Introductory Mathematical Methods for Biologists
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Lecture - 06
Images as 2D / 3D Functions

Hai. Welcome to this lecture, we have been discussing about functions we consider various graphs and wrote down equations for all those graphs. And we had functions that is increasing like y is equal to mx plus c; 2 x, 3 x and x square and x cube and all that. Then we also had functions that is decreasing y is equal to minus m x or y is equal to e power minus x. Then we had functions that is oscillating, it is increasing and decreasing we had sin x; cos x and some combination of this it is also possible.

So, we got used to be familiarize ourselves with various functions; now we are going to go one step beyond. So, today we are going to talk about images.

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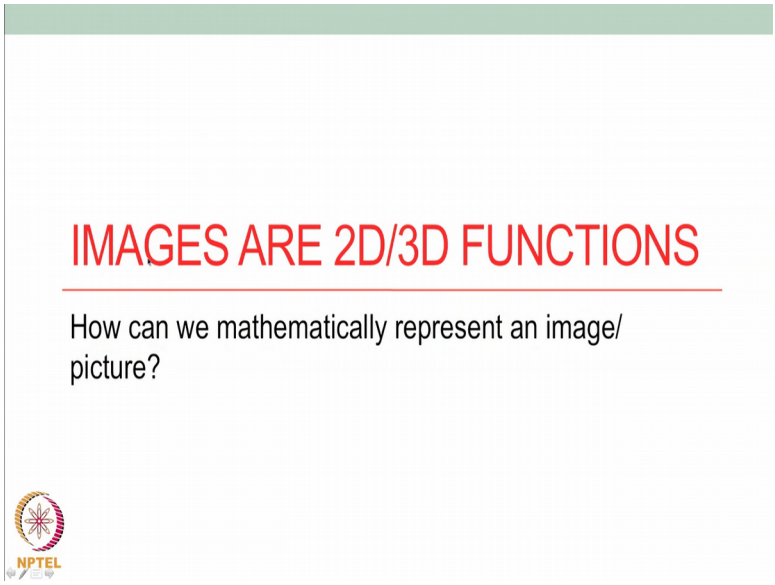

And images are essentially 2D or 3D functions and I will explain you what do I mean by this? So, the question that we will address today or we will try to answer today is that how can we mathematically represent an image or a picture as a function? This is what we will try to address in this lecture. How can we; if we get image, can we write a mathematical equation for this? Or is there a way of mathematically representing it as an image. The reason why we are thinking about this clear that; when we do experiments in

biology very often you will get data and you plot as a graph but it is also very common or in cell biologists and all that. It is very often you get images you get fluorescently labeled, you say fluorescently label cell; so, you get an image or if you think of some situations in biology.

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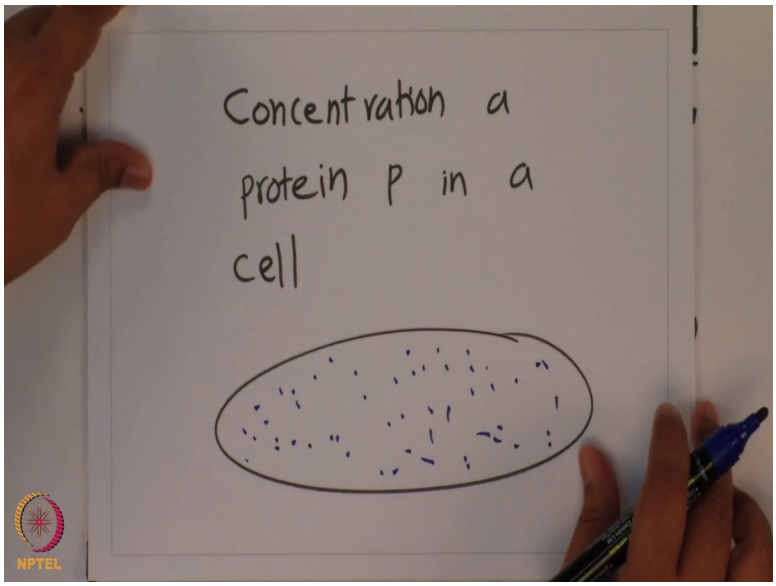

So for examples consider concentration of a particular protein in the cell; so, what we are going to consider? We are going to consider a concentration of a particular protein. So, what; it is called protein P in a cell. So, if you draw a cell like this it might; so, happen that the P is either uniformly distributed. So, what are the possibilities? Some possibilities that we can think of; how this a protein P might be distributed more. So, it could be equally distributed everywhere.

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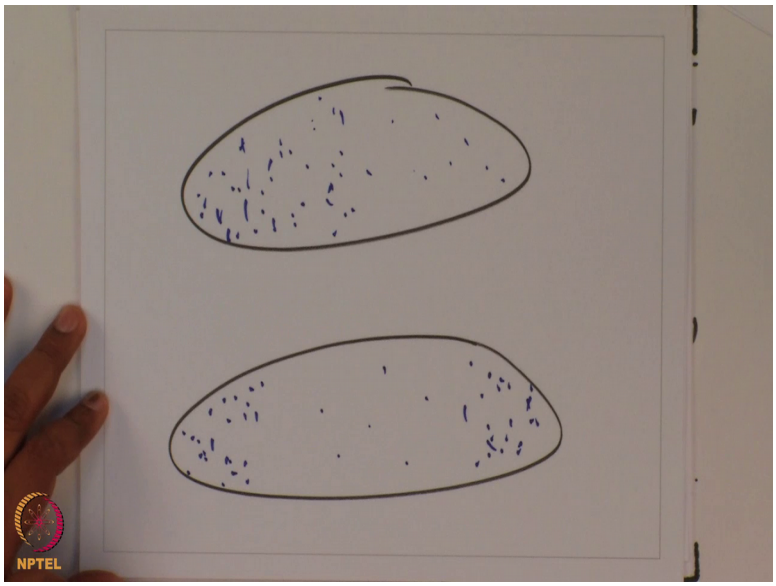

So, it could be uniformly distributed everywhere or it can so happen that this in a cell we can have the proteins distributed more towards one side of the protein and less towards the other side. Due to some reason, there are examples of such proteins it can so happen also that maybe; they are more in the edges and less in the middle. So, it can happen that the protein concentration is very high here and very high here and very little in the middle.

So, these are some possibilities; so now if you want to reserve some, if you take image that you get from an experiment; these are some 2D images in some sense. So, if you want to represent this mathematically; how will you represent?

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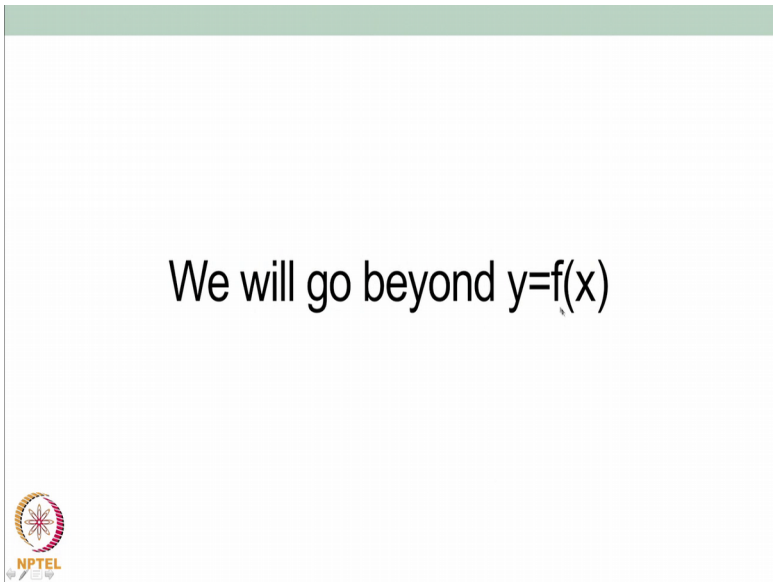

So, what we have been doing? So, far is y is equal to f of x. So, we had one x axis and the y value.

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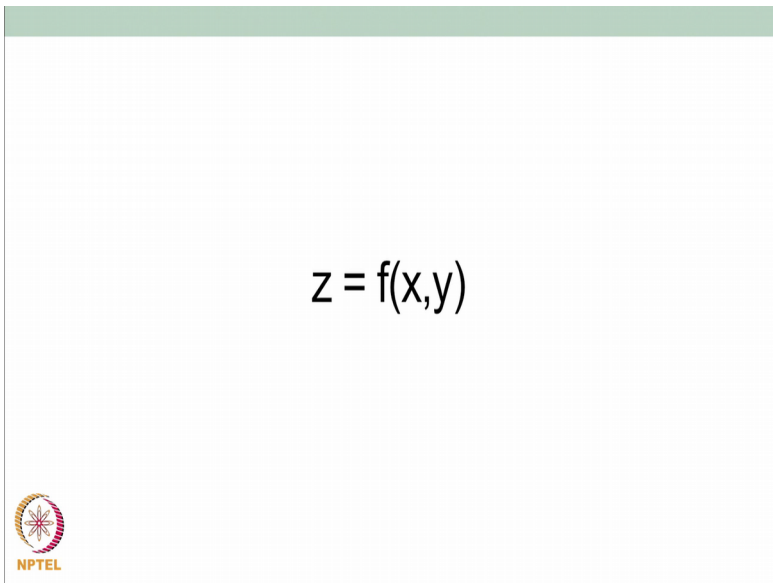

So, we have to, but you have to go beyond this and we have to do z is equal to f of x comma y. So, we have to calculate a function z which is a function of both x and y given x value and the y value we can get a z function. So, this will be the simplest way to represent a 2D image; so this function has 3 variables z, x and y. So, this is an equation having 3 variables.

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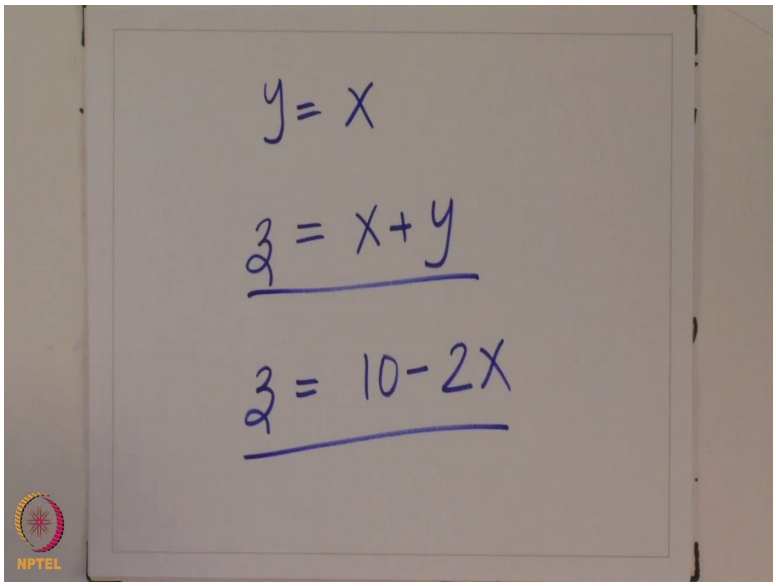

So, let us can think of"; so we said that in the simplest case we have an questions something like y is equal to x. We said this has the simplest equation possible, now if we want to go beyond this z is equal to what would be some function that you can think of? For example, I can think of z is equal to x plus y; this is some function that you can think of.

You can even think of even simpler function, but in 2D. So, let us say you can say that z is equal to some 10 minus 2 x; this is the another function possible, but here there is no y, but still you can plot this as a 2D function and I let us this think about this. So, let us think about this function first and then we will go to this function.

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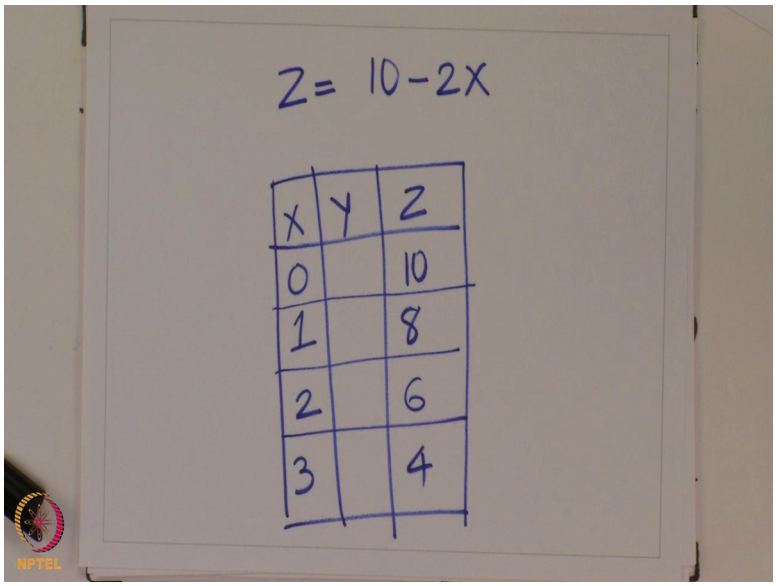

So, if we think about z is equal to 10 minus 2 x and plot it in 2D 10 minus 2 x. So, this function is independent of y; there is no y in this equation. So, whatever be the y value z; this z does not depend on y. So, if you make a table, so we can make a table for this. So, we have X, Y and Z; so if I just take x value 0 will be 10; 10 minus 2 times 0, which is 10.

If x is 1; does not matter whatever be the value of y, I can put any value of y; I want then this will be independent of y value. So, the z will be 10 minus 2 it will be 8; if I put 2 here, z will be 10 minus 2 times 2; 4. So, it will be 6 if I put 3 here; this will be 10 minus 6 this will be 4 and so on so.

I can make a table like this whatever y value I put the z will be independent of that y value, so we have such a function. Now, how will it appear if we plot it as a graph; so, I am going to show you how this will look like if we plot as a graph.

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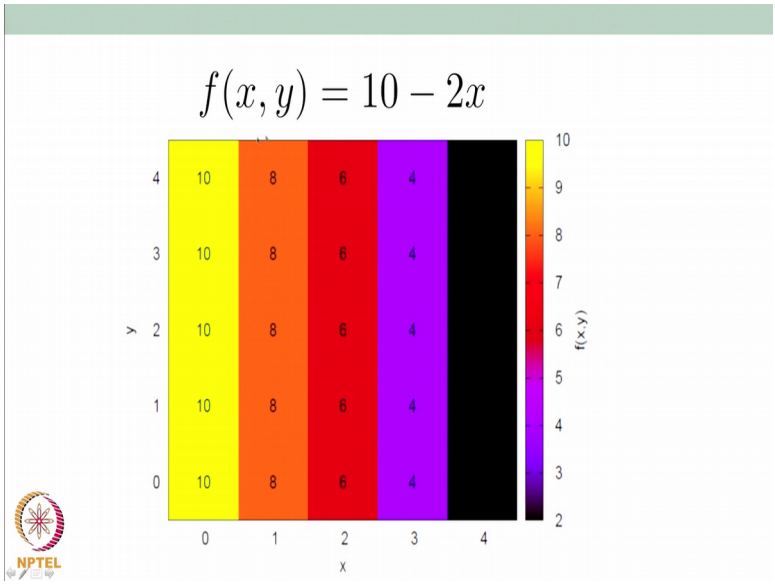

So, if I plot z; so, I can also represent this as f of x, y is equal to 10 minus 2 x or equivalently. So, I said that there are different ways we said there are different ways of there are different notations possible; so, one typical notation that we will use.

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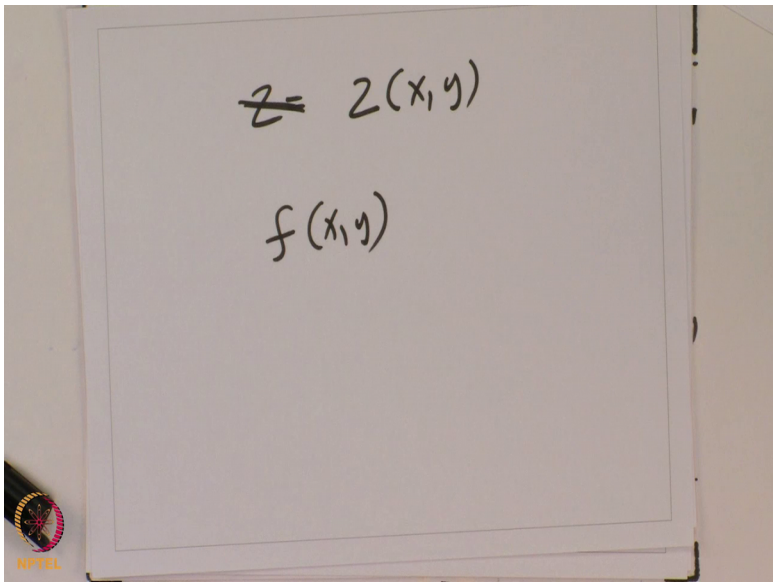

We said it could be z is equal to sum; we can say z as a function of x and y or you can say that f as a function of x and y there are different ways of representing.

So, what I have shown here is the function that we are interested is 10 minus 2 x. So, if I plot it is in 2D it will look like this. So, now let us look at this; when x is 0; 10 minus 0 is

10. So, all values is 10 independent of y whatever be the y value, when y is equal to 0; it is 10 and when y is 1; also is 10, when y is 2; it is 10, when y is 3; it is 10, when y is 4; it is 10. So, the yellow color it is like a stripe; which is independent of the y value, it depends on x; when x is 0, it is 10; so, 10 means yellow.

So, what this shown here is a color code; so, when this look at this small vertical bar here yellow color means value is very high understand, the black color means value is very small around 2 here. So, the red color means value is around 6 and 7 violet bluish color means value is around 3 and 4. So, the value the f value the function value can be shown as a color here we can think of them as a color.

So, this whole thing is yellow what does it mean? This means that when I substitute x is 0; 10 minus 2 times 0 is 10 and 10 is yellow; so, the whole thing is yellow whatever be the y value. Now, consider the next one when x is 1 as we showed 10 minus 2 times 1 is 8. So, 8 corresponds to some kind of reddish color; so, slightly orangish red. So, when x is 1, y is 8 everywhere and this is independent of y; again though 8 even if y is 3, y is 4 even if y is 1; everywhere it is 8, when x is 2; 10 minus 2 times 2, 4; which is 6. So, this is red color which is corresponding to 6; so, again it is independent of y.

So, I have taken x values in discrete like; so I have just divided this for convenience into some boxes that is why there is in from here to here I assume that x value is 0 here to here; I assume your x values 1 here to here; I assume. So, I discretized the space and I would wrote 0, 1, 2, 3, 4; so when x is 4; 10 minus 2 times 4 is 10 minus 8, which is 2 and it should be black color. And the point to notice that; this is independent of y whatever be the y value, you will get the same f values; so, the function is the independent of y and you get this and you can think of this in the experiments.

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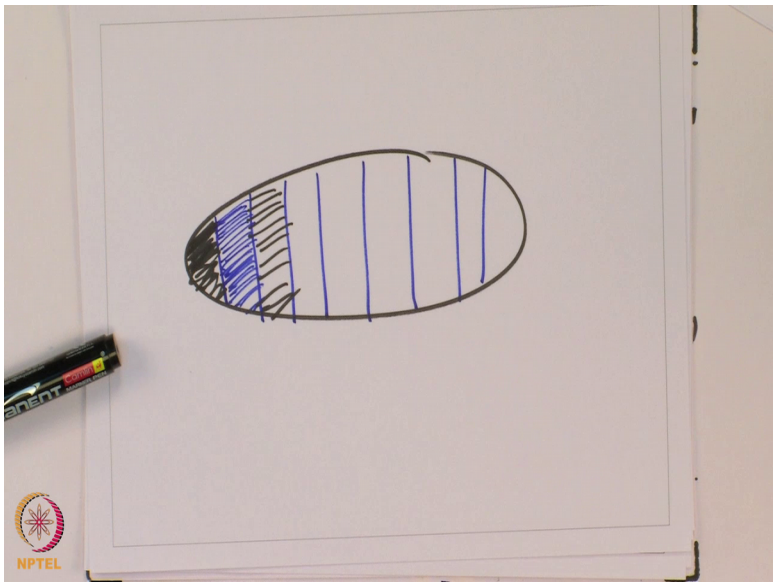

So, you can think of various context for example; if you have a cell you can think of some stripes, where you are well know for example, various organic; various like zebra line; we know very well about zebra line what does this represent? So, this represents some stripe.

So, there is some concentration here and there is a different protein has concentration here. So, you can have like a different color if you wish and you can have a separate color and so on and so forth. So, you can think of some stripes like this some context of side stripes on a 2D plane as something that is observable in biology. For example, in a tissue or even in a cell; you could think of protein concentration varying in a particular way which has some stripe like pattern.

So, the simplest example that we want to show is f of x comma y is 10 minus 2 x here; I wrote y, but it is independent of y does not matter it actually does not depend on y. So, say f of x only, but I just wanted to plot them 2D and therefore, I wrote this y here and I hope you have understood the color code. Think about this; if you have not understood, think about it carefully and try to understand this stripe graph as a 2D function.

Now, we will go to the next level as we said that the z or f has x plus y, where we will actually take y value.

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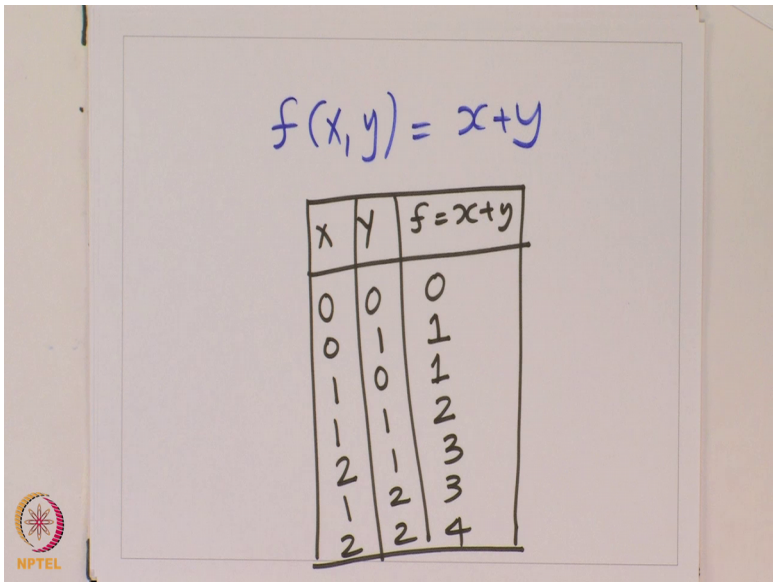

So, we will write f of x comma y is x plus y; now if we make a table for this, how will the table appear? So, let us make a table for this, so I will take x y f is equal to x plus y this is what I want to have. So, when x is 0, y is 0, f is 0; x plus y is 0, when x is 0 and y is 1; this is 1, when x is 1, y is 0, this is 1; when x is 1, y is 1; this is 2; you can write various values. I can write 2 and 1; this will be 3, you can have 1 and 2; this is 3, you can have 2 and 2; this is 4.

So, I can make a table like this for different values of x and y and I will get a nice table, which I can plot and the plot will look like this.

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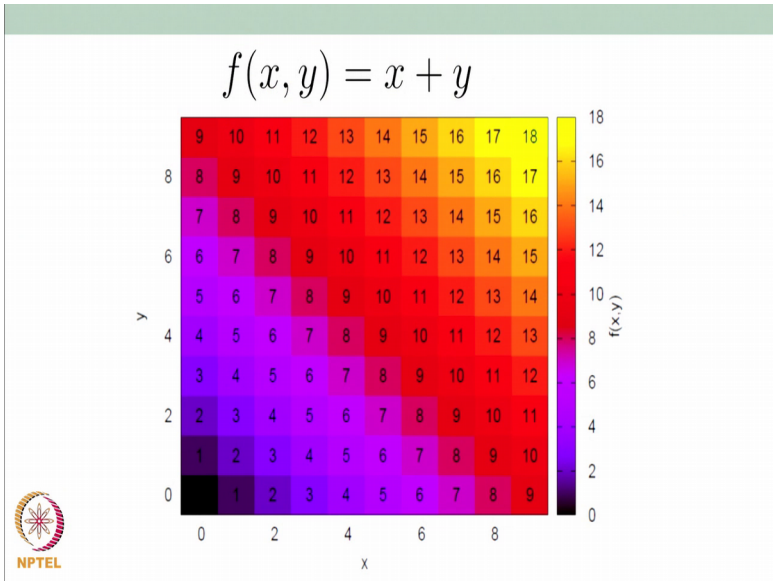

So, carefully slowly look at this graph; what I have plotted here is f of x, y is x plus y. So, if I take here the bottom left corner x is 0, y is 0; 0 plus 0 is 0 and this color code; this is thing this is also called heat map sometime. So, this color code here represent that the black is for very small values.

So, the blackish here means 0 or very small value. So, this is 0 then if I have x is 1 and y is 0; this is 1, this is also 1. So, for one it is slightly violetish, but black can violet combination; then I have here 2 1 x is; 1 y is 1; then the value is 2. So, actually will become more violet, when this is 2; the color code this represents what is the code for the color? So, when it is 2; it is a violet color.

Let us took this extreme here the top right corner. So, here x is after 8; this is 9. So, x is 9, y is 9; 9 plus 9, it is 18 and 18 is a yellow color, so 18. So, that is why yellow color here and somewhere here, so let us take here x is 6, y is equal; 6 plus 6 is 12 here; x is 6, y is 5; so is 11 here; x is 6 y is 4, so it is 10. So, 10, 11, 12 etcetera is slightly reddish color.

So, you can see that the color; so now you see a nice image with a color pattern emerging here and this color pattern has a simple equation; where f of x is y is equal to x plus y. So, this equation represents this image with this color pattern; you can think of removing all these numbers and just showing this color pattern alone. And that would mean that, there you have a nice image with you with some color pattern. This might be something

that you see in an experiment; so this is 0 of this color, you could think of some fluorescing; if you might have seen cells with fluorescent images.

So, let us say various this could be intensity; so instead of color I could think of intensity of some fluorescing protein. So, high intensity or low intensity depending on the color you want and you can imagine a 2D intensity pattern like this; that would represent an image. So, this is something which we want to get used to as a biologist, I want all of you to get used to thinking about 2D images as a function; now you can also plot this as z of x plus y like.

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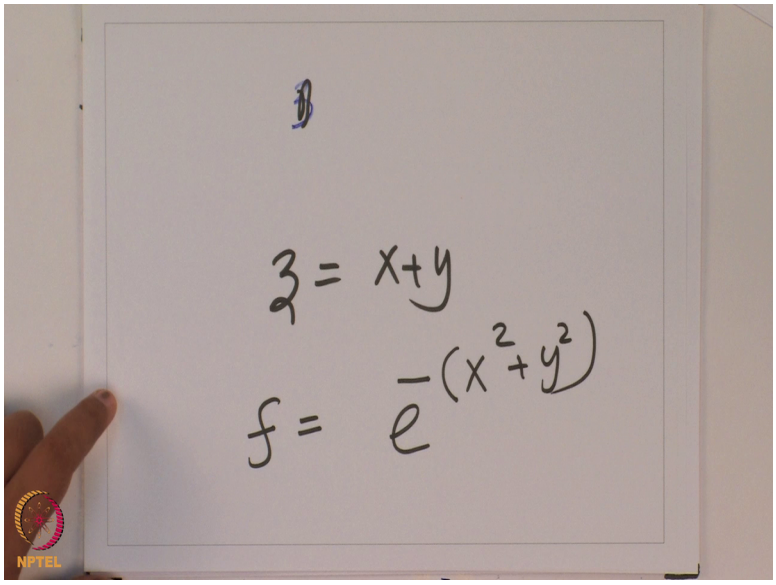

I can we wrote sometime the same thing has z is equal to x plus y. So, I can plot this as a graph with this x this is y and a 3D graph with some z value appropriate z value. So, xy in plane and the z value and if I plotted it will look like this.

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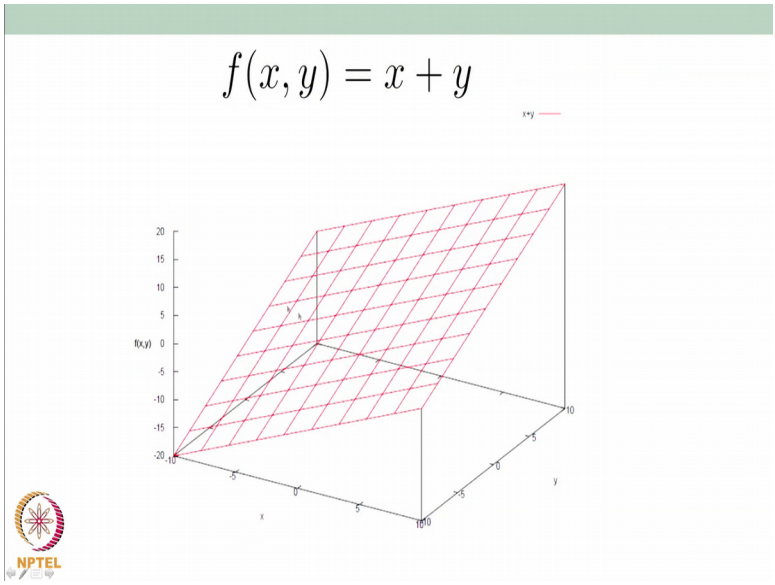

Even though you might have some difficulty in understanding it immediately, but you can think of x y and x plus y is z. So, you can think of this as a plane like this and just think about this even though this would need taking a paper yourself and holding as a plane and thinking about a little bit.

So, this is something I want you to get trained to think about 2 dimensional 3 dimensional functions like this. Now, we will think of slightly more complicated functions; so now we had simple function like z is equal to x plus y. Now we will have slightly more complicated function f is equal to e power minus x square plus y square, I can think of some sort of function like this. So, then for every x value and y value you can calculate this function and plot. So, again you can make a table and we can plot them; so let us say I have a table again.

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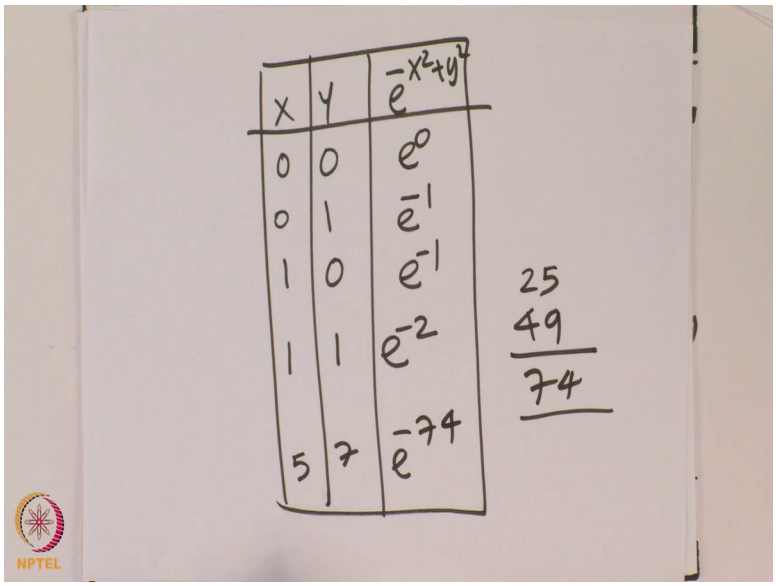

As we have said we have x y then e power minus x square plus y square is the via function that we have; so 0; 0 which is e power 0, which is 1 then you can put 0 and 1. So, e power minus 1; you can put 1 0 e power minus 1 will be the e power 1 square plus 0; square of 0 square plus 1 square, you will get e power minus 1; you can put 1, 1.

So, e power 1 square plus 1 square it will be e power e minus 2 and you can put any number; you are want like; let us say 5, 7. If I put 5, 7 like it will be e power minus 5 square, which is 25 plus 7 square; which is 49. So, this would be like 74; so it will be e power minus 74; it will be a very small value, if an x is 5 and y is 7.

So, there is a bracket here; this is a function that we want to plot e power minus x square plus y square, so we will see how this will appear.

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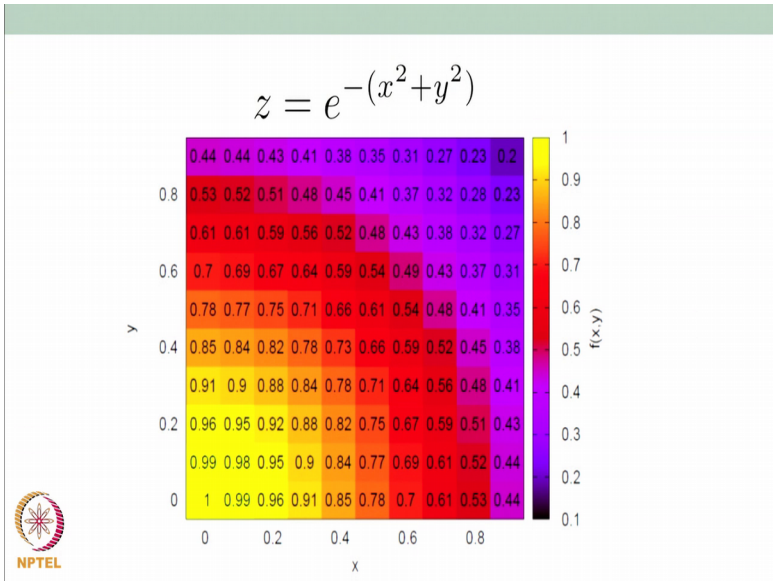

So, have a look at here; so, when this is again a pattern you can first look at the color pattern then x is 0, y is 0; you have e power 0 which is 1, then e power minus 0.2. So, e power minus 0.1, e power minus 0.2 will be this and then e power minus 0.3, 0.4 and somewhere here you will have e power minus 0.9 square plus 0.9 square. So, it says point 0, 0.2 and if I take more and more value it will be very small.

So, again what you are seeing here is a nice pattern of a color pattern that color patterns the matrix. So, you can think of this as a matrix or 2D function and what I want you to think about that given any image you can get a 2D matrix like this, which may or may not be able to write it as an equation. But he can get a data a 2D data like this or a given a 2D data, you can plot a color pattern like this.

So, this is something that we should get trained as a biologist; who was trying to learn mathematics. So, the bottom line that I want all of you to know is that any image you get you can think of it as a 2D; 2D function and it would be very complicated function. There we may not be able to write an equation, but we whatever be in a function that you get what all be the image that you get.

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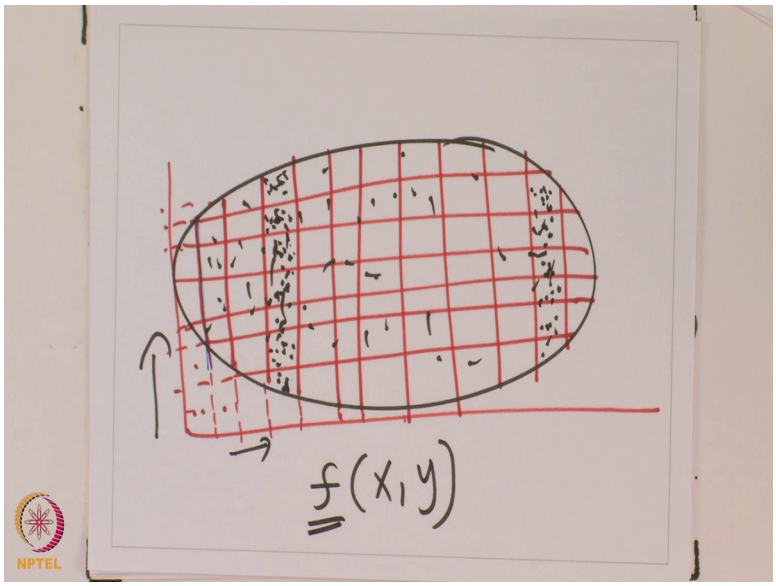

Any complicated image you get, so I got some pattern image. So, let us say the some concentration pattern I got; I can divide this cell into small, I can divide the cell into small grids like this. So, I can divide the cell into grids like this and ask a question. So, let me divide these into grids and ask a question in this first grid. So, I can draw this x and y; so I can extend this grids for a 2D purpose, if you want and I would extend this grid this way also and so on so forth.

So, I can asked the question the first grid how many black dots you see? The second grid in this grid in each of this grid; how many black dots you see? Maybe there are some grids; where there are many, many, many, many black dots. So, if you have many, many, many, many black dots in a particular grid. So, there is a very high concentration; there is a stripe of black dots here. So, then you will write down some particular function even here let us there us many, many, many black dots here in this cells.

So, you can divide the cells like this into grids and get nice black dot and this is some function of x comma y. So given x value and a y value so I have a function f; which is essentially the number of black dots, here the number of black dots is my f for a given x value and a y value. I can count how many black dots are there and that will give me some f of x comma y, which is a mathematical function. So, you can also; we just talked about this function here e power minus x squared plus y squared.

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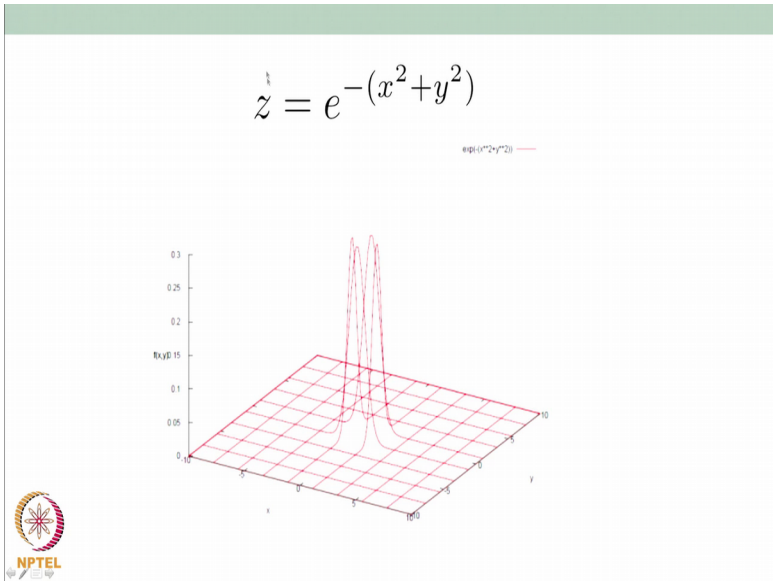

You can also plot it as z is equal to e power minus x square plus y square with some graph like this; depending on various values of x and y, I can get various interesting graphs.

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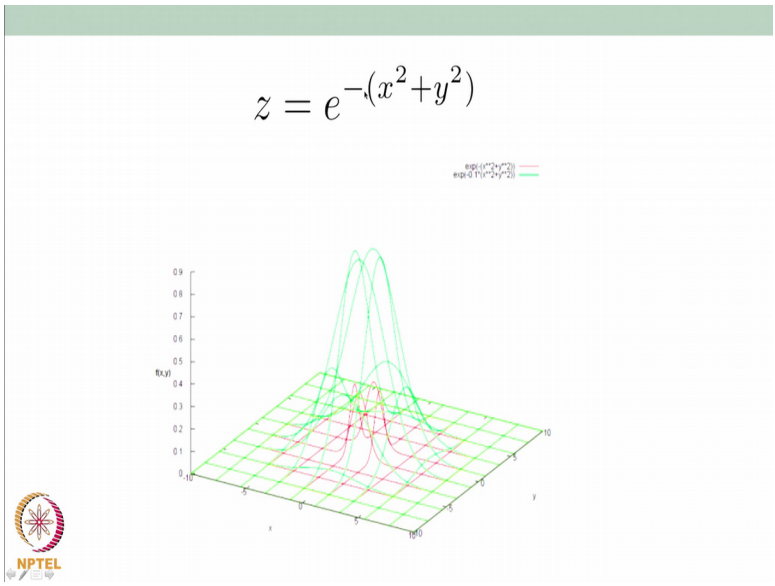

There is some coefficient I can put here, I can write e power minus.

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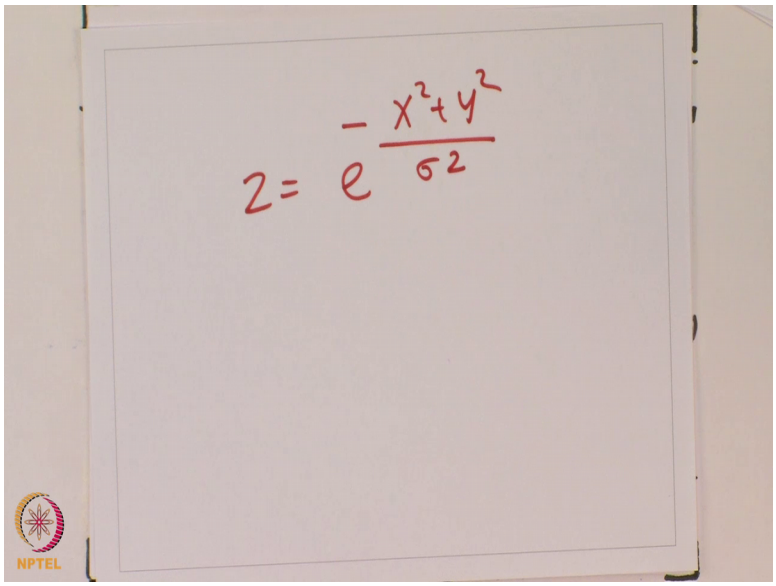

I can write z is equal to e power minus x square plus y square by some sigma square and depending on different values of sigma, I can get different kinds of plots and I want you to. So, the general equation for here is e power minus x square plus y square divided by sigma square and different values of sigma I will get different plots.

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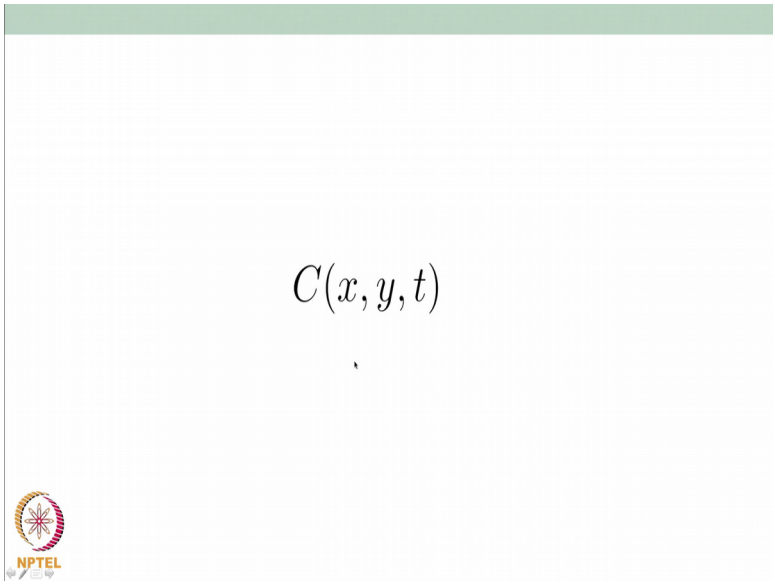

So, this is something that I want you to think about; now I want you to also think about something as a some concentration let us; as a function of x, y and t. So, you have something which is a function of x, y but it is also changing with time.

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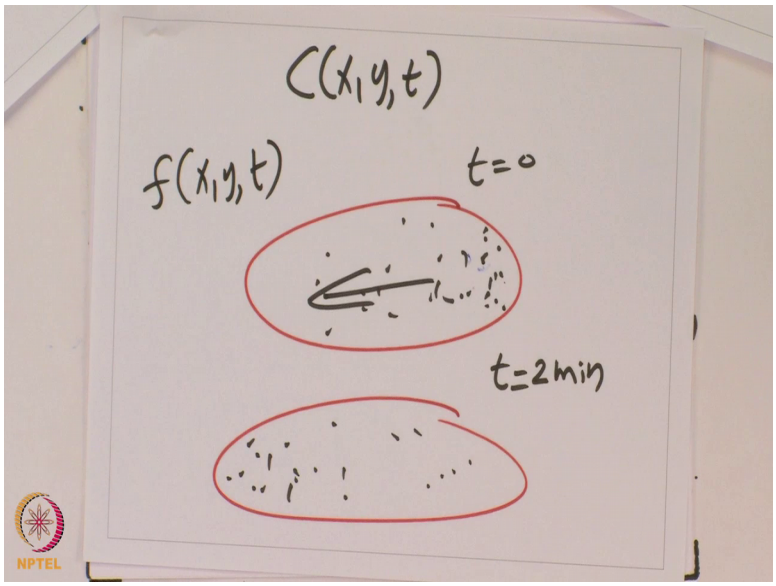

So, you have a cell which is this is in red and you are some concentration of some proteins which is here. But this concentration is at t equal to 0 is like this; that is high concentration here low concentration here.

Now, after t is equal to 2 minutes this could change such a way that it is high concentration here and low concentration here. So, the concentration has changed with time, so it is like this things as flowing here; so this is like a flow. So, c of x comma y comma t if you write some function; like this which is somewhat representing flowing things along 2D. So, if you have a video of a 2D picture that is if you have a 2D video, then what you will essentially have as a flow and that is some function c which is x, y and t or in general I can write f of x, y and t some function of x, y and t.

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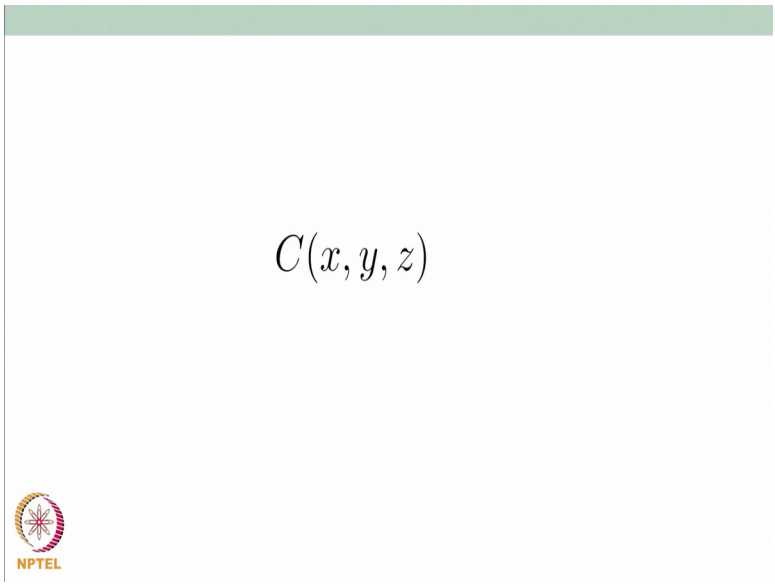

You can also think of c of x, y and z some 3D like a double helix which is some function of x, y and z. So, I want you to think about helix and in equation for a helix; what is an equation for helix I want? All of you to think about it; this will be some function of x, y and z and double helixes are very important thing in biology. So, I want you to all of think of what is the mathematical function for a helix maybe it is some point we will discuss this.

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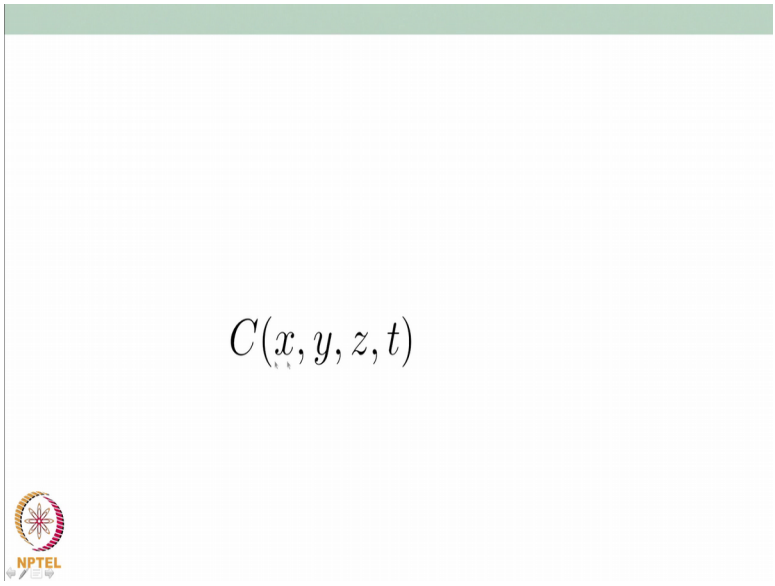

You can also think of c as a x, y, z and t which is something flowing in 3D. So, these are functions, so both flow and images can be thought of as some mathematical functions that is the message of today's lecture. So, I will just summarize what we discussed so far today.

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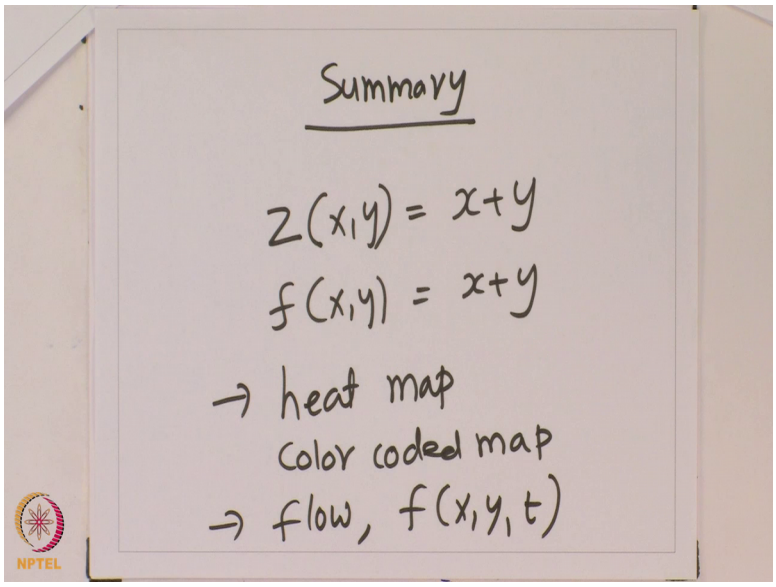

So let me to summarize the summary; so, we had some function z is x of some z is a function of x comma y some simplest thing. We said or f x plus y or any other function that we can write; we wrote this as a heat map or a color coded map and correspondingly I can think of this as a fluorescently; different fluorescent at different locations. And then I also said that 3D functions and flow also can be thought of as f of x, y and t.

So, this is to go beyond the simple function we learnt train yourself to think about function as in 2D and 3D and f of x comma y f of x comma y comma z and f of x comma y comma z comma t. So, train yourself for just think about it and we will discuss this more detail in detail later, bye.