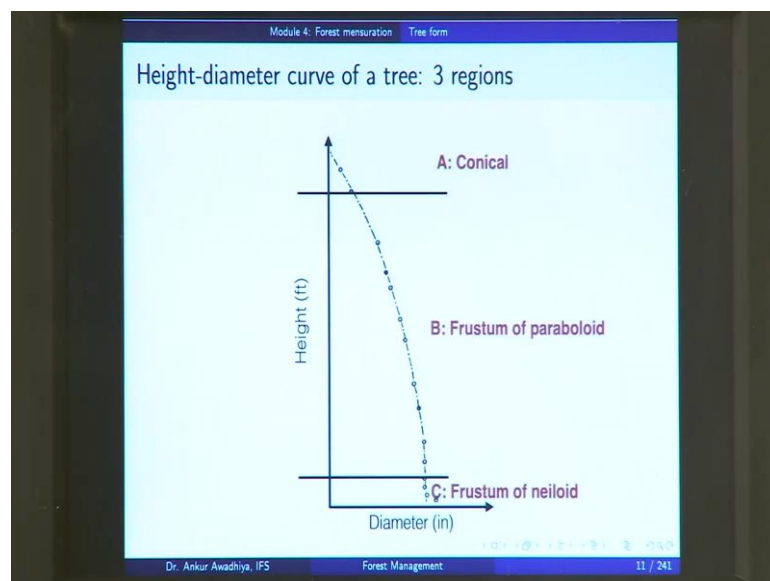
So, how do we describe the shape of a tree? If you look at actual trees; so, this is a tree from Kanha, and you will find that you have a very big sized crown. But, then the shape looks a bit odd, if we look at the stem the stem, it starts with a broad base, then it reduces then it increases and so on.

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If you look at another tree, this has a much simpler bole. So, it looks sort of conical, but then not every portion is conical, because in this portion you will see that it is tapering and becoming and coming either with a broader base. So, trees come in a variety of shapes and sizes, but if we were to generalize the shape of a tree, we could do that by plotting a few curves.

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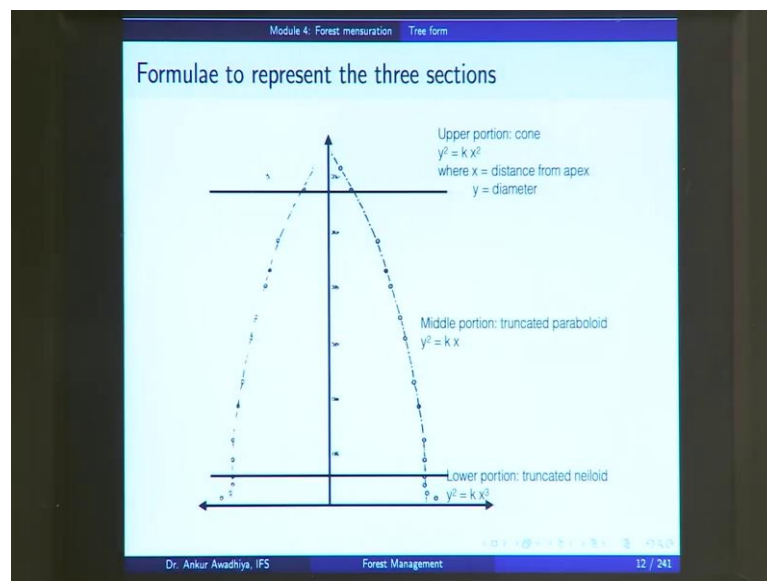


So, let us plot a diameter versus height curve, for a particular tree. So, this is actual data. So, here on the x axis you have the diameter in inches; on the y axis, you have the height

and feet. So, as we go up, as the height increases, we find that the diameter is reducing, But then, this height diameter curve, it is not a cone, because in that case, it will form a straight line; it is not even elliptical, because this lower portion is showing a sigmoidal sort of shape.

So, we can divide this curve into three portions, and what we find is that the topmost portion is conical in shape. So, this portion more or less makes a straight line in the height diameter curve. The central portion is the frustum of a paraboloid. So, what is the frustum of a paraboloid? You take a parabola; you move it along the axis and then it forms a three-dimensional shape, and then you cut it at the top and the bottom. So, that becomes the frustum of a paraboloid. The bottom most portion is the frustum of neiloid because of this shape.

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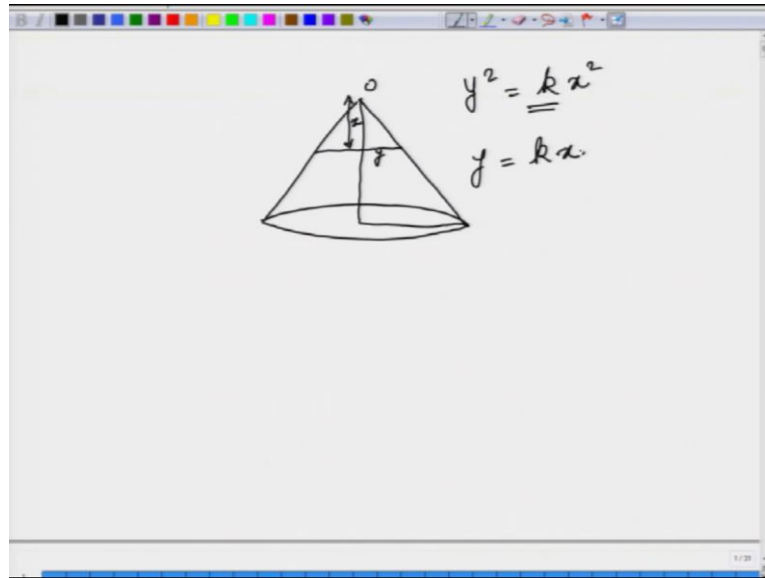


If we were to represent it mathematically, the upper portion which is conical can be represented

$$y = k x^2$$

where x is the distance from the apex and y is the diameter.

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So, in this case, what we are saying is that for the uppermost part which is conical; so, here you have the point 0, and for any point on this cone; this distance is  $x$  and this matches  $y$ . So, what we are saying here is

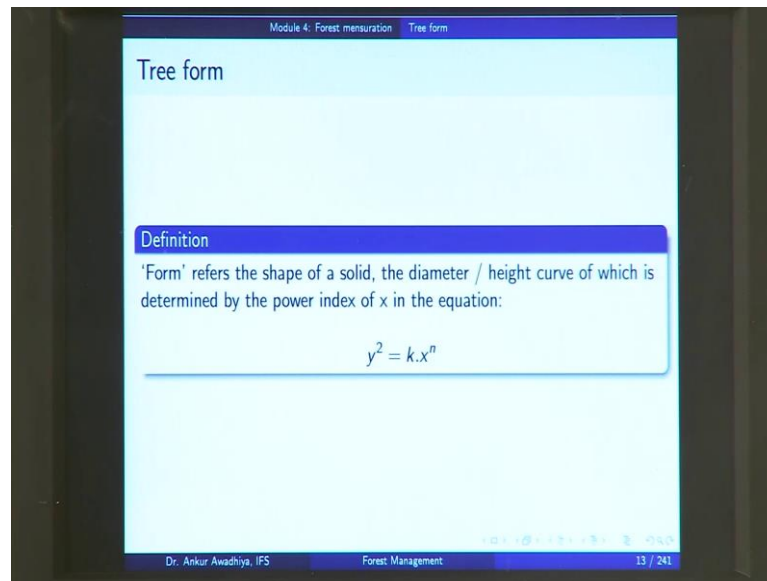
$$y^2 = k x^2$$

where  $k$  is a constant. We have seen it in the form of  $y$  is equal to  $k x$  in a number of textbooks, but then to generalize these equations for all these three portions we are using  $y$  square.

So, the upper portion or the cone is why is it  $y$  square is equal to  $k x$  square, where  $x$  is distance from the apex, and  $y$  is the diameter. Now, the middle portion is a truncated paraboloid, and here in place of  $y$  square is equal to  $k x$  square, you have  $y$  square is equal to  $k x$ , and the lower portion is a truncated neiloid; the equation for which is

$$y^2 = k x^3$$

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Module 4: Forest mensuration Tree form

### Tree form

**Definition**

'Form' refers the shape of a solid, the diameter / height curve of which is determined by the power index of x in the equation:

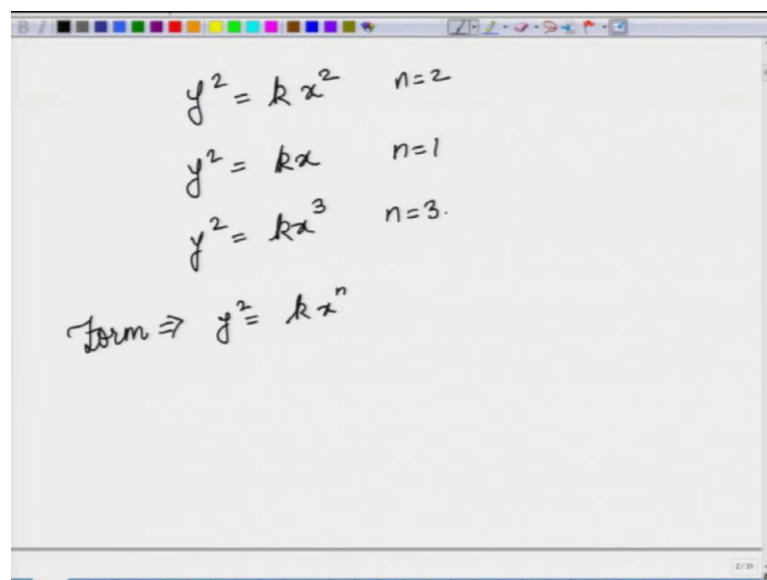
$$y^2 = k \cdot x^n$$

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Now, the shape of the tree is also known as the tree form. So, form refers to the shape of a solid, the diameter or by height curve of which is determined by the power of x in the equation,

$$y^2 = k x^n$$

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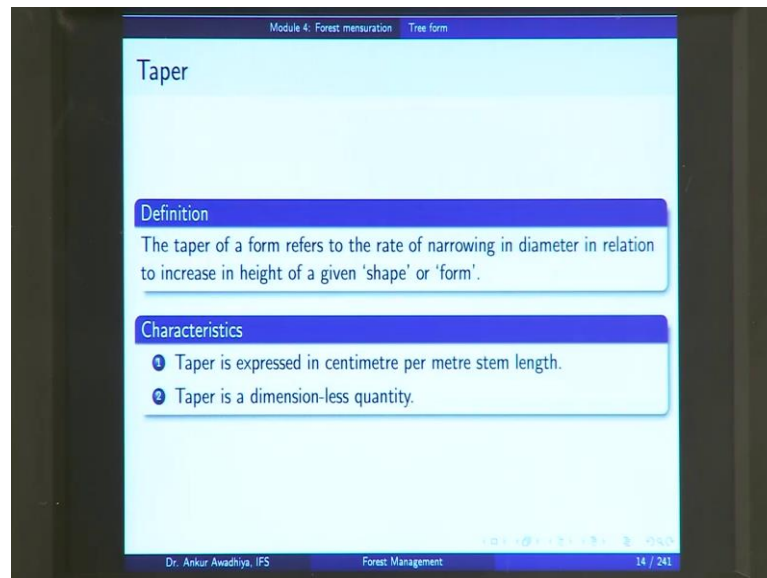


$y^2 = k x^2 \quad n=2$   
 $y^2 = k x \quad n=1$   
 $y^2 = k x^3 \quad n=3$   
form  $\Rightarrow y^2 = k x^n$

So, what we saw in the previous slide is that the topmost portion was represented as y square is equal to k x square; the middle portion was y square is equal to k x; and, the

bottom most portion was  $y^2$  is equal to  $kx^3$ . And, what we are saying is that the form of the solid is represented as  $y^2 = kx^n$ . So, here  $n$  is equal to 2, here  $n$  is equal to 1 and here  $n$  is equal to 3. So, that is the tree form. The shape of the solid represented by the equation  $y^2 = kx^n$  where  $n$  as we have seen in the case of trees is 2, 1 or 3.

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Module 4: Forest mensuration Tree form

## Taper

**Definition**  
The taper of a form refers to the rate of narrowing in diameter in relation to increase in height of a given 'shape' or 'form'.

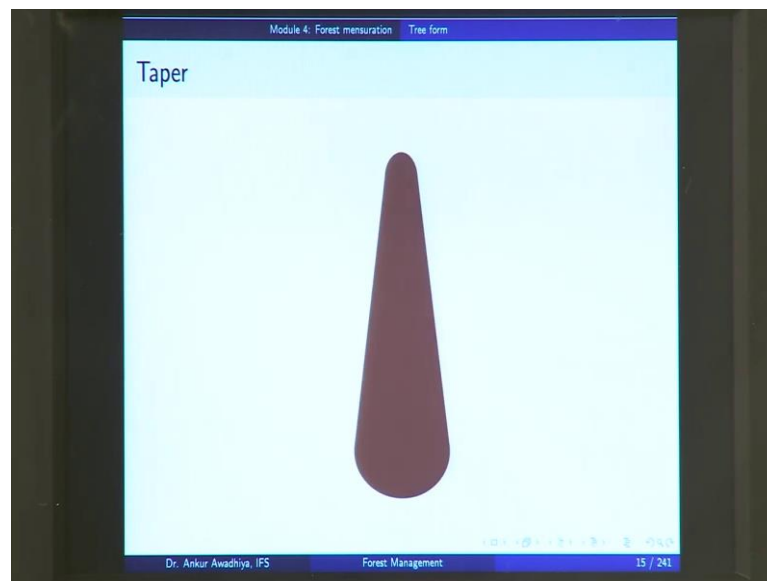
**Characteristics**

- 1 Taper is expressed in centimetre per metre stem length.
- 2 Taper is a dimension-less quantity.

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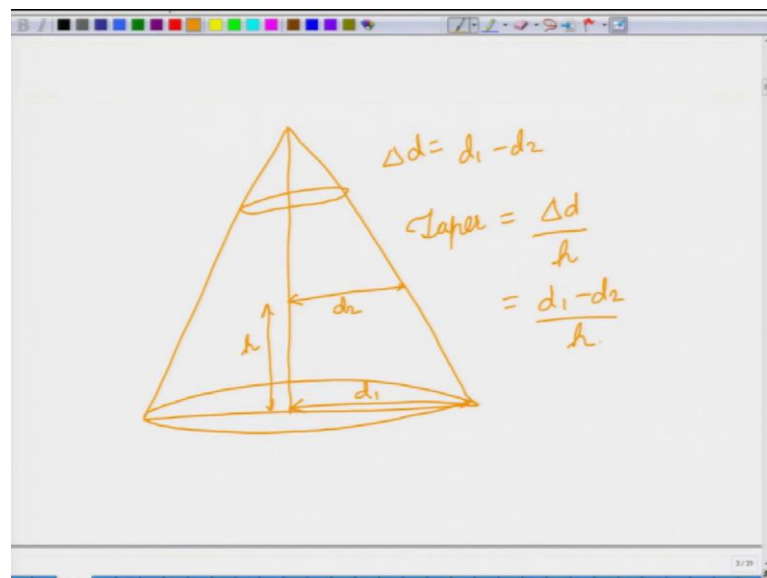
Now, we also saw that the height that the diameter of the tree reduces as we go up. Now, this reduction in diameter is known as taper. So, taper is defined as, the taper of a form refers to the rate of narrowing in diameter in relation to increase in height of a given shape or form. So, for any shape or form the change in diameter divided by the change in the height, is referred to as the taper. So, it is expressed in centimeters per meter of stem length, and it is a dimensionless quantity.

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So, what we are saying here is that if you look at this particular form, it is a frustum of a cone. Now, as you are moving up, the diameter is reducing, and in this case.

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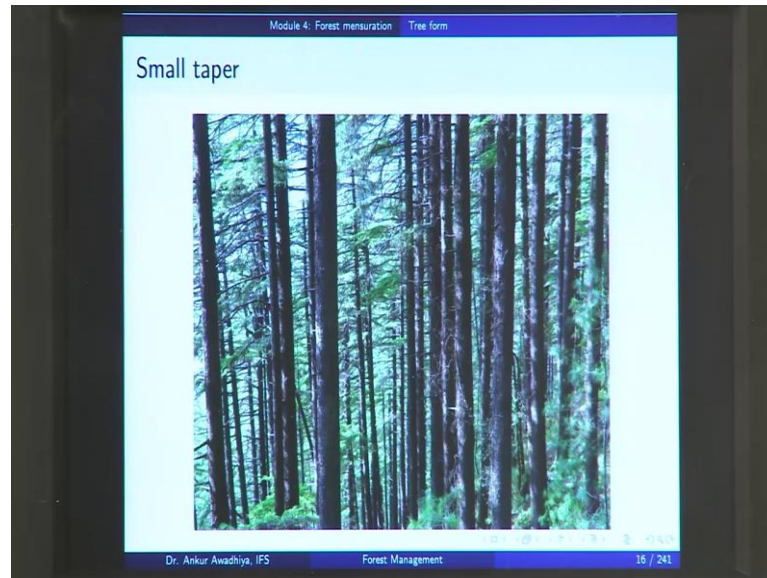
So, let us draw this shape. Now, the diameter at this point is  $d_1$ , the diameter here is  $d_2$ . Now, for a change in height of  $h$ , we are seeing a change in diameter of  $d_1$  minus  $d_2$ , which we can write as  $\Delta d$ . So, the taper is the change in diameter divided by the change in height.



$$\text{Taper} = (d_1 - d_2) / h$$

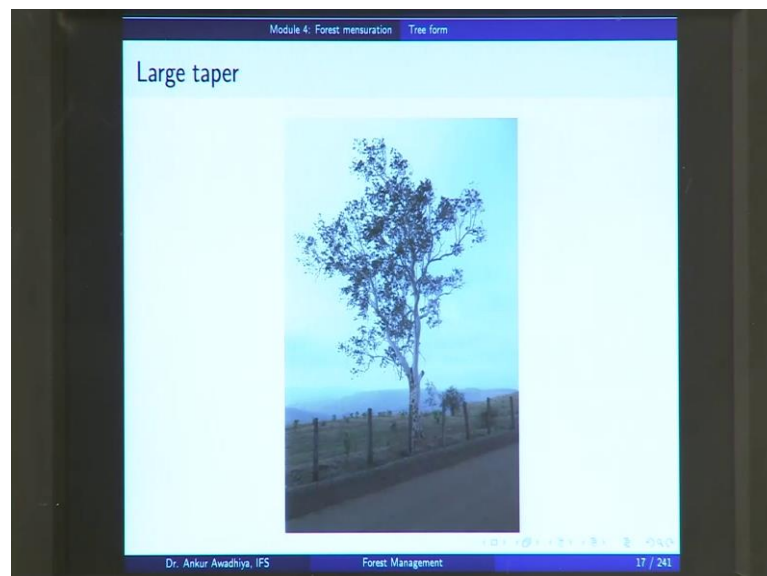
So, this is the taper of this particular solid. Now, this taper can be large or small.

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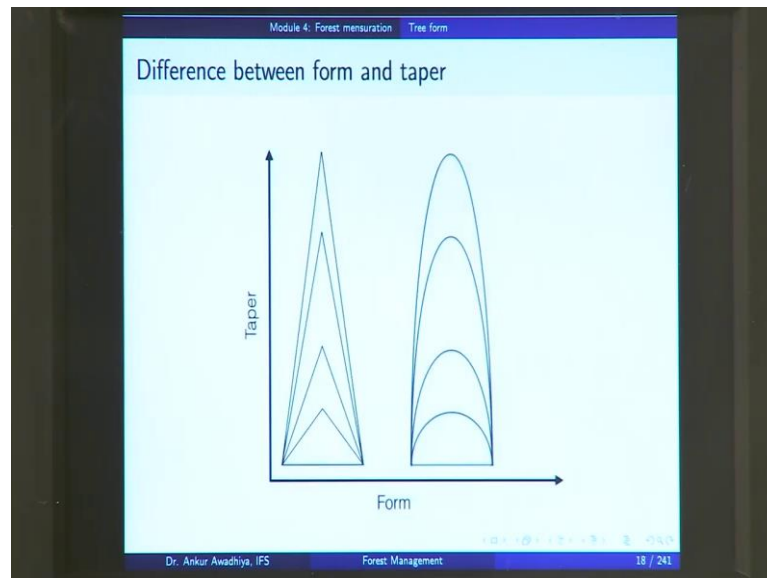
So, for instance, in the case of these trees, here you can see that the bole is roughly cylindrical in shape. So, there is hardly any taper; taper is very close to 0, because when you are increasing in height as you go up the tree, there is hardly any change in the diameter.

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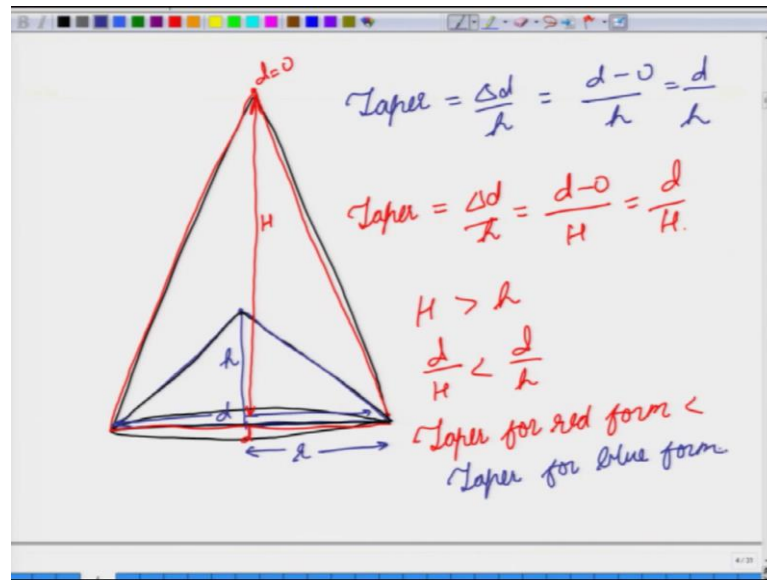
On the other hand, for this tree, the taper is a bit larger because here you can see that the stem at the bottom has a larger diameter, and as you go up the diameter reduces. Now, taper and form are two different things.

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So, you should not confuse between taper and form. Form refers to the shape of a solid. So, for instance, in this case, you have a conical solid, and in this case, you have a paraboloid solid. So, these are the two different forms, whereas taper would refer to the change in diameter divided by the change in the height. So, in this case, for this small height, you are seeing a much greater change in diameter as compared to this one.

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So, what we are saying here, is that you have these two solids with the same form. So, both of these solids are cones. Now, in the case of this blue solid, and let us say that this is the diameter  $d$  at this point, and this match is the height  $h$ . So, you have the taper for the blue solid is given by the change in diameter. So, as you increase by a height  $h$ , you have the  $\Delta d$  is given by  $d$  minus  $0$  because here you have a diameter of this is the radius this is the diameter.

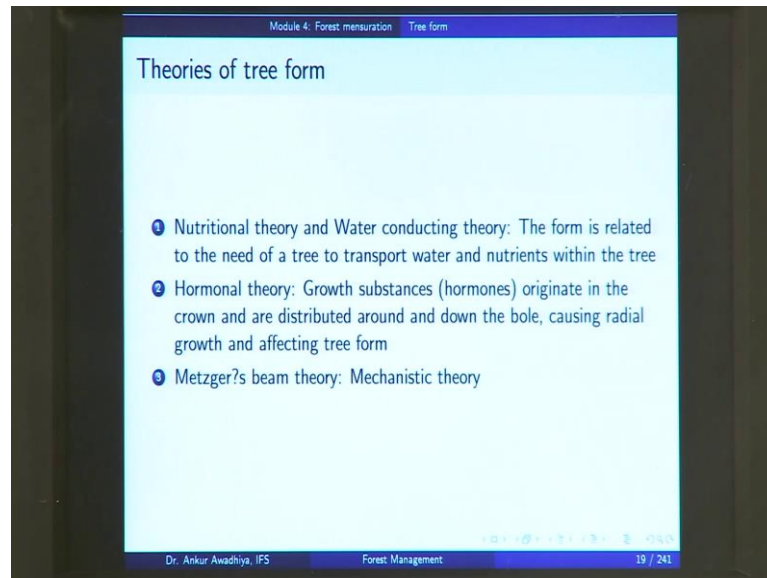
So, here you have a diameter of  $d$ , and at this point you have a diameter of  $0$ . So, you have  $d$  minus  $0$  divided by  $h$  is equal to  $d$  divided by  $h$ , whereas for this red solid, it was called it as capital  $H$ . You will have the taper which is given by  $\Delta d$  by  $h$ , and here the diameter is changing from here you have a diameter of  $d$  and at this point you have a diameter of  $0$ .

So, you have  $d$  minus  $0$  divided by the height is given as  $H$ , is equal to  $d$  by  $H$ . now, because  $H$  is greater than small  $h$ ; So,  $d$  by  $H$  is less than  $d$  by small  $h$ . So, we can say that the taper for the red form is less than taper for the blue form, and this is what we are representing in this figure as well. So, here the taper in this solid is much greater than the taper in this solid, and if we continue it till infinity, then this will roughly resemble a cylinder in place of a cone.

Then the next question is, how do we explain these forms? So, the question is why does a tree have a taper? Why does a tree have this particular form, in which the bottom portion

is the thrust is a frustum of neiloid, the central portion is the frustum of a paraboloid, and the topmost portion is a cone? Why do we have this particular shape of a tree?

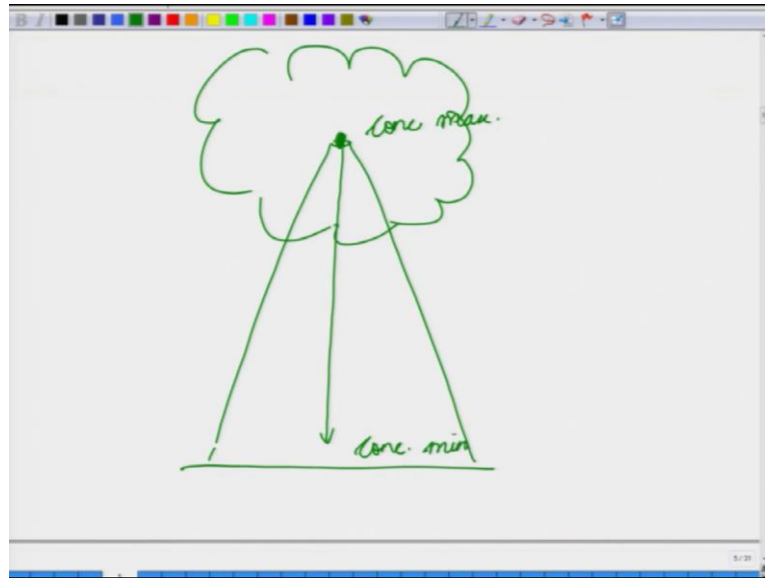
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So, there are three different theories of tree form that have been devised to explain these shapes. The first one is known as the 'nutritional theory' and or the 'water conducting theory.' So, this theory see is that the form is related to the need of a tree to transport water and nutrients within the tree. And, this theory says that the form of the tree has been derived through evolution, in such a manner, that it optimizes the ability of the tree to conduct water and nutrients throughout its body.

The second theory is the 'hormonal theory' which states that the growth substances or hormones originate in the ground, and then they are distributed around and down the bole, which causes the radial growth and affects the tree form.

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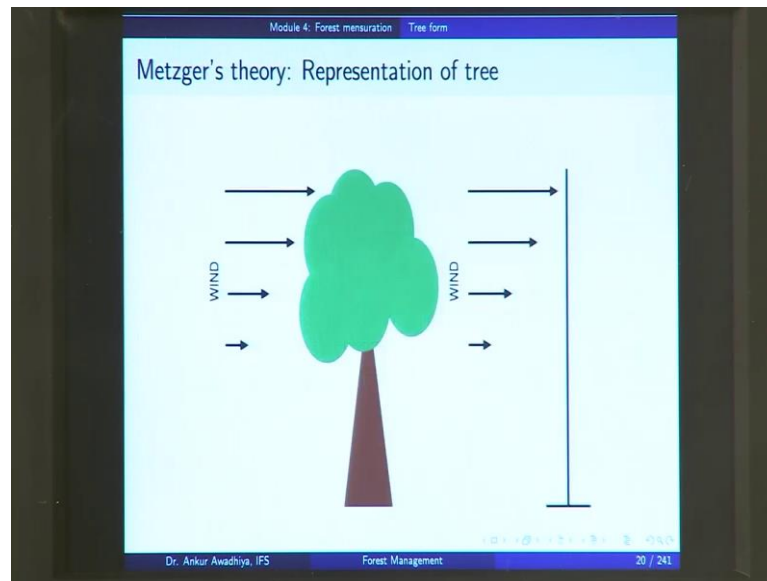


That is what it is saying is that, if you consider a tree, this is the bole of a tree, here you have the crown. So, it says that the hormones originate in the ground. So, here you have the highest concentration and then they move downwards. So, the concentration here is the maximum and the concentration here is the minimum.

And, when you have a maximum concentration of this particular hormone, the radial growth is less. When you have less of this concentration, then the radial growth is more. So, the hormonal theory says that you are having a hormone that is being generated or originated in the ground, then it is distributed around and down the bole, and this sort of inhibits the radial growth of the bole.

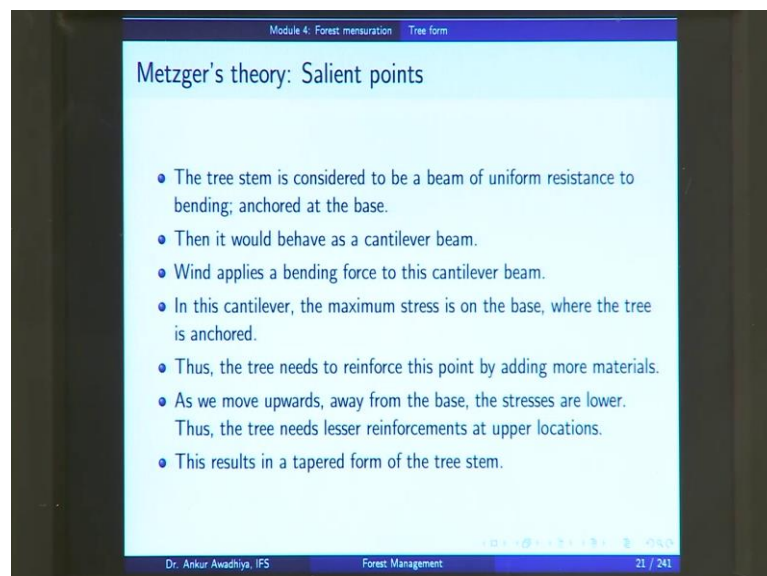
So, if you have it in a lower concentration, you will have more radial growth. And, the third theory is a mechanistic theory that is known as the 'Metzger's beam theory' which is the one that is the most prevalent.

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Now, the Metzger's beam theory represents a tree as a straight line. So, here you have a tree, and in this theory, we represent it as a straight vertical line, and here is the ground and because any tree will be facing some amount of wind pressure. So, we are representing that wind here in this tree as well.

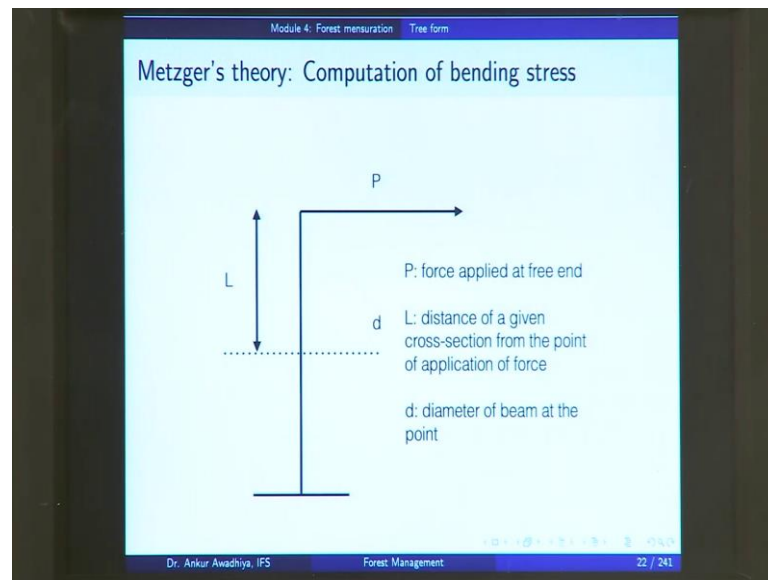
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So, how does Metzger's theory explain the form of the tree? In Metzger's theory, the tree stem is considered to be a beam of uniform resistance to bending which is anchored at the base. So, it says that the material of this beam is the same whether you consider the

bottom or the top portion, and so, the resistance that this material can offer is uniform across the whole of the stem. So, this is a beam with uniform resistance to bending, and this beam is anchored at this point. So, in this case, this beam behaves as a cantilever beam.

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So, a cantilever beam is a beam that is anchored at one point, and the other point is free at which you are applying some particular force. Now, this force, in the case of Metzger's theory, is the force of the wind. So, wind applies a bending force to this cantilever beam. So, this is the force, we represent it as  $P$ .  $P$  is the force that is applied by the wind, and it is applied at the top portion of this beam. Now, in this cantilever, the maximum stresses on the base where the free is anchored.

So, when you are applying this force at this point, the stress is maximum at the base. And so, there is a greater chance that this tree will break at this particular point, and thus that tree has a need to reinforce this point by adding more materials. So, because you cannot change the material, the material is uniform; what you can do is that you put more material here at the bottom where the stress is more, and you require less material here. So, you deposit less amount of material on the top.

Now, as we move upwards away from the base, the stresses are lower, and thus, the tree needs lesser reinforcements at the upper locations. So, as we move up, the stresses are

lowered, and the amount of material that is deposited at any place is proportional to the amount of stress that is there at that particular place, and so, if there are lesser stresses then you have a lesser amount of material that is deposited, and this results in a tapered form of the trees.

So, if we put it up mathematically; so, this is our tree, we are applying a force P, because of the wind. So, P is the force applied at the free end. L is the distance of any given cross section from the point of application of force. So, this is L, when at the bottom portion you have L is equal to the height of the tree, at the topmost portion L is equal to 0, and d is the diameter of the beam at the point.

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Module 4: Forest mensuration Tree form

### Metzger's theory: Computations

The bending stress is given by

$$S = \frac{32P.L}{\pi.d^3}$$

P is given by

$$P = W \times A$$

where W = wind pressure per unit area  
A = crown area

Thus ,

$$S = \frac{32P.L}{\pi.d^3} = \frac{32W.A.L}{\pi.d^3}$$

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Now, with this formulation, if we compute the bending stress, then the bending stress at any point will be given by,

$$S = 32 \times P \times L / (\pi d^3)$$

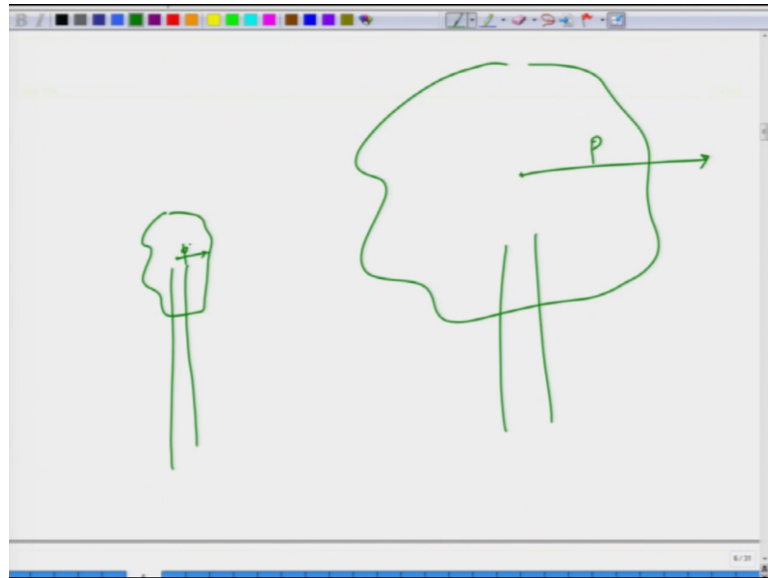
where P is the force of the wind L is the distance from the top, where d is the diameter of the stem at that particular point.

Now,  $P = W \times A$ ,



where wind where  $W$  is the wind pressure per unit  $A$  and is the crown area. So, if you have a larger sized crown, you will have more amount of force that is being applied.

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So, if you consider two trees, and this has a larger sized crown and this has a smaller sized crown. So, the amount of force that comes up in this capital  $P$  will be much greater than the small  $p$  that we have in this place. So,

$$S = 32 W \times A \times L / (\pi d^3)$$

So, this is the equation that we get.

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Module 4: Forest mensuration Tree form

### Metzger's theory: Computations

$$S = \frac{32P.L}{\pi.d^3} = \frac{32W.A.L}{\pi.d^3}$$

Since the material is considered homogeneous, S is constant.  
Thus,  $d^3 = k \times L$  (Equation for a cubic paraboloid)  
Hence, as per Metzger's theory, the shape of a tree is given by the equation for a cubic paraboloid:

$$d^3 = k \times L$$

Metzger confirmed this for many stems, particularly of conifer species.

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Now, since the material is considered to be homogeneous; so, remember we said that the material is uniform, it does not change as we move along the tree. Now, since the material is uniform; so, S is constant, because the same amount of bending stress will be there at every point. Because the tree is trying to put in materials or deposit materials in different places; so that the amount of bending stress is the same at all the locations.

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Module 4: Forest mensuration Tree form

### Metzger's theory: Computations

$$S = \frac{32P.L}{\pi.d^3} = \frac{32W.A.L}{\pi.d^3}$$

Since the material is considered homogeneous, S is constant.  
Thus,  $d^3 = k \times L$  (Equation for a cubic paraboloid)  
Hence, as per Metzger's theory, the shape of a tree is given by the equation for a cubic paraboloid:

$$d^3 = k \times L$$

Metzger confirmed this for many stems, particularly of conifer species.

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So, S is a constant. So, what we are getting here.

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The image shows a whiteboard with handwritten mathematical equations. At the top, the equation  $S = \frac{32 PL}{\pi d^3} = \frac{32 \cdot W A L}{\pi d^3}$  is written. Below this, the word "constant" is written under the  $S$  term. The next line shows the rearranged equation  $\Rightarrow S = \frac{32 W A L}{\pi d^3} \Rightarrow d^3 = \frac{32 W A L}{\pi S}$ . The terms  $32 W A L$  and  $\pi S$  in the denominator are circled. The final equation is  $\Rightarrow d^3 = k \cdot L$ , which is underlined.

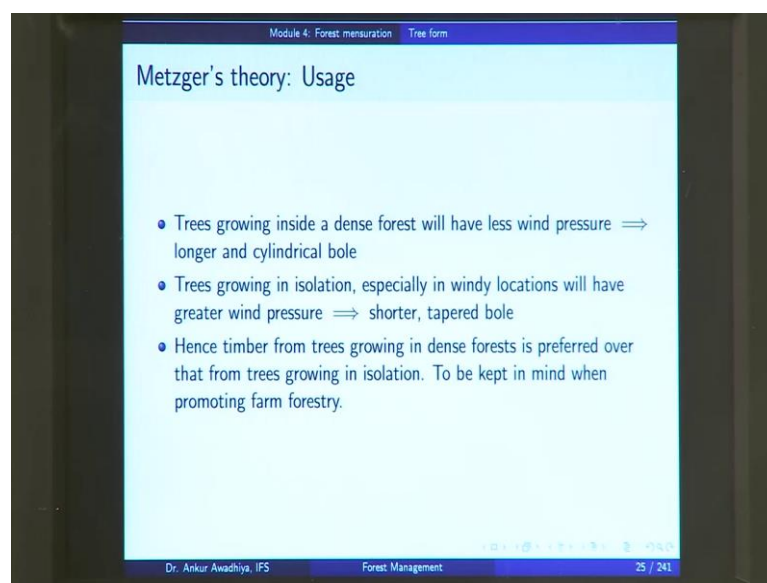
So, we said that  $S$  is equal to  $32 P L$  by  $\pi d$  cube, which is equal to  $32$  into  $W A L$  divided by  $\pi d$  cube. Now,  $s$  is a constant. So, in this case, we will have that  $S$  is equal to  $32 W A L$  by  $\pi d$  cube, which will give you that  $d$  cube is equal to  $32 W A L$  divided by  $\pi$  times  $S$ . Now,  $32$  is a constant;  $\pi$  is a constant;  $S$  is a constant; the  $A$  which is the area of cross section of the vertical cross section of the crown is a constant; the amount of wind pressure that we are having in a place is constant.

So, we can write that  $d$  cube is equal to  $k$ , which is a constant. So, what we are getting here is that  $d$  cube is equal to  $k$  times  $L$ , which is the equation for a cubic paraboloid. Now, hence as per the Metzger's theory, the shape of the tree is given by the equation for a cubic paraboloid or  $d$  cube is equal to  $k$  times  $L$ , and Metzger's confirmed this for many steps particularly of the conifer species.

Now, the Metzger's theory is able to tell us that the form of a tree can be represented as a cubic paraboloid, which is given by the equation that we saw just now. So, it can explain that in the case of a tree, we why do we see a taper, but then there are also a few drawbacks because as we saw before. The shape of the tree is not completely given by the shape of a cubic paraboloid. In fact, what we have seen is that the lower portion is a truncated neiloid, the middle portion is a truncated paraboloid; even this is not a truncated cubic paraboloid, and the top portion is given by a cone.

So, while the Metzger's theory is able to explain to some extent the form of a tree, but it is not completely able to explain why these different portions are having the different equations of the form. Now, one point to consider here is that while Metzger's considered that the beam has a that this beam is made of a uniform material. In actual case, the plants have an option to deposit different materials at different places. So, this is one drawback of the Metzger's theory, but still this theory is able to explain the form of a tree to quite an extent.

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Now, how do we make use of this theory? Now, some practical applications are, for a tree that is growing inside a dense forest, you will have less pressure. And so, you will have a longer and a cylindrical bole, because the amount of wind pressure is less. So, there is little difference between the bending stress that we have at the bottom and the bending stress that we have at the top.

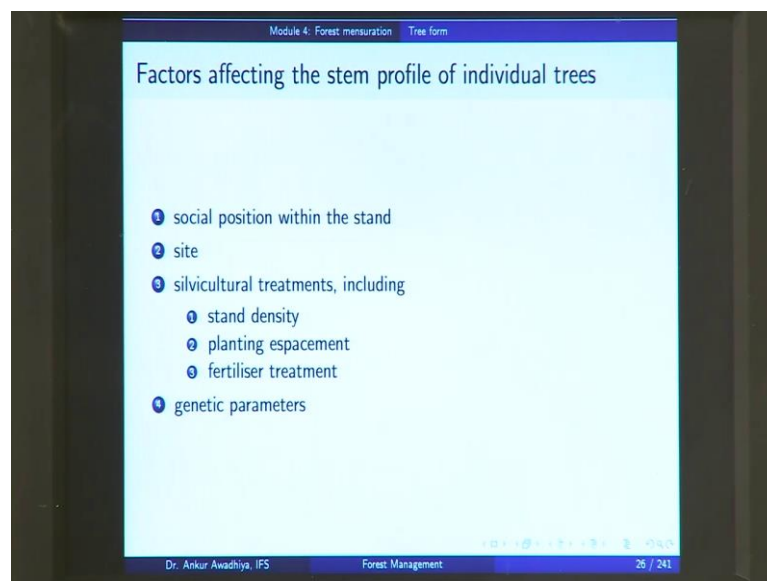
And so, in this case, the Metzger's theory would predict that the tree would deposit similar amounts of materials at the bottom as well as at the top, in which case, you will have a cylindrical bole with a very less amount of taper. Whereas, if you consider a tree that is growing in isolation, especially in many locations, they will have greater amount of wind pressure, and so, there will be a short and tapered bole. Because in this case, the amount of bending stress that you will have in the case of a cylindrical bole will be different.

And so, to equalize the bending stresses the tree will put up more materials in the bottom and less materials at the top. At the same time, these trees will be shorter in height because there is a limit to which your tree will be able to deposit the materials. And, because the wind forces less, so, it makes much more sense to have a shorter height of your cantilever beam.

And so, we can say that trees that are growing in dense forests are preferred over that from trees growing in isolation. Because, in the case of a tree in a dense forest, you have a long and roughly cylindrical bole. So, it is easy to work with, Whereas, in the case of a tree that is growing in isolation, you have a much more tapered bole, and so, there is a lot of wastage when you want to work that material, and so, people generally prefer trees that are growing in the dense forest.

So, when we are promoting farm forestry, which is the growing up of silvicultural trees in the farmlands as well; so, this is something that needs to be kept in mind that the amount of profits that people might receive could be lesser than the amount of profits that they will receive, if it was a completely dense stand. And so, we have to make some policy interventions to ensure that the people who are growing trees in a farm are also able to gain adequate amount of remuneration.

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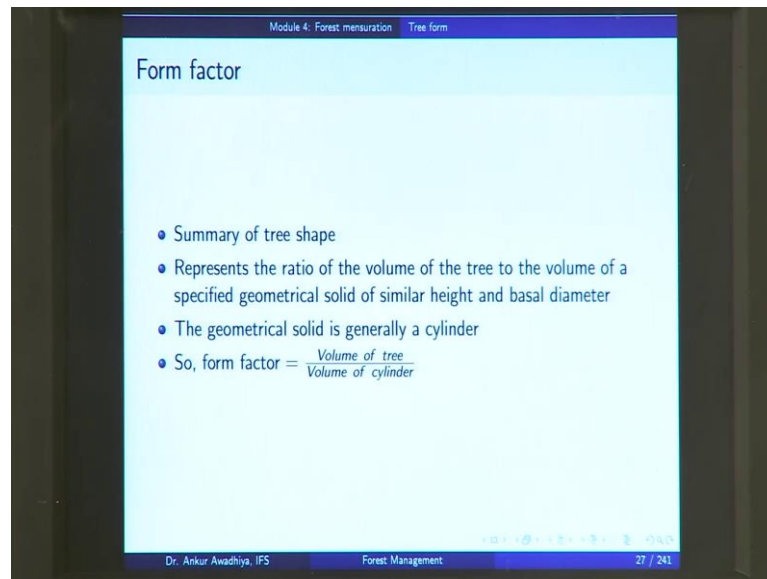
So, what are the factors that affect the stem profile of individual trees? So, as we saw, that the position within the stand can determine to a large extent, the profile of individual trees. If you have a tree that is inside a stand, which is in a centralized location. So, the amount of wind pressure that will that it will face will be lesser than the amount of wind pressure that the other trees that are at the periphery would be facing.

So, social position within the stand, is a factor that affects the stem profile of individual trees. the site or the site quality also affects the stem profile. Because, if you have a site that is fertile, so in that case, there will be enough amount of nutrients that are available for your tree to put up its height. Whereas, if you have a site that does not have good amount of nutrients for the plant, then your trees will not be able to grow to a large extent, and so, you will be having shorter trees in a poorer site quality.

Then, silvicultural treatments including the stand density, so, if you have a greater density, in that case, you will be having trees that are having lesser taper. And, probably trees that are having a greater height, because the surrounding trees are competing with it for sunlight. And, in this case, the tree will just grow up and up, so that it is able to gain the maximum amount of sunlight.

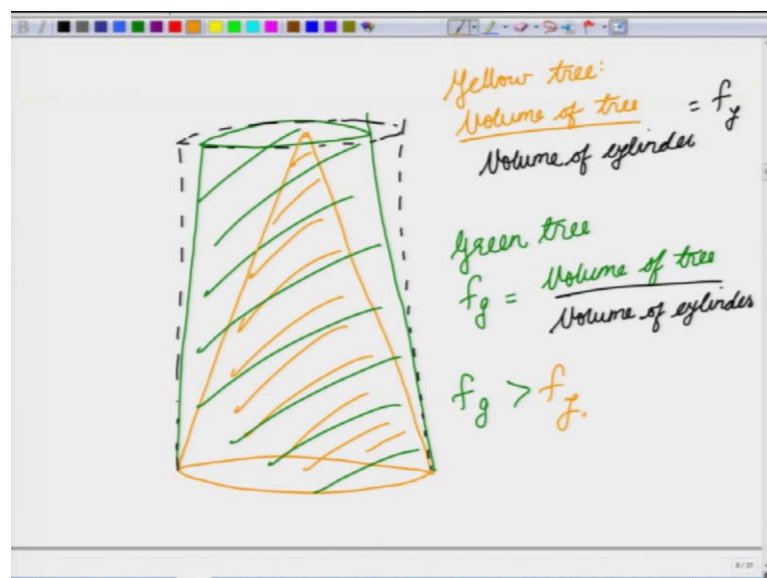
The plantings basement and the fertilizer treatment, which in turn provides the amount of nutrients that your tree requires to grow. Now, stem profile also depends on genetic parameters, which is something that Metzger did not consider in his theory. So, if you have trees that have a very cylindrical bole, and if you consider the progeny of those trees, then they have also probably will be having much more cylindrical boles as compared to other trees, because it is there in their DNA.

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Now, when we talked about the form of a tree, but then if you require a single number with which you can represent the form of a tree, then that number will be called as the 'form factor'. So, form factor is a way of summarizing the tree shape. It represents the ratio of the volume of the tree to the volume of a specified geometrical solid of similar height and the basal area, which is generally a cylinder.

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So, what we are saying here is that suppose, you consider a tree which is a taper tree, and then you consider another tree, which has a much lesser taper. And, both these trees are

of the same height, then the form factor would represent, would be a measure of, what is the volume of your tree to the volume of a cylinder of the same height. So, in the case of the yellow tree, the volume of tree divided by the volume of cylinder.

Let us say that this is  $f$  for the yellow tree. And, in the case of the green tree, the  $f$  for the green tree is given by the volume of tree divided by the volume of cylinder. Now, as we can see the volume of the yellow tree - this one is much less than the volume of the green tree, which is this much. So here, we will find that  $f_g$  is greater than  $f_{yellow}$ . So, in this case, what we are saying is that even though you have these two trees with very different shapes, we can use the form factor as just a single number to give you an idea of how much is the taper of the tree.

Now, this is very important because, in the case of silvicultural operations, we are we are very much interested in the volume of timber that we get out of a tree. Now, if there is a tree that has a huge amount of taper, in that case, the amount the volume of timber that we will get will be much lesser than a tree that has a very less amount of taper and has a cylindrical bole, considering both these trees are of the same diameter and the same height.

Now, we have defined the form factor as the volume of the tree divided by the volume of the cylinder, with the same height and the same diameter, but now the next question is where do we measure this diameter?



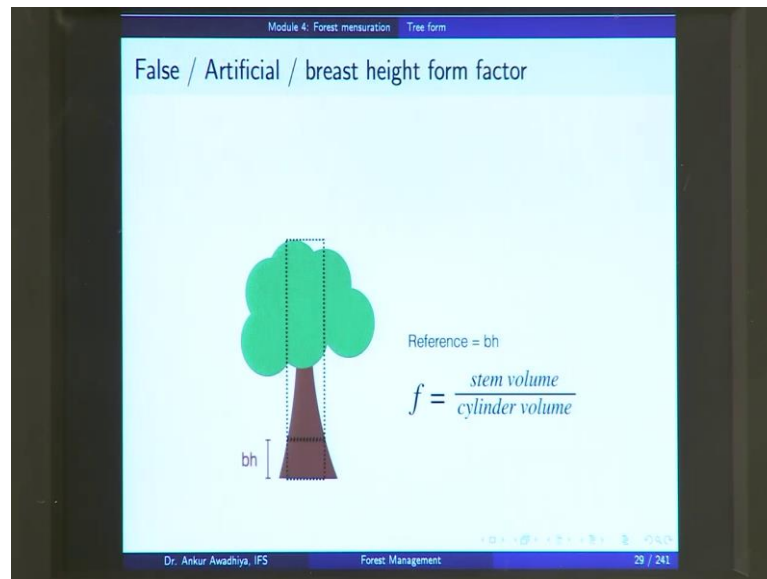
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The slide is titled "Absolute form factor" and is part of "Module 4: Forest mensuration" and "Tree form". It features a diagram of a tree with a brown trunk and green canopy. A dashed rectangular box is drawn around the trunk, indicating the reference point for the cylinder volume calculation. To the right of the tree, the text reads "Reference = base of the tree" and the formula  $f = \frac{\text{stem volume}}{\text{cylinder volume}}$ . The slide footer includes "Dr. Ankur Awadhya, IFS", "Forest Management", and "28 / 241".

So, depending on where we are measuring the diameter, there can be different form factors. The simplest of which is the absolute form factor. Now, absolute form factor takes the diameter as the base of the tree, and this is absolute because this is measuring absolutely the whole volume of the of the tree bole. So, we have the stem volume divided by the cylinder volume, where you have the base at this point.

Now, the issue with an absolute form factor is that, because at the base of the tree, you have earth; you might even be having some stones. So, it is difficult to take the diameter at that particular point. So, even though absolute form factor theoretically will give you the a very good idea about the about the form of the tree, but in practice it is difficult to take the diameter at the base of the tree.

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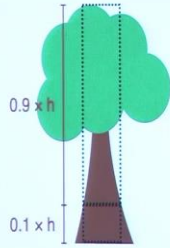
So, we devised another form factor which is known as the false form factor or the artificial form factor or the breast height form factor. Now, in this case, we take in the measurement at the breast height. Now, breast height is defined as 1.37 meters, and if you stand up in front of a tree; So, this height the height of your hands will roughly be the breast height.

So, in this case, you can just use calipers or you can use tape, and get the diameter or the girth of your tree at this height, and it is also very easy to see. So, this is a form factor that is mostly used, but because it is taking the height as the breast height or 1.37 meters, which is not a natural value, it is something that we have devised out of our convenience. So, it is known as an artificial form factor or the false form factor or the breast height form factor.

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Module 4: Forest mensuration Tree form

### True / Normal form factor



0.9 x h

0.1 x h

Reference = 10% of height

$$f = \frac{\text{stem volume}}{\text{cylinder volume}}$$

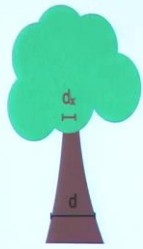
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Now, there is also another form factor which is known as the true form factor or the normal form factor. Now, in the case of the normal form factor, the reference is taken as 10 percent of the tree height. So, if this is h, this is 10 percent of h. So, you will measure your stem volume, the volume of your tree, and you will divide it with the cylinder with the height of h, and diameter taken at this particular point. So, this is the true or the normal form factor because you have normalized it for trees of different heights.

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Module 4: Forest mensuration Tree form

### Form quotient



$d_x$

$d$

A single number depicting the rate of decrease in stem diameter: a ratio of diameter at two different places on the tree

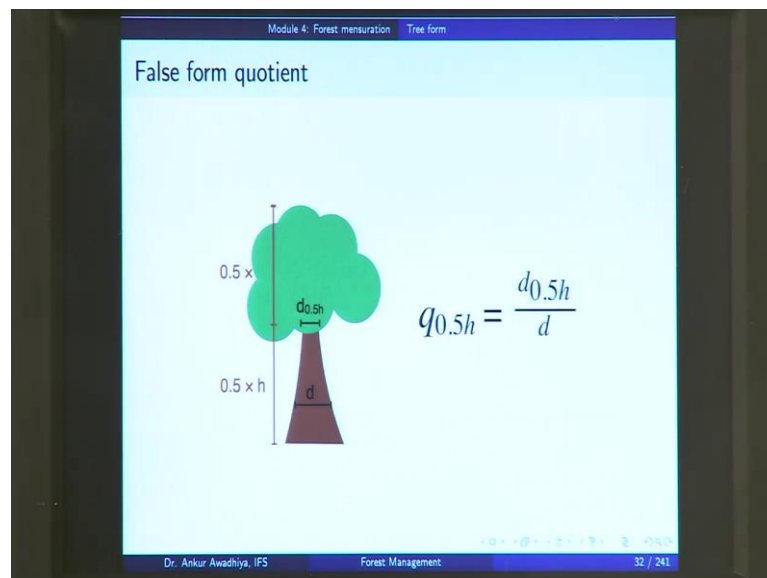
$$q = \frac{d_x}{d}$$

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Now, another value with which we can represent the form of a tree is known as the form quotient. The form quotient is a single number depicting the rate of decrease in stem diameter or the taper as a ratio of diameter at two different places on the tree. So, we are taking diameter at an upper location, diameter at a lower location, and we define form quotient as diameter at the upper location divided by diameter at the lower location.

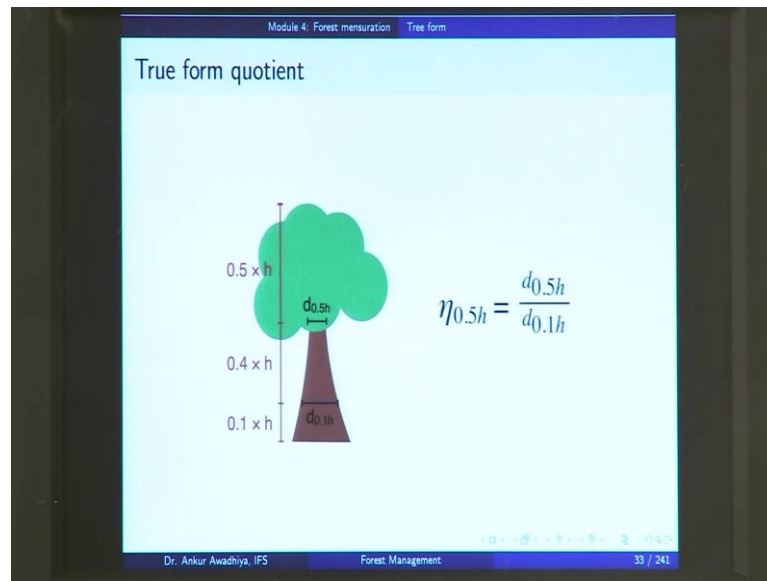
So, this can also be used to discern the form of a tree, if you have a tree with a great amount of taper. So,  $d$  at this point,  $d$  at  $x$  will be very small, and so, the form quotient will also be small. Whereas, in the case of a cylinder the form quotient will be greater. So, now, the next question is where do you measure this  $d$ , and where do you measure there the  $x$ ?

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So, here we define the form the false form quotient as, the diameter at 50 percent height divided by the diameter at the breast height.

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So,  $q_{0.5h}$  or the form quotient or the false form quotient is given by, the diameter at 0.5 h divided by the diameter at the pressed height. And, we also have the true form quotient that is given by eta, and eta is given as diameter at 50 percent height divided by diameter at 10 percent of height.

So, in this case, we are not using the breast height which is an artificial value, and we are normalizing it, for any tree as 10 percent of the height. So, you made a diameter at 10 percent of the height, you measure diameter at 50 percent of the height. And, diameter at 50 percent height divided by diameter at 10 percent height will give you the true form quotient.

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Module 4: Forest mensuration Tree form

## Problem

For a *Pinus patula* tree with dbh = 45.6 cm, height = 27.4 m and total stem volume of 1.782 cum, the bole diameters at various heights are given in the next slide. Find the

1. True form quotient
2. False form quotient
3. True form factor
4. False form factor

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So, let us now look at how we compute all of these in real life situations. So, this is the problem for a *Pinus Patula* tree with diameter at breast height is equal to 45.6 centimeter.

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$d = dbh = 45.6 \text{ cm}$   
 $h = 27.4 \text{ m} \rightarrow 0.1h = 2.74 \text{ m}$   
 $0.5h = 13.7 \text{ m}$   
Stem volume = 1.782 cum.

$d_{0.1h} = d_{2.74 \text{ m}} = 40.2 \text{ cm}$   
 $d_{0.5h} = d_{13.7 \text{ m}} = 30.7 \text{ cm}$

False form quotient  $\frac{d_{0.5h}}{d} = \frac{30.7 \text{ cm}}{45.6 \text{ cm}} = 0.673$   
True form quotient  $\frac{d_{0.5h}}{d_{0.1h}} = \frac{30.7 \text{ cm}}{40.2 \text{ cm}} = 0.76$

So, we have  $d$  or  $dbh$  is given by 45.6 centimeters. The height of the tree is given as 27.4 meters.  $h$  is equal to 27.4 meters, and the total stem volume as 1.782 cubic meter.

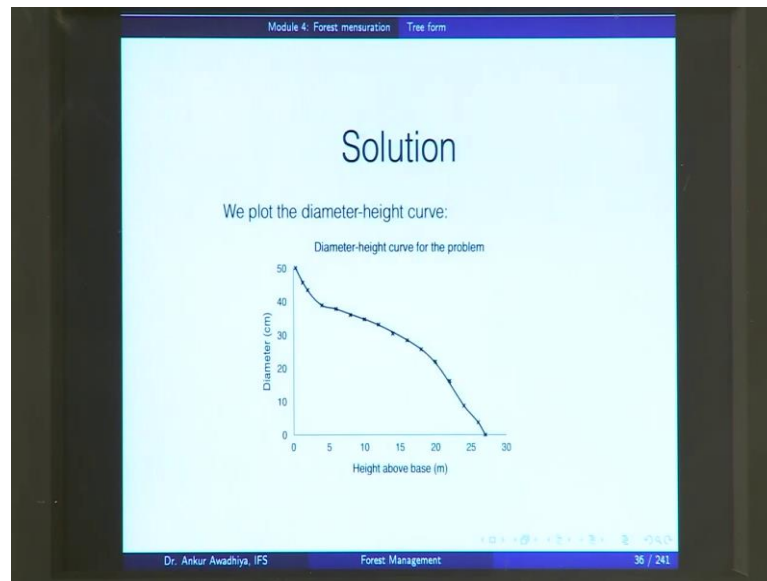
So, you have the stem volume is equal to 1.782 cubic meter. Now, the bole diameters at various heights are given, and we have to find out the true form quotient, the false form quotient, the true form factor, and the false form factor.

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Height (m)	Diameter (cm)	Height (m)	Diameter (cm)
0.3	50	14	30.3
1.3	45.6	16	28.3
2	43.4	18	25.6
4	38.8	20	21.9
6	37.7	22	16.1
8	35.9	24	8.7
10	34.6	26	3.7
12	33	27	0

Now, in this case, we are giving the height at different points and the diameter at those points. So, at a height of 0.3 meters, you have a diameter of 50 centimeter. As you go up, at 1.3 meters you have 45.6 centimeter and so, on. And, at the height of 27 meters you have diameter that is roughly equal to 0, because the height of your tree is 27.4 meter. So, how do you compute the form factors and the form quotients?

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So, we can begin by plotting the diameter height curve of this tree. So, on the y axis you will have the diameter, on the x axis you have the height. So, in this case, if you plot these points, you get a curve like this. Now, remember the top portion looks like a cone, and this portion will look something like a paraboloid. Now, in this case, the height is 27.4 meters, which will give you that  $0.1 h$  is given as 2.74 meters.  $0.5 h$  is given as half of this that is 13.7 meters.

Now, we want to find out the diameter at  $0.1 h$ , the diameter at  $0.5 h$ . and, we can use this curve to find out these diameters. So, the diameter at  $0.1 h$  is the diameter at 2.74 meters. So, at 2.74 meters roughly at this point the diameter would be roughly this value, which is 40.2 centimeters. Diameter at 2.74 meter; you draw a straight line and you get the value of the diameter at this point at this height. Now, the diameter at  $0.5 h$  is equal to the diameter at 13.7 meters.

Now, what is the diameter at 13.7 meters? So, you take this point 13.7 meters; Draw a straight line, and this is the diameter that you will get. So, in this case, it comes out to be 30.7 centimeters. And, the diameter at the breast height we already know. So, what is the form quotient? Now, you have the false form quotient  $q$   $0.5 h$  is given by  $d$   $0.5 h$  divided by  $d$ , is equal to 30.7 centimeters, divided by  $d$  is 45.6 centimeter, and is equal to 0.673. So, this is the false form quotient.



Next, you have the true form quotient, which is given as  $\eta$  at 0.5 h; is defined as  $d$  at 0.5 h divided by  $d$  at 0.1 h. Now,  $d$  at 0.5 h this value is 30.7 centimeters divided by  $d$ , at 0.1 h is 40.2 centimeters, this comes to be 0.76.

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Vol of tree = 1.782 cum.

False form factor =  $\frac{V}{V_f} = \frac{1.782}{4.47} = 0.399$

$d = dbh$   
 $h = h$

$V_f = \frac{\pi}{4} d^2 h = \frac{\pi}{4} \times \left(\frac{45.6}{100}\right)^2 \times 27.4m$

$\Rightarrow V_f = 4.47 \text{ cum.}$

True form factor =  $\frac{V}{V_t} = \frac{1.782}{3.48} = 0.512$

$V_t$   $d = 40.2 \text{ cm}$   $V_t = \frac{\pi}{4} d^2 h = \frac{\pi}{4} \times \left(\frac{40.2}{100}\right)^2 \times 27.4$   
 $h = 27.4m.$   $= 3.48 \text{ cum.}$

Next, we want to measure the volume. So, you have the volume of the tree now this volume is given as 1.782 cubic meters. Now, in the case of the false form factor, it is given by volume of the tree divided by the false volume. Now, this false volume is the volume of the cylinder with diameter is equal to  $d$  bh, and height is equal to the height of the tree. So, the volume of a cylinder is  $\pi$  by 4  $d$  square  $h$ .

So, this is  $\pi$  by 4  $d$  we have it as 45.6 centimeters. So, 45.6 centimeter you convert it into meters divided by 100 into height of the tree, is given as 27.4 meters. So, you have the false volume is given by when you do this computation; you have it as 4.47 cubic meter. So, the false form factor will be given by  $V$  or the volume of tree is 1.782 divided by 4.47 is equal to 0.399. Now, remember that the false form factor is a dimensionless quantity because you have a volume divided by another volume.

Next, have a look at the true form factor. The true form factor is given by  $V$  divided by  $V_t$ . Now,  $V$  is now  $V$  is 1.782; and  $V_t$  is the volume of a cylinder with  $d$  is equal to 0.1 h this one 40.2 centimeters. 40.2 centimeters in height is equal to the height of the tree which is 27.4 meters.

So, the true volume is given by  $\pi \times d^2 \times h$ , is equal to  $\pi \times 4 \times d$  here is 40.2, divided by 100 meter into height is 27.4. So, this comes to be 3.48 cubic meter, and so, the true form factor is 1.782 divided by 3.48, is 0.512. The true form factor again is a dimensionless quantity, because you are dividing a volume with and by another volume.

So, in this lecture, we had a look at what is the form of a tree, how does a tree look like what is the shape of a tree. So, form and shape are the same thing, and we saw that if you take a tree, and you plot its height versus diameter curve. So, the top portion will look like a cone, the middle portion will look like a paraboloid, the bottom portion will look like neiloid.

Next, we ask this question, why does a tree look like this? So, we had three theories of tree form - the first one is the water conduction or the nutritional theory, which says that this is the form that optimizes the flow of water and nutrients throughout the bole of the tree. So, it looks at it from a biological point of view.

The second theory is the hormonal theory which says that you have some hormone that is generated at the crown of the tree, and then this hormone moves down the bole, and is then distributed at other parts. And, if you have more amount of this hormone at the top, so, you will be having a lesser diameter. If you have lesser amount of the hormone at the bottom, you will have a greater diameter.

And, the third theory was the Metzger's, a mechanistic theory, which considered that your tree is a cantilever beam which is anchored at the base, and this tree is made up of uniform materials, and so, the amount of bending stress that this tree has, so, the stress is greatest at the bottom and the stress is the minimum at the top. But at any point you can compute this stress, and because this material is uniform, and this tree has to preserve itself in the face of a wind pressure. So, it deposits more material at the bottom, and it deposits lesser amount of material at the top, so that, it is able to resist this wind force both at the bottom and at the top and in between as well.

So, if you have this wind pressure, you can compute the form of the tree by using mathematical equations. And, it said that the form of a tree is a cubed paraboloid. But then, as we have seen that in the case of a tree, it is not made up of just one material. The material is not uniform, and so, in actuality though it is able to predict why we have a

taper, but it is not completely able to give us the correct form of the tree, but still it is a good approximation.

Then, we saw that one usage of this theory is that if you have a tree that is standing as a single tree the amount of wind pressure will be so high that it has to have a greater amount of taper. Whereas, if you have a tree that is surrounded by the trees say in the centre of a stand. So, in that case, the amount of wind pressure that this tree will face will be very less.

And, in which case there will be a lesser need to deposit more materials at the bottom and less when you will set at the top, and so, in that case you will have a straight bole. You will have a cylindrical bole of that particular tree. Now, when we are you - when we are doing silviculture to harvest trees or to harvest timber, in that case, we prefer a cylindrical bole because it is easy to work with it is uniform everywhere. So, if you cut it any it in at any place, the diameter will be the same.

So, those trees are preferred, whereas trees which have a very great amount of taper are not that much preferred. So, this is practical applications. In a way that if you have a stand, the tree is at the at the outer periphery will show you amount of taper as compared to the trees at the center of the stand. And, those trees that are growing individually say in a field, because of farm forestry, they will be having a much greater amount of taper, and so, much lesser prices as compared to trees that are being grown naturally in the forest.

Then, we looked at those values through which we can describe this tree form in a single measure. So, we looked at two things. We looked at form factors and the form quotients. Now, form factor is the volume of the tree divided by the volume of a cylinder with the same diameter and the same height. So, the height of a tree is a constant, but then where do we measure the diameter.

So, depending on whether we are or depending on where we are measuring the diameter, if we measure it at the base of the tree, this is the tree anchored here; you are measuring it at the base you get the absolute form factor. If you are measuring it at the breast height, you get the false form factor or the breast height form factor, and if you measure the diameter at 0.1 height, in that case, you will be having the true form factor.

Now, another value through which we can represent the form of a tree is the form quotient. And, a form quotient is defined as the diameter at a point on the top of the tree and divided by the diameter at a point at the bottom of the tree. Now, the diameter at the top is generally taken to be diameter at 50 percent height of the tree. And, if you take the bottom diameter as the diameter at breast height, then you will get the false form quotient.

And, if you take the diameter at 0.1 of the height, and you get the true form quotient. And, then we looked at how do we compute these values actually in the field. So, if you want to measure these values for any tree, you take the diameter and the height readings throughout the bole. You plot them up and then you get all these values of  $d - 0.1 h$ ,  $d - 0.5 h$ , and then use these values to compute your form factors and the form quotients. So, that is all for today.

Thank you for your attention [FL].