Introductory Mathematical Methods for Biologists
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Lecture – 09
Rules for Calculating the Derivatives

Hai, welcome to this lecture on mathematical methods for biologists, we have been learning; how to compute derivatives we learned how to compute derivatives of finding slopes of simple functions like x y is equal to z x and y is equal to x square and even y is equal to x cube and we found that finding delta y by delta x for small value of delta x is the local slope right. It is the local slope, locally, we can find the slope and that is the derivative and while we do this whenever we do in this we should always think why are we doing this why are we learning to find derivative why are we learning this calculus differentiation.

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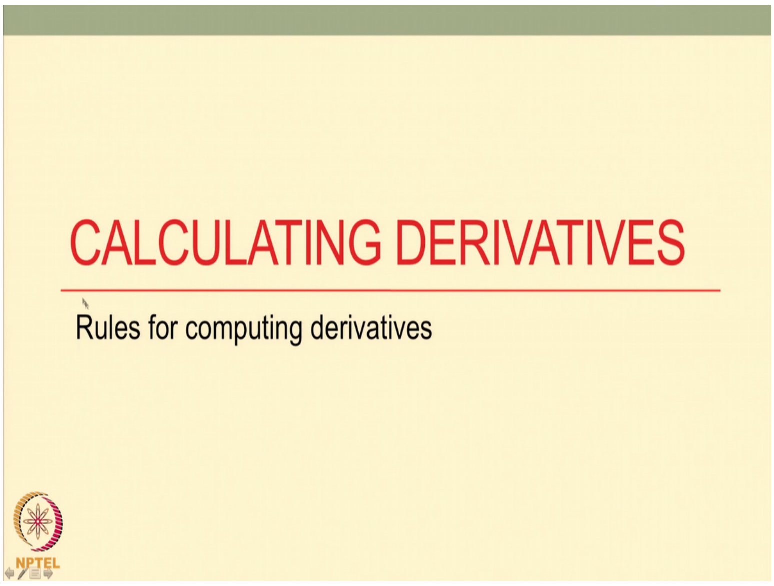

So, why this is something that we will think about it today. So, while today's topic is calculating derivatives and we will discuss rules for computing derivatives. So, today's topic would be calculating derivatives and rules for computing derivatives.

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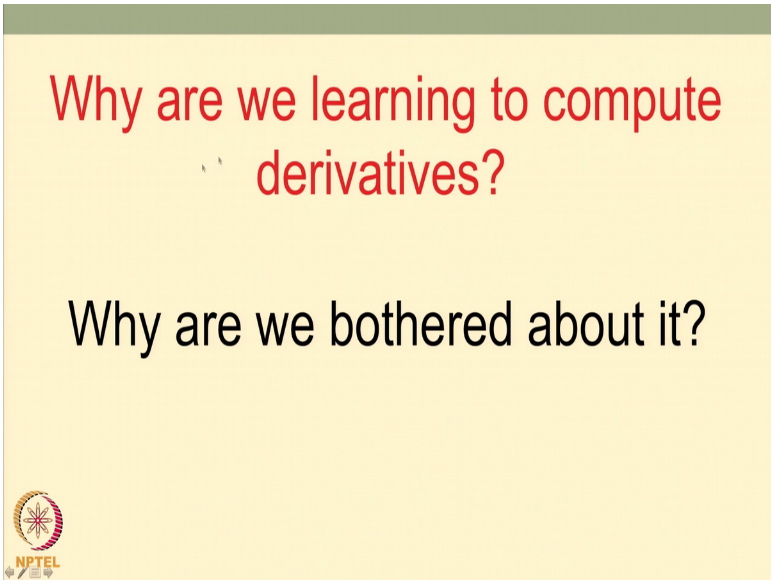

And we will bother we will worry about first we will little bit think about why are we learning to compute derivatives why are we bothered about it at all right this is always good to understand good to think about it. So, that we get some sense of what we are doing and as supposed to blindly following something and learning something because is there the textbook or because it is there because somebody teaches you; you want to learn this, always good to ask why are we learning this.

So, it turns out that we do experiments as biologists and from experiments we learn that we will get data and we can plot the data as a graph. So, everything that we see around us in nature we plot us graphs this quantitative way of describing what is going on around us that is something that we discussed in the earlier lectures. Now when we have some data we can it is not enough to know how the data looks like how does the graph looks like it is also important to know; how does that quantity that we measured changes. So, the change in that quantity how fast they changes, how slow it changes, how within this minute distance would it change, would it change as a function of time, as a function of space, how would it change, this is important to understand.

So, let me say; this once more it is important to understand not how the data looks like it is also important to understand how the data changes, how fast it would change or how slow it would change or in space would it change quickly, in space that is as you go along a certain distance or as you go from one part of the cell to the other part would your protein concentration change, right, for examples, this is something the change in concentration or how fast the number of cells change. So, the change is something that we always want to know in to understand the nature around us or to understand any phenomenon. So, calculating derivatives or computing derivatives is the way to compute how things change.

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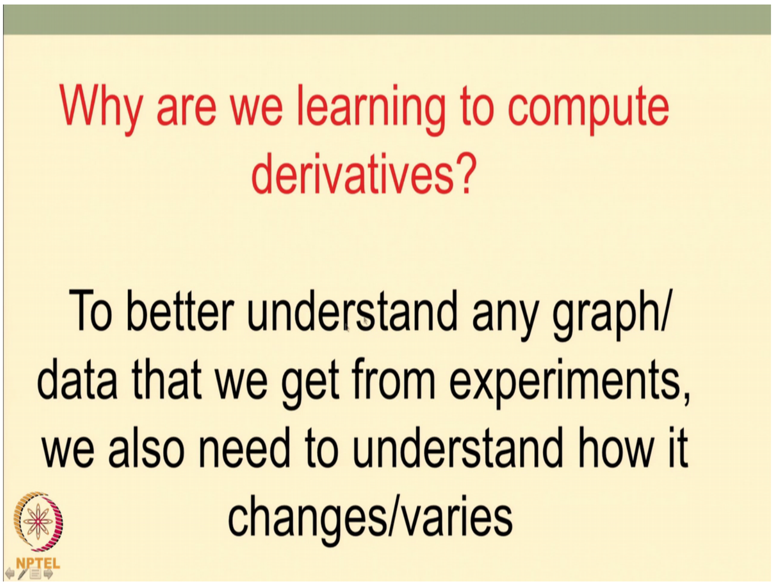

So, therefore, why are we learning to better understand any graph or data that we get from experiments we also need to understand, how it changes or how it varies both in time and how fast it would vary and also in space that is one reason why we are learning derivatives.

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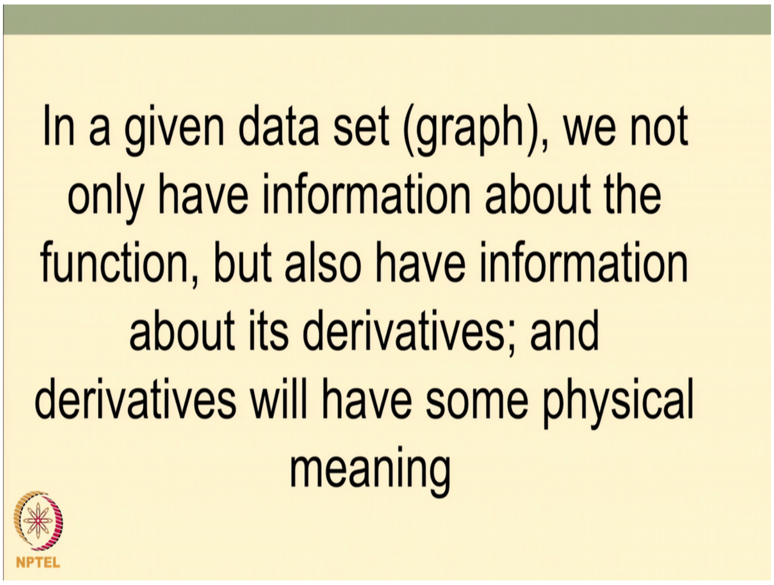

So, in a given data set that is if we have a graph we not only have information about the function, but also have information about its derivatives and derivatives will have some physical meaning? So, the information hidden in the data that we experimentally measure will have not only just the data how it looks like, but also something in hidden in it which is in terms of its derivatives. So, therefore, it is important to understand how things change that is the reason why we learn derivatives ok.

So, that is it is always important to think why we are learning it. Now let us come back to what we learnt.

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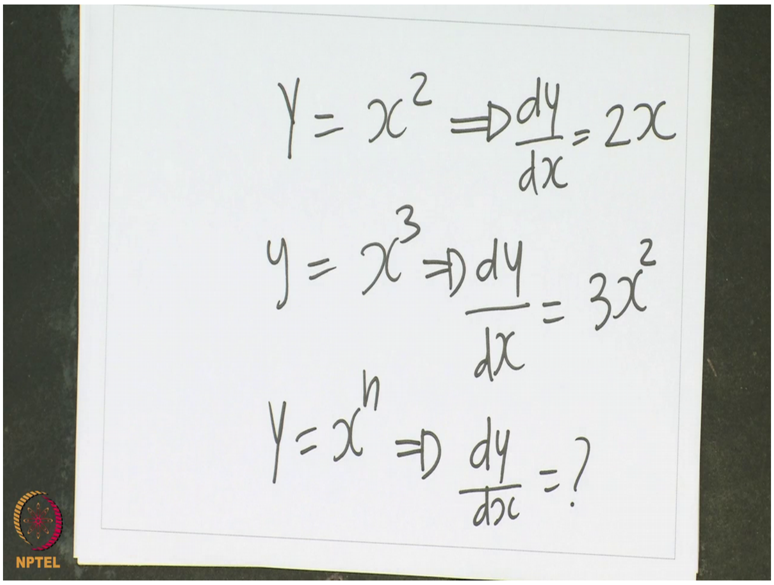

So, yesterday we learnt we have function Y is equal to x square, we had Y is equal to x cube and when we calculate the derivatives of this dy by dx, we calculated and we got 2 x and here we calculated dy by dx and we got 3 x square.

So, now in the general question is; if we have Y is equal to x power n what would be the dy by dx right if we have instead of x power 2 x power 3 if it is x power n what would be the dy by d x. So, that is what one of the rules of derivatives that you should learn.

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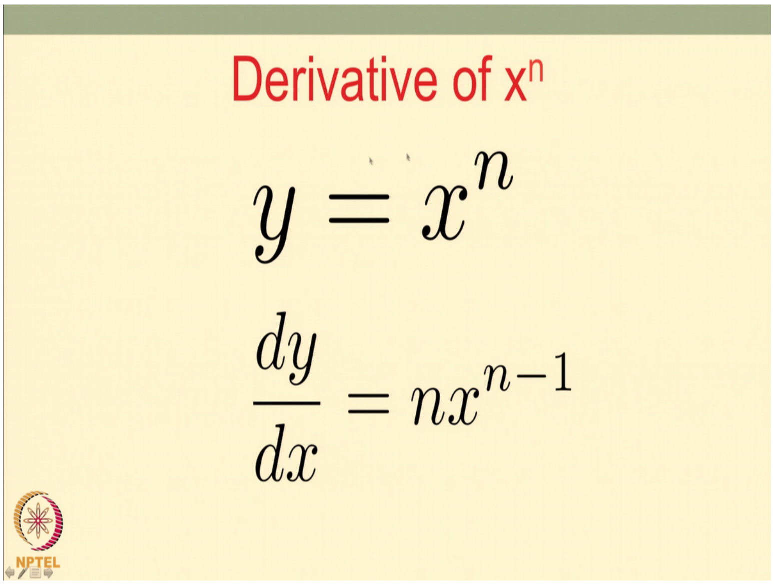

If we have y is equal to x power n the dy by dx the derivative is n x power n minus one. So, u n is any number. So, if n is 2 here it is 2 x power 2 minus 1, just 2 x when is 3; 3 x power 3 minus 1 which is 2 which is 3 x square.

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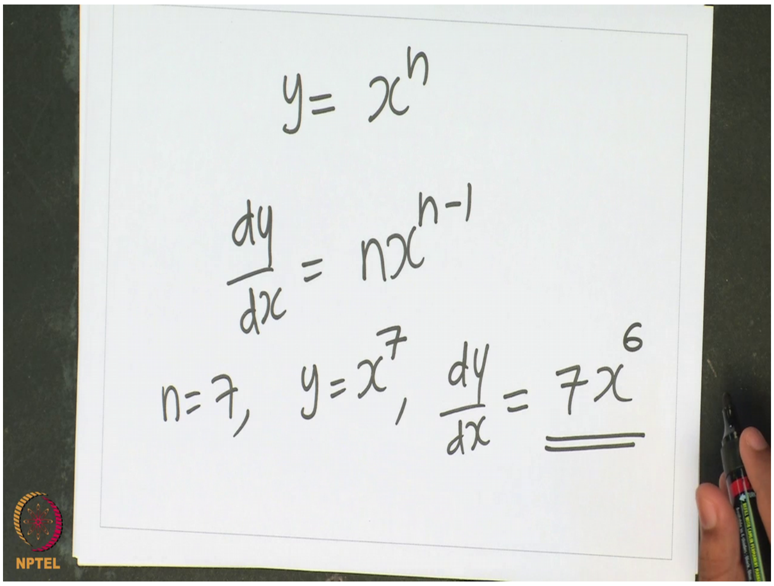

So, in general if we have n x power n minus 1. So, if we have y is equal to x power n dy by dx is n x power n minus 1, we can put any value of n if we let us put to n is equal to 7 dy is y is x power 7 and dy by dx corresponding dy by dx is 7 x power 7 minus 1; 6. So, this is applicable for any n and it has also applicable for fractions. For example, if first of all before that let us take n is equal to 0, something that you would want to take you would want to learn, right, if n is 0; what happens? So, if n is 0, it is a different case that you should understand.

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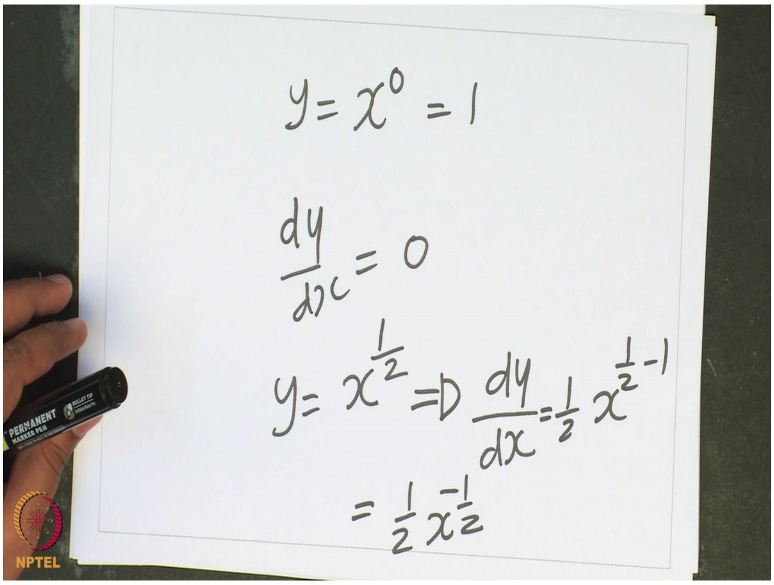

So, let me write here if y is x power 0 x power 0 is 1. So, derivative of that is 0. So, that is something which you should know dy by dx of the one which is the constant is 0, but if you have y is x power half then the corresponding dy by dx is half x power half minus one which is half x power minus half. So, this is applicable for fractions also if n could be a fraction also. So, please keep in mind that this rule that we learned that y is equal to x power n.

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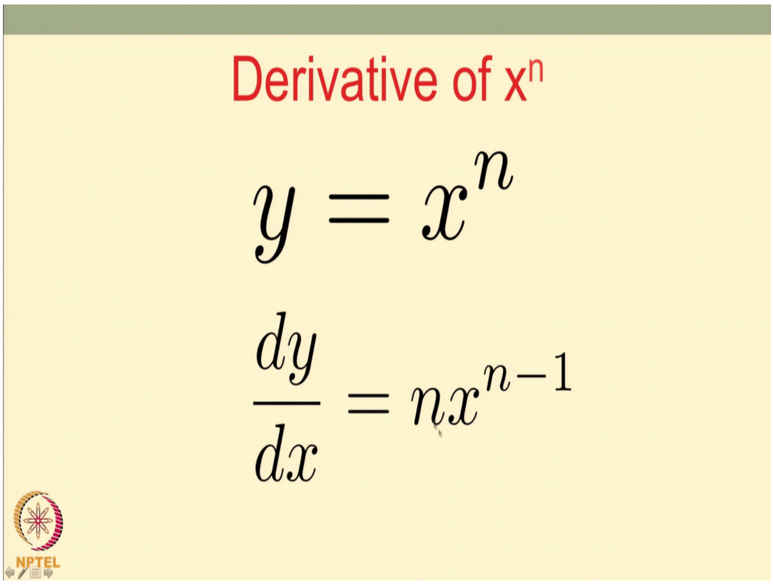

Where dy by dx is nx power n minus 1, this is applicable for n integers and n fraction and this is some rule that we should learn.

The next rule you would worry; you would want to know is what happens if you are adding 2 functions you would for example, you would have for example, 2 functions f and g.

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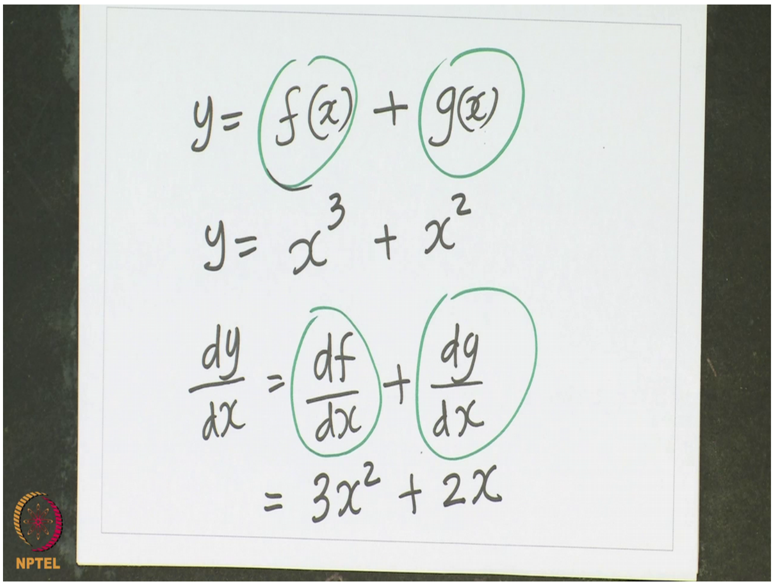

So, you have some function f of x and some function g of x and you have a sum of this for example, you have you have your y is sum of 2 functions, let us a x cubed plus x square. So, that is your y as you if your y is x cubed plus x square then dy by dx is basically the derivative of f which is df by dx plus the derivative of this dg by dx.

In other words, derivative of x cube which is 3 x square according to the formula that we learned nx power n minus 1 and derivative of x square is 2 x power 2 minus one which is 1. So, this is going to be the derivative of this. So, you have derivative of f which is df by dx derivative of g which is dg by dx and the sum of this would give you the derivative of this function. So, that is the next rule that I want to specify here.

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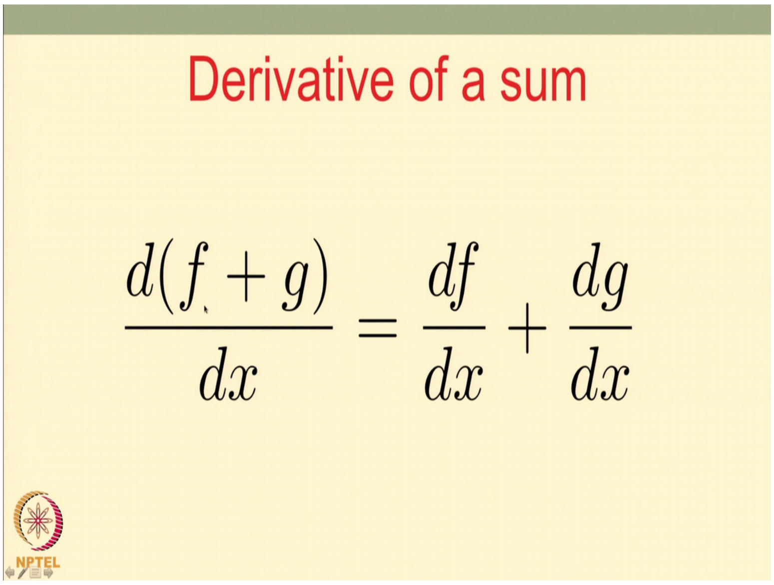

If you have 2 functions f and g; the derivative of a sum that is the derivative of x f plus g is derivative of the function f plus the derivative of this function g.

Actually if you think about this you can extend this to many interesting simple things for example, if you have 2 x.

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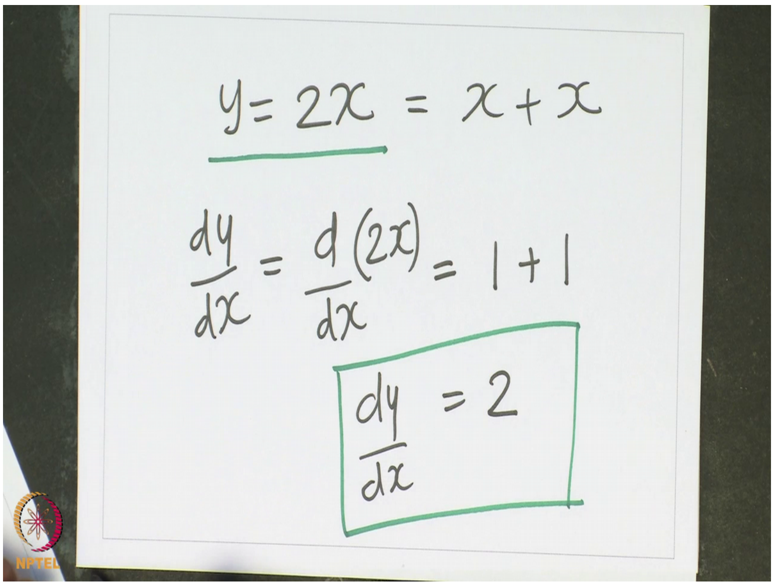

So, let us say the function that you have is y is equal to 2 x this y is equal to 2 x can be written as x plus x. Now if you want to calculate dy by dx you want to calculate the derivative of 2 x, you know that the slope of 2 x is 2 which is the straight line and we know about the slope is 2, but we can also see from this rule that we applied this derivative of x which is the line with slope one plus the derivative of this which is one. So, this y is equal to this x and x are 2 functions which is slope 1 and 1. So, therefore, this will have a slope 2. So, the dy by dx for 2 x is 2 something that we know from what we learned if. So, if you have a function y is equal to x it can be written as sum of x plus x and the derivatives we can calculate we can apply this rule and see for yourself that dy by dx is indeed 2.

So, this rule is a simple and that it has a fundamental nature that it will be useful to think about what we learned also do think about what we learned see if this rule is applicable wherever we have learned things right. So, do think about it and then that is what this rule is d by dx of f plus g is df by dx plus dg by dx. Now, once you have this; obviously, the next question would be if you have a product right if you have a 2 functions let us say f times g, if you have product of 2 functions. So, let us try about that later. So, let us think about it now.

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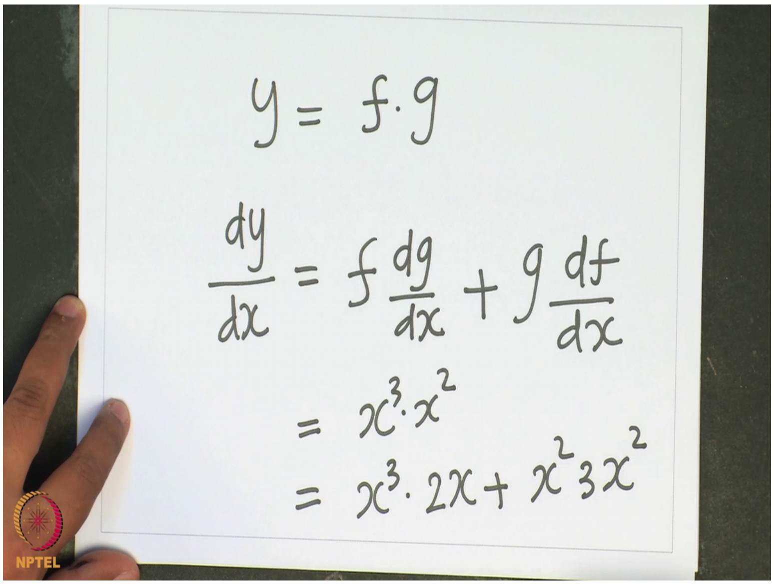

So, if you have a your y is product of 2 functions f times g what is dy by dx. So, it turns out that the way to calculate dy by dx is keep f constant and find the derivative of g plus keep g constant find the derivative of f. So, f dg by dx plus g df by dx. So, this is the derivative of y. So, if you have product of 2 functions, this would be the answer you could think of any 2 functions like we did you could think of x cube times x square who is x power 5 we know, but let us do it as separate 2 functions this would be keep x cube as it is and the product of x square derivative of x square is 2 x plus keep x square at this as it is and derivative of it is 3 x square. So, this is something that this way you can write and you will get this answer the appropriate answer one can write down.

So, this can we can also think about this in much simpler way. So, let us take x square itself. So, let us take the example of y is equal to x square.

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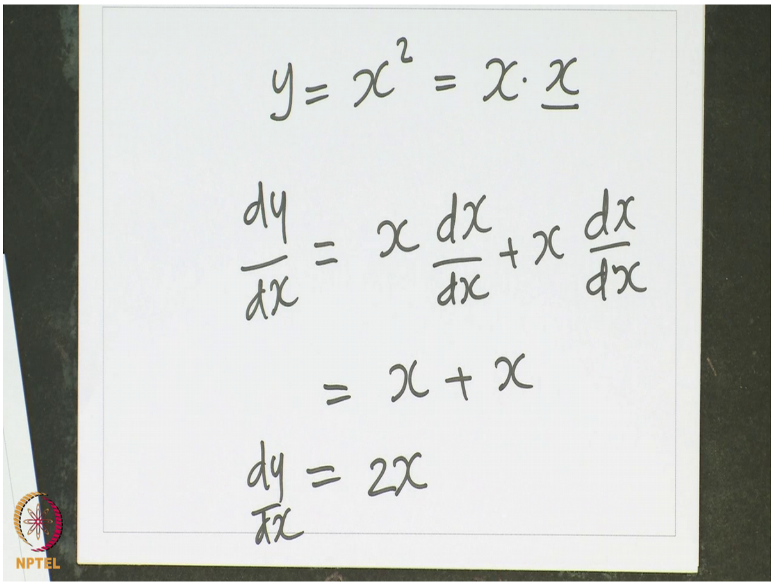

So, you have y is equal to x square which can be written as x times x. So, then you want to calculate dy by dx. So, which is keep the x constant and find the derivative of this x plus keep this x constant and find the derivative of this other x. So, what is this this is x the dx by dx the derivative of x is one and here also derivative of x is 1. So, the answer is 2 x. So, derivative of x square is 2 x that we know. So, this is another way of finding the same thing that is you can use the product rule to find the derivative of x square or any complicated function that we have.

So, once we understood these 2 simple rules f plus g and f times g we can essentially find out derivatives of any function that we want essentially and we will see some more complex more complicated functions that we will have learnt, we will try an exam will go and examine some other functions and what are the relatives of this.

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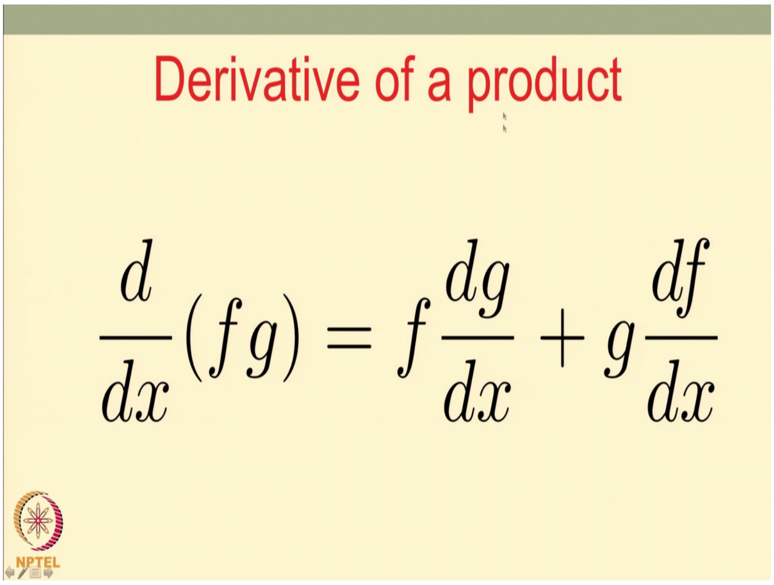

So, the rule here is this is a derivative of product d by dx of f times g is f times dg by dx plus g times df by dx. So, this is the product rule of finding derivatives. So, you have to just know these 2 rules the sum rule and this product rule and we can learn many of the things here now when this product rule when we have this product rule imagine that if f is a constant what does that mean. So, let us say if you have a constant here which is let us say you have you have y is equal to 3 x power 7.

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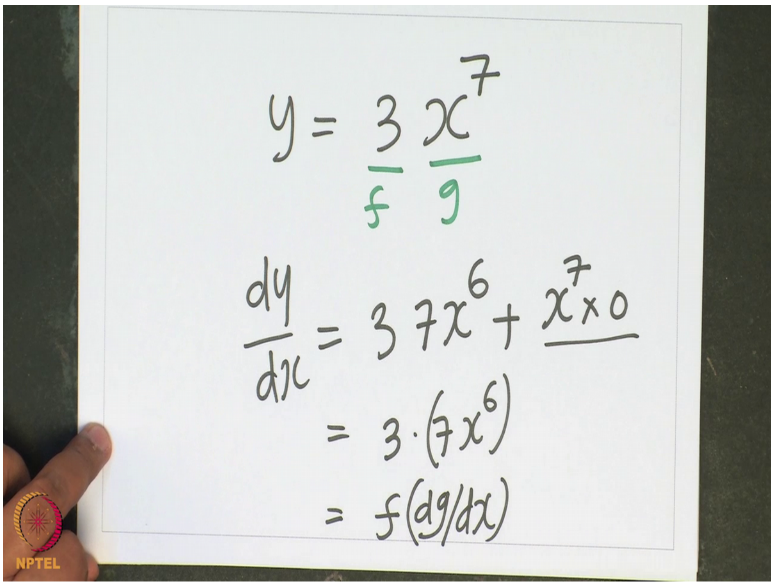

So, here what we have is this is your f if you wish this is your g if you wish right. So, according to that rule f times dg by dx. So, you have dy by dx is 3 times derivative of this which is 7 x x power 6. So, x power 7 derivative is 7 x power 6 plus g which is x power 7 times derivative of 3 which is 0 3 is a constant. So, derivative of f which is derivative of 3 is a 0. So, this is nothing, but 3 times derivative of x power 7. So, 3 times 7 x power 6 which is nothing, but the derivative of x power 7.

So, whenever you have a; f is a constant the second part of it will always be 0 the. So, only you have to do keep the constant as it is and find the derivative of this. So, if f is a constant the answer will be f times dg by dx if f is a constant dy by dx will be just f times d with g by dx. So, keep this in mind that if f is a constant you would come across many situations where this f is a constant or one of it is a constant and you will come you can calculate the derivatives using the same rule, but just we applying some common sense knowing this it is enough to know as a derivative of pretty much anything.

So, let us now go and learn the derivative of some other function let us say e power x. So, we have learned what e power x is.

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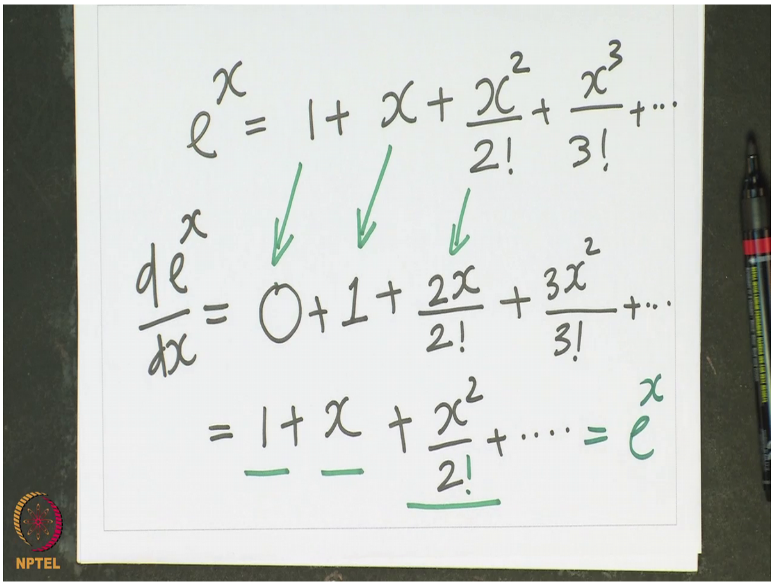

So, e power x we wrote we said that d power x can be written as infinite series one plus x plus x square by 2 factorial plus x cubed divided by 3 factorial plus dot, dot, dot. So, what is 3 factorial 3 factorial means 1 times 2 times 3; 2 factorial means one times 2 times 1 times 2. So, n factorial in general is one times 2 times 3 times up to n. So, that is n factorial if you do not know what this factorial means do go and look some math books or search for in google for this understanding what this factorial is.

Now, we are interested in finding out derivative of e power x what is the derivative of e power x. So, we know that e power x is sum of many terms. So, now, derivative of a sum is individual derivative of each of this and the sum of it. So, if i can find the derivative of one plus derivative of x plus derivative of x square by 2 factorial plus derivative of x cube by 3 factorial plus dot, dot, dot is your derivative of this whole function. So, what is derivative of 1? So, derivative of one is 0. So, derivative of one is 0 then we will rewrite derivative of one which is the derivative one is 0.

Now what is the derivative of x? So, plus, you are a derivative of x derivative of x is 1. So, derivative of x is one now what is derivative of x square divided by 2 factorial. So, one by 2 factorial is a constant it is just a number 2 factorial is two. So, one by 2 factorial is a constant. So, let me write plus 1 by 2 factorial is a constant I said. So, I wrote here one divided by 2 factorial I kept as it is and the derivative of x square is 2 x. So, this is the derivative of this term now plus derivative of this term is 1 by 3 factorial is a constant. So, I kept as it is and derivative of x cubed is 3 x square plus dot, dot, dot. So, there will be an x power 4 it will have corresponding some term here and all that.

Now, if you; if I rewrite this what will I get 1 plus 2 x by divided by 2 factorial 2 factorial means 1 times 2. So, this is 2; 2 divided by 2 is 1. So, this is just x. So, this we simplify this 2 divided by 2 factorial is 1; therefore, this whole term is just x plus 3 divided by 3 factorial 3 divided by 3 factorial 3 factorial is 1 times 2 times 3. So, 3 divided by 3 factorial is 2. So, this is x square divided by 2 plus dot, dot, dot.

So, if you look at here and compare these terms with this original term here it is one, it is 1 x x x square by 2 factorial here, it is also x square by 2 factorial x it will be a factorial here which is 2 factorial is 2 itself plus dot, dot, dot, if I do more and more term what I will end up finding is that derivative of e power x is e power x itself. So, e power x is a very interesting function where its derivative is the function itself. So, this is something interesting about e power x why you any of you would wondering why are we. So, particular about e power x we could have used 2 power x or 3 power x or 4 power x we could use any exponential functions.

Why are we so much interested in e power x e power x s main interesting properties and one of the interesting property is that the derivative of e power x is e power x itself. So, this is something that we would want to understand.

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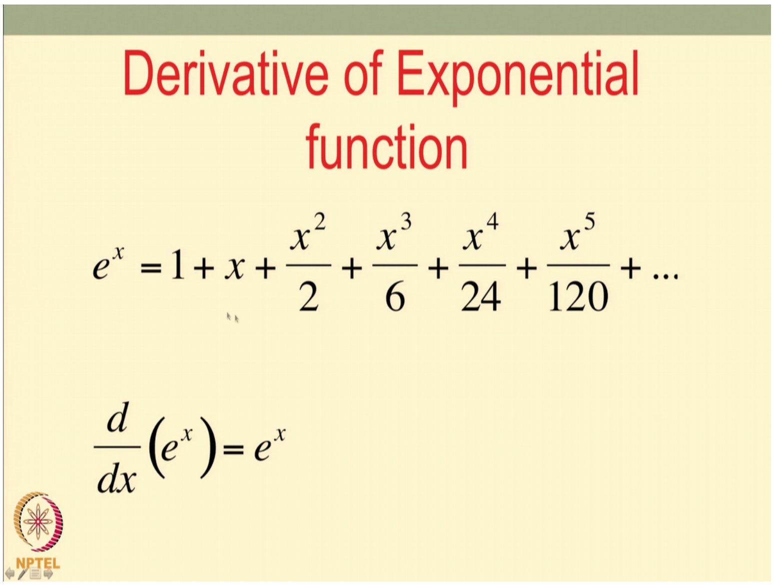

So, I urge you to do this derivative of all these terms. So, take e power x as 1 plus x plus x square divided by 2 plus x cube divided by 3 factorial which is 6 x power 4 divided by 4 factorial which is 24 x power 5 divided by 5 factorial which is 120. So, plus dot, dot, dot. So, I would urge you to take as many terms as possible do take more terms x power 6 x power 7 and so forth.

Then calculate the derivative of each of this term. So, whatever this denominator is can be kept as constant and you find the derivative of the numerator and essentially, you will find that d by dx of e power x is e power x itself. So, this is an interesting property of e power x that the derivative of e power x is e power x itself. So, this is something that all of you should learn.

Now, we could think of e power minus x right. So, let us think of this e power minus x. So, 1 e power minus x is 1 minus x plus x square by 2 minus x cube by 6 plus x power 4 by 24 minus x power 5 by 120 and so on and so forth. So, I just let us try this once more find the derivative of e power minus x. Let us try this our self to find the derivative of e power minus x and see what we would get. So, let us quickly try this.

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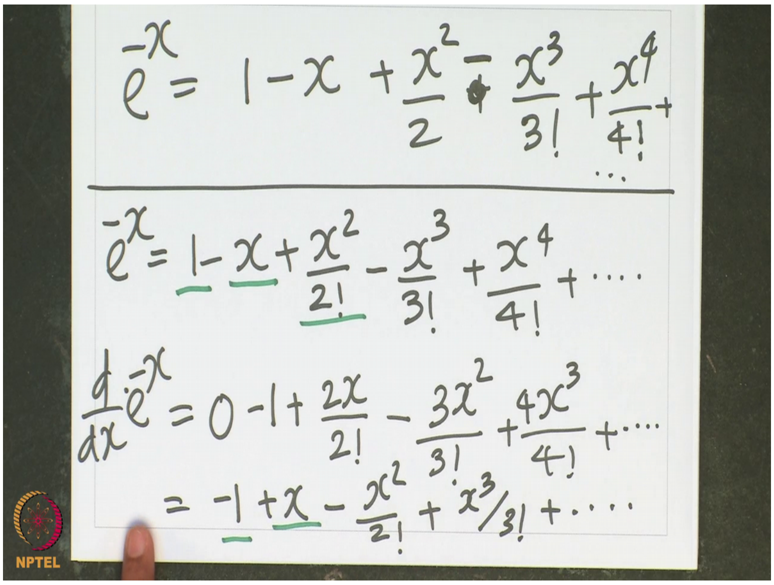

So, e power minus x is one minus x plus x square by 2 plus sorry minus. So, minus sign here minus x cube by 3 factorial plus x power 4 by 4 factorial plus dot, dot, dot, let me rewrite that here, it did not write it very neatly, let me rewrite it here e power x is one minus x plus x square by 2 factorial minus x cube divided by 3 factorial plus x power 4 divided by 4 factorial plus dot, dot, dot, this is e power minus x.

Now, if I want to find the derivative of e power minus x derivative of 1 is 0 derivative of minus x is minus one derivative of x squares is 2 x. So, 2 x by. So, I will keep one by 2 factorial is a constant and you have 2 x and you have minus x cube. So, basically you have minus 3 x square divided by 3 factorial plus 4 x cubed divided by 4 factorial and so on and so forth.

So, if I do this and compute this derivative and compute the next simplify this. So, if I simplify this, it is minus 1; 2 and 2 cancels. So, plus x 3 and 3 factorial is 2 factorial. So, minus x square by 2 factorial plus x cube divided by 3 factorial plus dot, dot, dot and if you look at it, if I compare this here, it was one here, it is minus 1 here, it is minus x here, it is plus x here it is x square by 2 factorial here it is minus x square by 2 factorial here is minus x cube by 3 factorial here it is plus x cube by 3 factorial.

So, it turns out that this is exactly if I multiply with this with a negative, if I multiply this e power minus x with the minus 1, if I take all these terms and multiply with the minus 1, I will get this. So, it turns out that derivative of e power minus x is minus e power minus

x. So, I urge you to work this out properly do this take a pen and paper and do this yourself only by doing this you will learn this. So, by doing this you will figure out how to how this would be how the answer will be what you will learn the answer has indeed this.

Similarly, you can do for any function. So, I also urge you to do for sin x and cos x. So, sin x we can write sin x as x minus x; x cube by 6 x power 5 by 128 so on.

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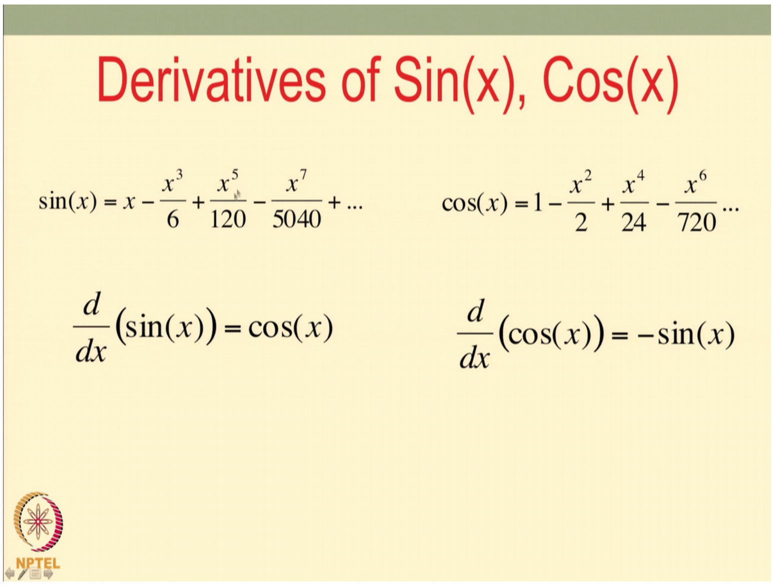

And you find the derivative of it you will find cos x do cos x you will find the derivative as minus sin x. So, if you expand sin x and cos x, you will find its derivative as appropriately derivative of sin x is cos x and derivative of cos x is minus sin x do this and in the next lecture, we will discuss more about this.

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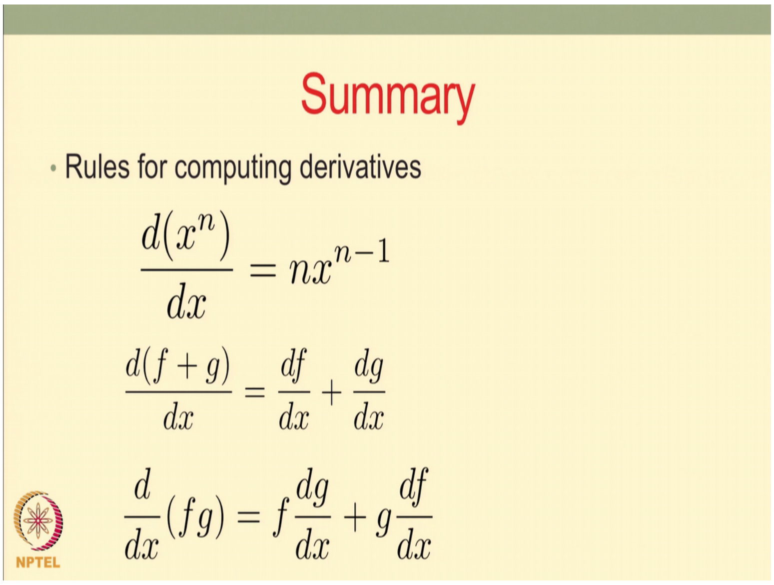

To summarize what we have learnt so far, the most important things that we learned are 3 rules; one is derivative of x power n is n x power n minus 1 derivative of sum which is f plus g is df by dg dx plus dg by dx and derivative of product fg is f dg by dx plus g df by dx with this, we will stop this lecture and continue in the next lecture, see you, bye.