Introductory Mathematical Methods for Biologists
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Lecture – 04
Graphs : Exponential and Periodic Functions

Hello, welcome to this lecture. We will continue discussing functions and graphs. In the previous lectures we discussed simple functions like y is equal to x, x square, x cube and so on so forth and we also said that we can think of functions which are combinations of x, x square and x cube. So, that is where we said last time that we can have a function that is a combination of x, x square and x cube.

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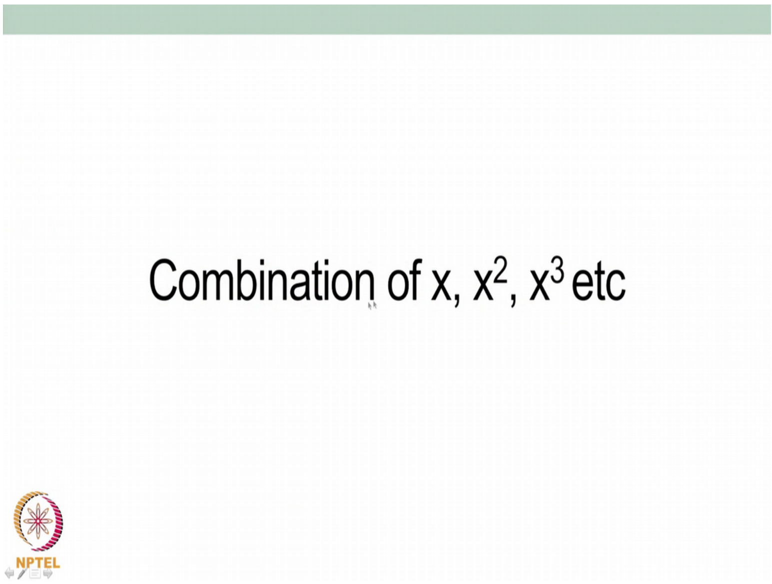

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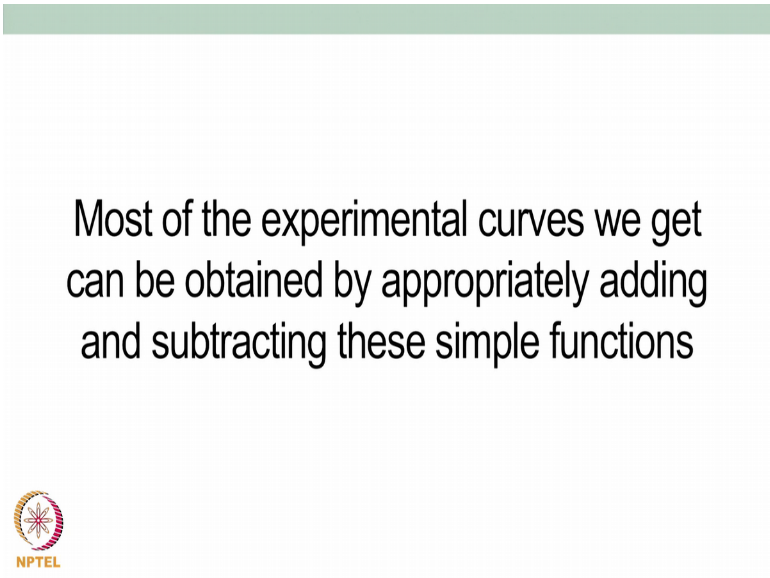

And we also said that most of the experimental curves we get can be obtained by appropriately adding and subtracting these simple functions such as the x, x square, x cube etcetera.

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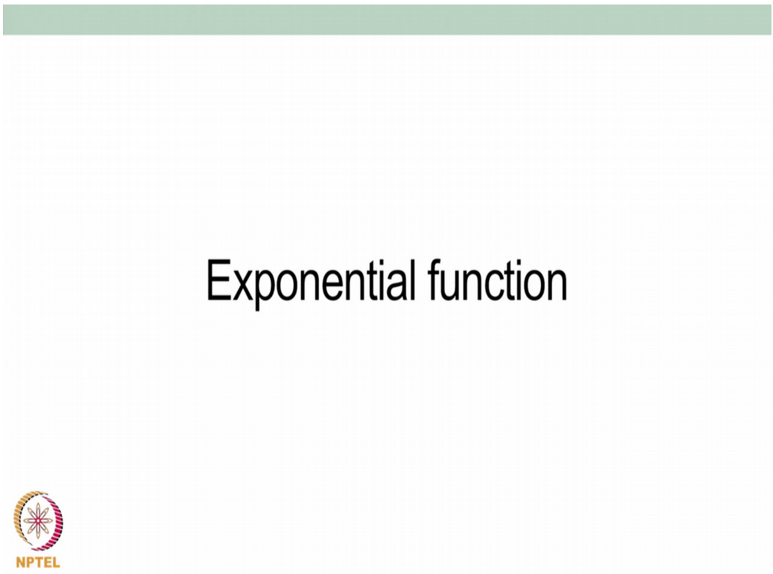

And one example we said is an exponential function, and it is Y is equal to e power X.

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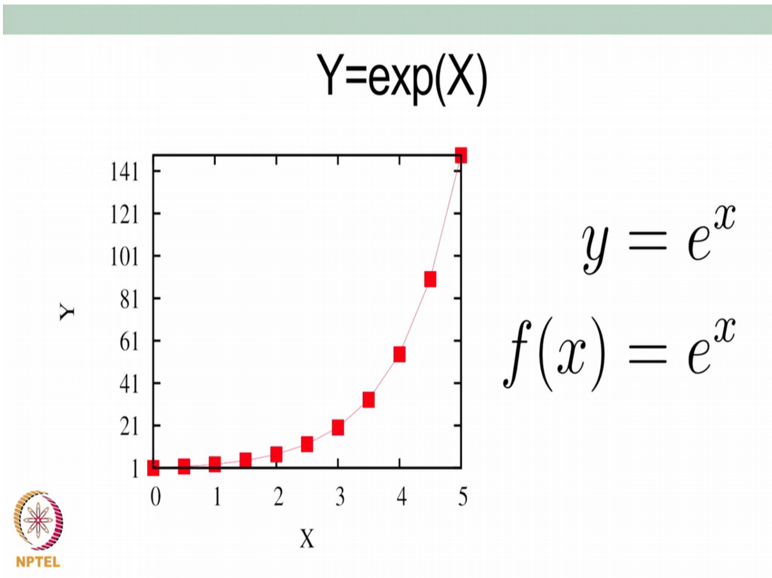

If we plotted by tabulating we will get a curve that is increasing with respect to x. So, this is also just like x, x square. All of these functions are increasing functions like Y increases with when X increases and we said the way to write y as a function of x is an infinite series. So, that is you take many terms, shows as shown in the computer here.

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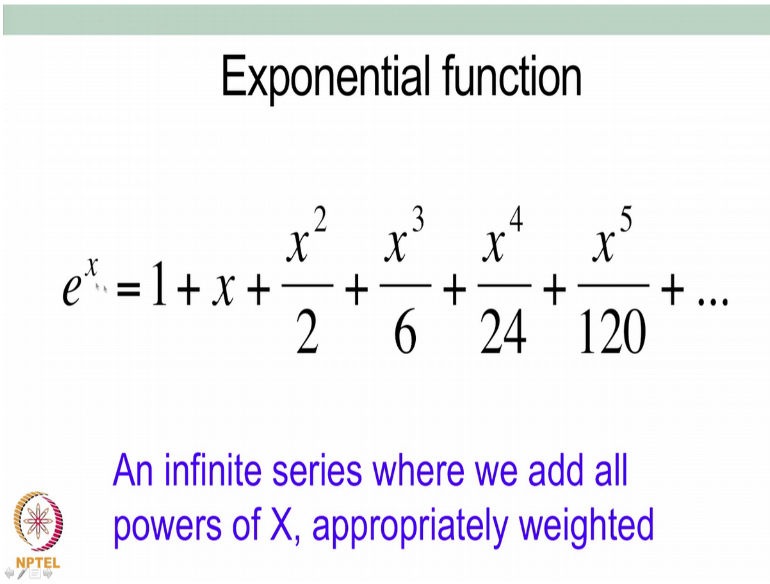

e power x can be written as 1 plus x plus x square divided by 2 plus x cube divided by 6 plus x power 4 divided by 24 plus x power 5 divided by 120 plus x power 6 divided by 6 factorial plus so on and so forth.

So, you can write this is a sum of many many many terms and this 3 dots here indicate that you have to have many more terms which I have not specified here which we will get to know later, but e power x can be written as a sum of many terms where each of this term is some x, x square, x cube, x power 4 etcetera divided by some number how this number is coming etcetera we will discuss later. But at the moment is enough to sufficient to understand that an infinite series where we add all powers of x appropriated weighted by various numbers will give us e power x.

One of the thing that we want to note is that x is a dimensionless quantity, what do I mean by that.

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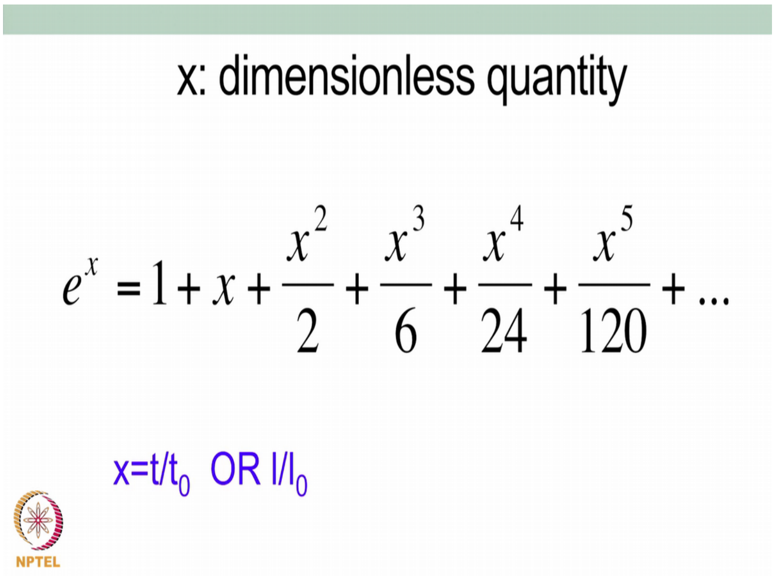

So, when we learn math for biology we should think of this not as simple just mathematical equations, but as some physical quantities. So, when we say e power x, this x will be always some physical quantity.

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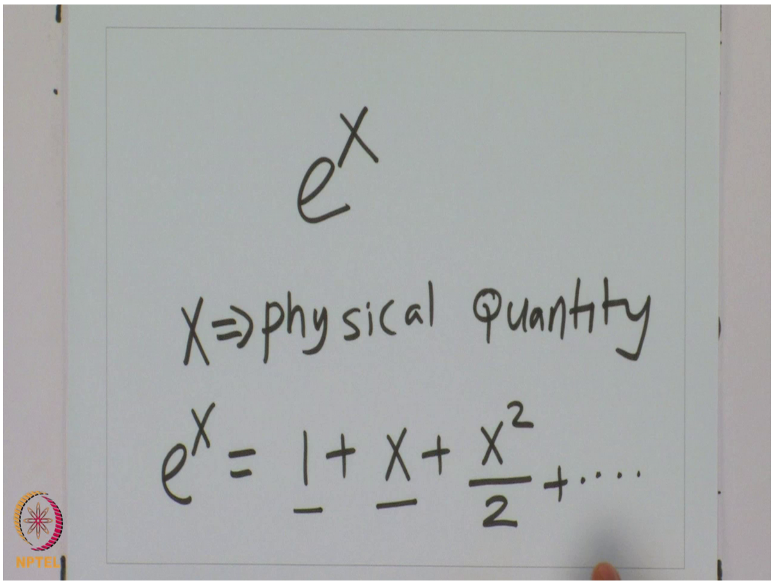

So, sometime x will be some physical quantity that is something that you can measure in experiments, but if you look at this expansion here e power x is 1 plus x plus x square divided by 2 plus dot dot dot. So, you can see that 1 plus x.

So, some very one fundamental thing physical rule is that when you add 2 quantities here you are adding one and x they should have same dimension. So, x and 1 should I have said, 1 is a number which has no dimension therefore, x also should not have any dimension in this definition of e power x. So, wherever, whenever we write x as a quantity we have to write it as a dimensionless number only then this that is what this implies and we will see that x is indeed always written as a dimensionless number and we will see that.

Typically for example, if t is x is some time divided by some another time. So, if we can write some ratio of 2 times x can be written as ratio of 2 times so some very often x will be written as t by t 0 or sometime it will be written as t by tau, t tau t 0 they are all some times in characteristic time which we will understand.

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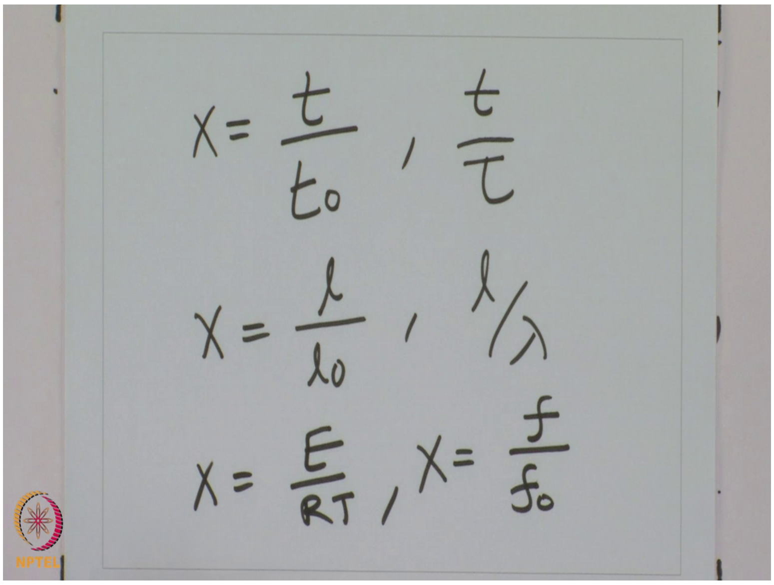

And some other time x will be some ratio of lengths. So, l by l 0 or l by lambda all the lambda l 0 are some kind of lengths. So, when we write this as ratio this will not of course, have dimension therefore, very often x is a you will also see some other times like it will be written as E by RT, RT has dimension of energy, e also has dimension of energy. So, this essentially will not have ratio of this or it can be written as f by some f naught this is some force this is some other force.

So, ratio of some two quantities which are same dimension very often, whenever you see e power something whenever you see e power something the quantity in the power e power x that x will always be ratio of 2 quantities such that the x has not dimension. Always in biology you will see that and there that therefore, this also we implies that e power x has 1 plus x plus x square by 2 and so on and so forth implies that x and 1, if you want to add x and one x should not have any dimension and indeed we will see that x does not have any dimension in all the cases that we would discuss. So, this is something which you should keep in mind I would urge you to look at the dimension of any equation that you see.

So, beyond mathematics when we learn mathematics for a biology we should go slightly beyond mathematics in the sense that we should always think about the dimension of left hand side and right hand side, we will train that, we will train ourselves to do that and first case is this that x will not be having any dimension and we will come discuss that.

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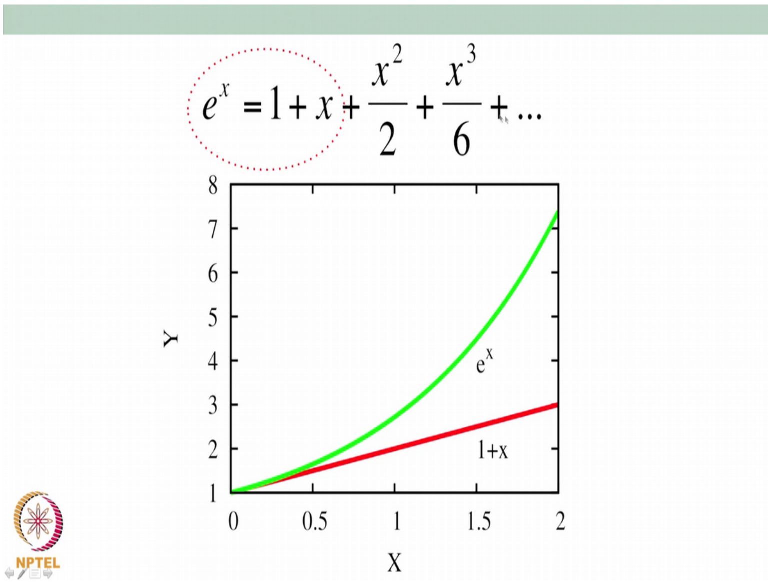

So, now, e power x we said that e power x can be written as 1 plus x plus x square by 2 plus x cube by 6 plus many terms.

Let us now, why what is this infinite series means and what are the implications of this. So, there are 1 2 3 4 terms here 1, x, x square by divided by 2, x square divided by 6, there are 4 terms here. In that if we plot only 2 terms just 1 plus x only what will we get. So, if you plot y is equal to 1 plus x we will get this red line if you plot y is equal to e power x we will get this green line and when you compare this. So, what we are doing here is we are comparing two functions.

So, what we said, we said that y which is e power x and we said that this is equal to 1 plus x plus x square by 2 plus x cube by 6 plus many more terms, that is what we said.

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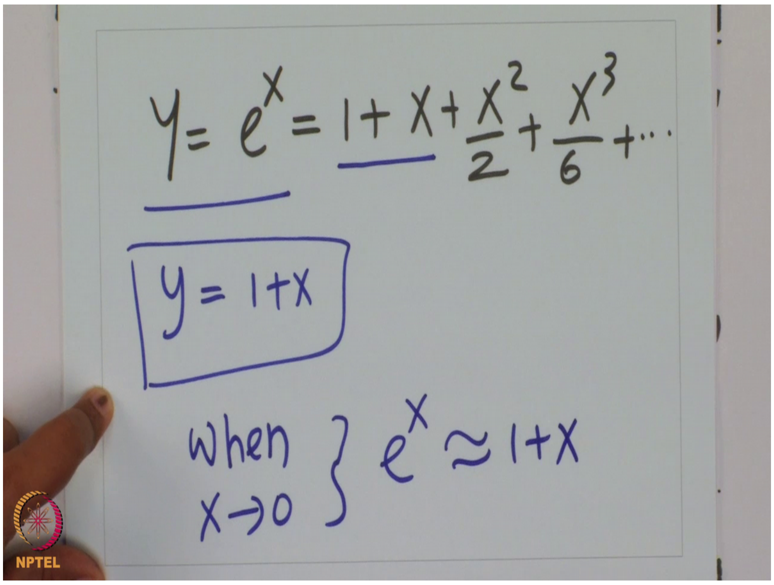

Now if we just take only two terms. So, we just take only first two terms and if we just plot y is equal to 1 plus x only these two taken neglect all other terms what will we get. So, we are going to compare y is equal to 1 plus x with y is equal to e power x and that is what is shown here, y is equal to 1 plus x is a straight line this is just y is equal to mx plus 1 where m is one and therefore, it is a straight line and e power x is this green curve what you can see is that very close to 0, below 0.5 somewhere here very close to 0 this two curves coincide.

But for larger values anything larger than 0.4 or 0.3 things start diverging and then e power x and 1 plus x are very different. So, what is this tell us? This tell us this tells us that when x is very very small, when x is close to 0 e power x is approximately 1 plus x. So, here very close to x is equal to 0 e power x is 1 plus x. So, that is what it is written here, when x is very close to 0 e power x is approximately 1 plus x that is what this comparison tells us. Only when x is very small e power x all other cases this will not be true.

Why since, why do we get, why do we get this correct why e power x is approximately 1 plus x when x is very small, because when x is let us say when x is 0.01, when x is 0.01, 1 plus 0.01, 1.01 plus 0.01 square right. So, if you look at here if you take x is equal to 0.01 y is 1 plus 0.01 plus 0.01 square which is very small number divided by 2 plus 0.01 whole cubed divided by 6 and so on and so forth.

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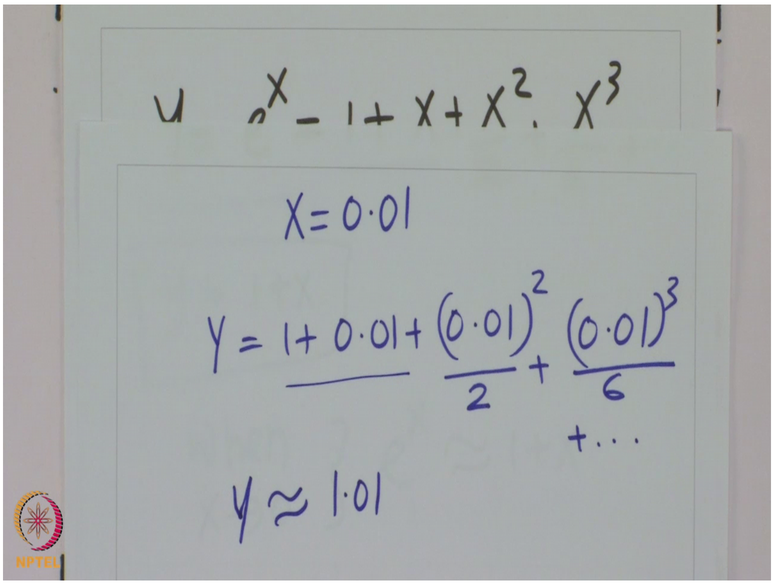

So, when you square this small number it becomes smaller and this is much smaller. So, approximately this 1.01 plus when small number, y will be approximately 1.01. So, this is the reason that why this works, why when x is very close to 0 e power x is approximately 1 plus x.

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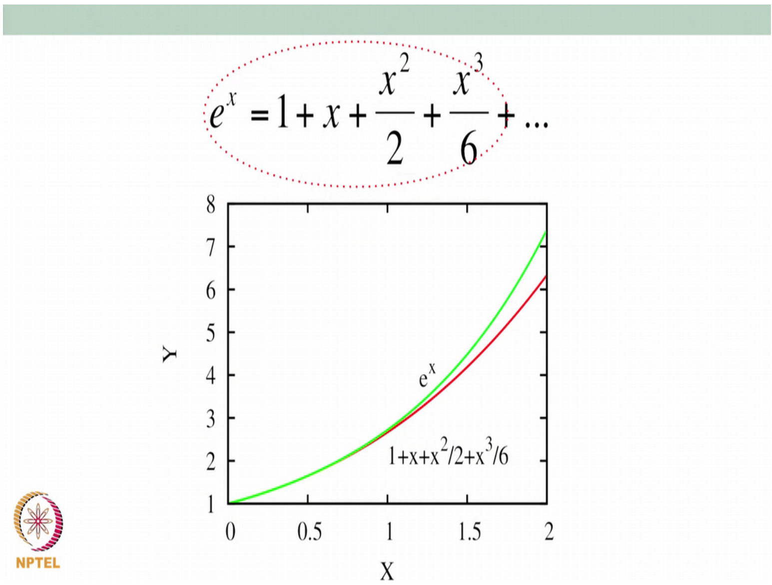

Now let us try another, let us take a few more term. So, in this infinite series in this series if we take 4 terms if we take 1 plus x plus x square by 2 and x cube by 6 and plot this 4 terms that is what this red curve here is this 4 terms I have plotted.

So, I calculated I took an x value calculated 1 plus x plus x square by 2 plus x cube by 6, for different values of x and plotted it here that is what the red curve is then I plotted e power x. What can you see? You see that it fix pretty well even around 0.5, 0.6 somewhere even here much for much larger values of x where this is below one you can see they match, but in again you dead start deviating for larger values of x. So, this approximations will work for small values of x and for large values of x you would need to take more and more and more terms in this infinite series.

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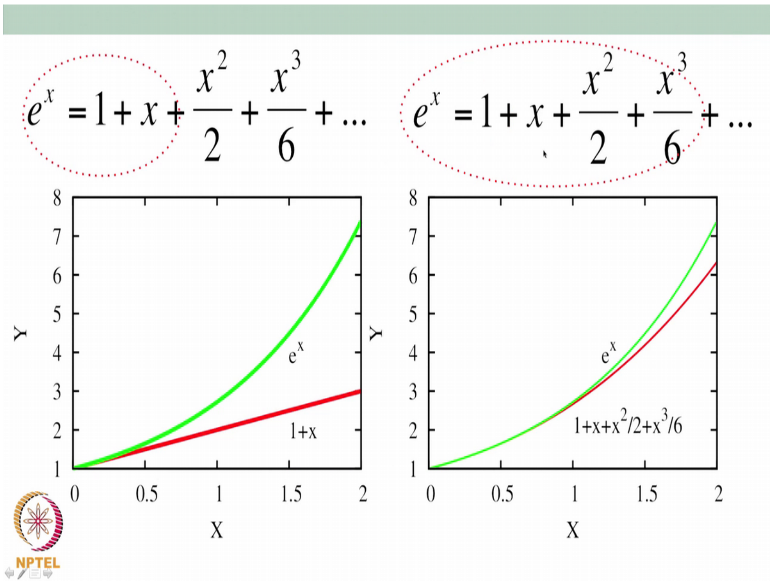

So, here is comparison, if you just take only two terms. So, the green in both the left and right side green is y is equal to e power x, in the red left hand side red is y is equal to 1 plus x that is I take only 2 terms I will get this. If I take 4 terms the red curve becomes closer to the green curve, if I take 5 terms it will become closer, if I take 6 terms it will become further closer and if I take many many many terms e power x will be equal to this sum of these terms.

So, what is this implies that this tells us that as you add more and more terms e power x is indeed sum of these powers and one can demonstrate this just by plotting a graph. Now, we learn many functions all of these functions are increasing when x is increasing they all increasing all of this function increases with x. Now, in nature if you want to use mathematics as a language you would also need functions that would decrease with respect to x because we might want to describe a phenomena where things are decreasing with respect to x.

So, what are those some example functions where things will decrease; in some other cases things will not be either decreasing or nor increasing it will be oscillating. So, the these quantities so far represent, these functions represent quantities that are increasing with X.

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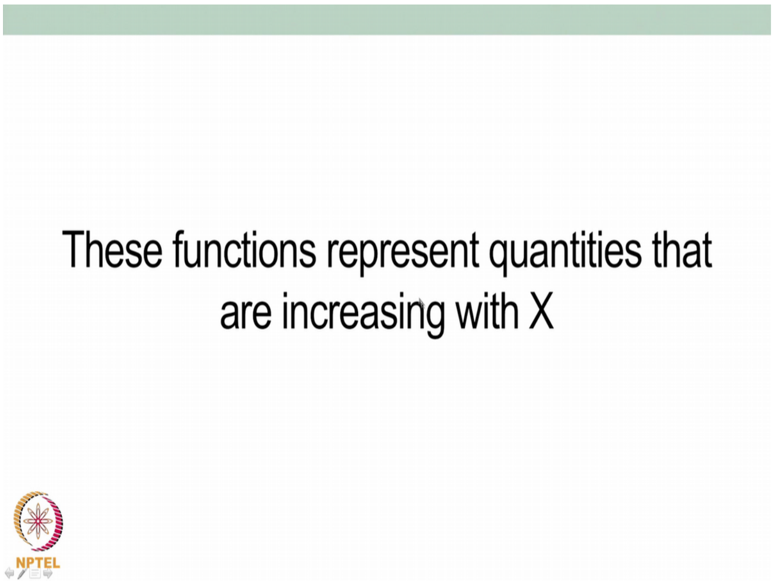

In nature there are quantities phenomena where one should use equations to represent quantities that are decreasing or even oscillating, they could increase and decrease.

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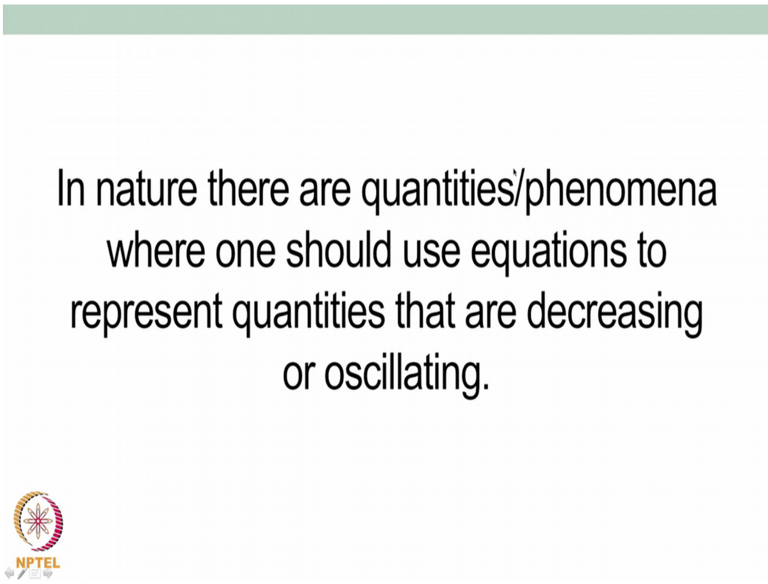

So, we will discuss some example functions first some function that would decrease then some functions that would oscillate and once we learn these functions we can use them to describe some phenomena that we would see.

So, first function is e power minus x.

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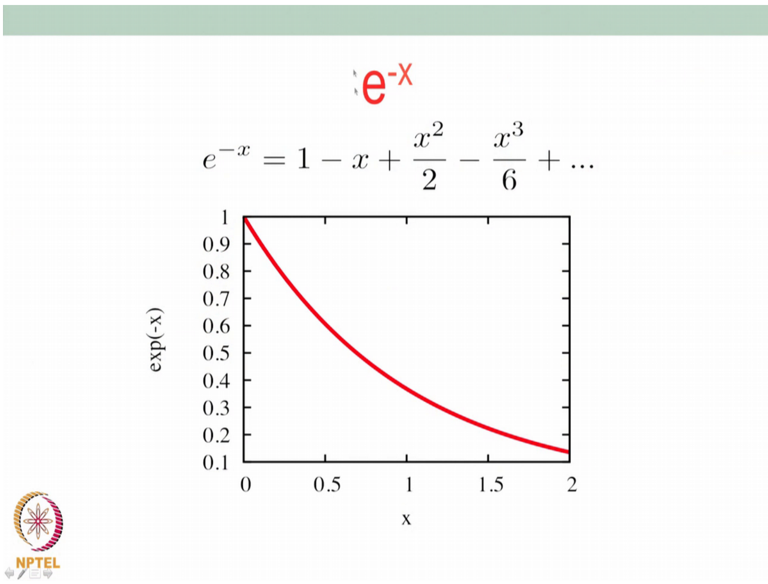

So, this e power x we learnt e power minus x. So, the expansion e power minus x can be written as 1 plus x plus x square divided by 2 minus x cube divided by 6 plus dot dot dot many other terms. So, again this is also an infinite series and when x is 0, you put x equal to 0, this is 0, this is 0, this is 0, if x is 0 e power minus x is 1, it starts with 1. So, when x is 0 the value y value is 1 and then you put some x value positive value it will decrease. So, this is a decreasing function. So, as x increases y will decrease.

So, when you can see that when x is 0 y is one when x became 2 the y already became 0.2 way or close to 0.1. So, it is decrease a lot when x increase up to 2. So, this is a decreasing function. We can see that we said here that why for e power x we wrote that y is equal to e power x and e power x we wrote 1 plus x plus x square divided by 2 plus x cube divided by 6 plus dot dot dot.

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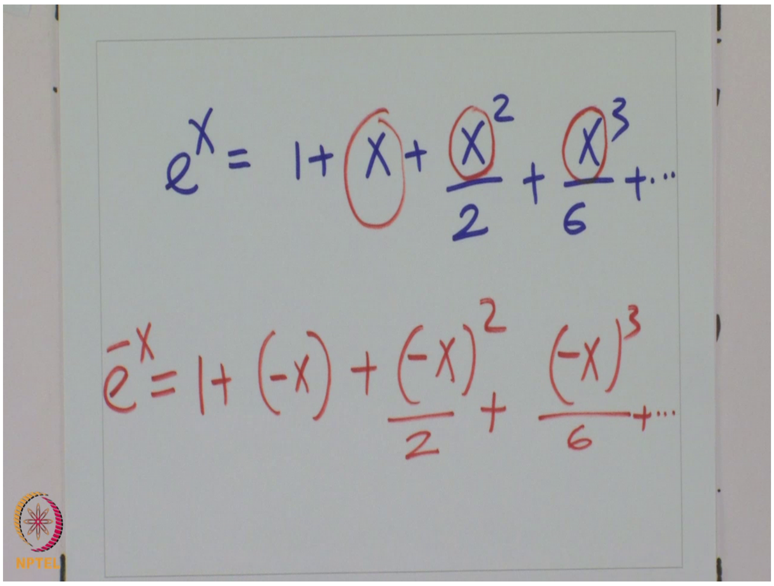

Now, instead of this x here we substitute minus x. So, here wherever there is an x there we will put minus x then we would get 1 plus minus x plus minus x square by 2 plus minus x cubed by 6 dot dot this will give you e power minus x. So, e power minus x is same expansion wherever x you put minus x, here e power x this x you substitute minus x you get the expansion for this.

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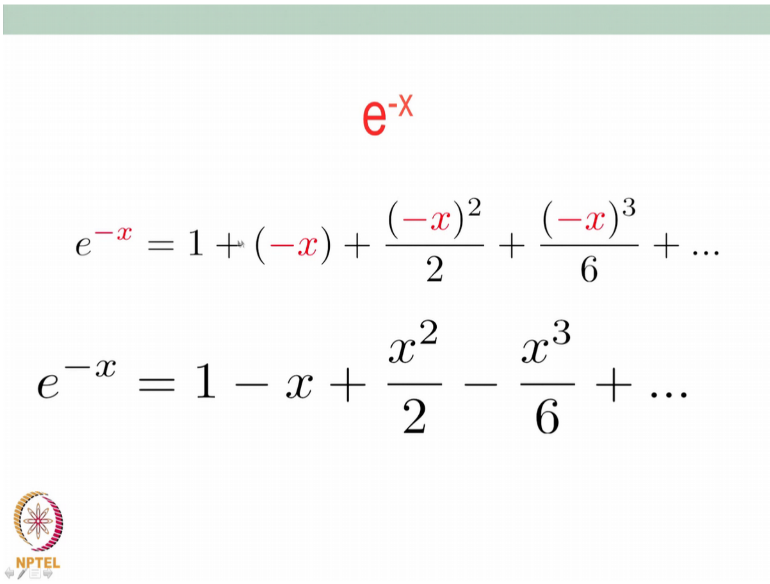

So, this is what I want to describe here e power minus x is nothing, but 1 plus instead of x we put minus x minus x square by 2 minus x cube by 6 plus dot dot dot, same way we had written e power x where wherever x you substitute minus x.

Then if you expand this you will realize that this is indeed this and the thing should remember is that this is a decreasing function and it decreases as x increases. Where are we would see e power x in biology there are many examples where we would see e power x some of them is we have heard this word exponential growth.

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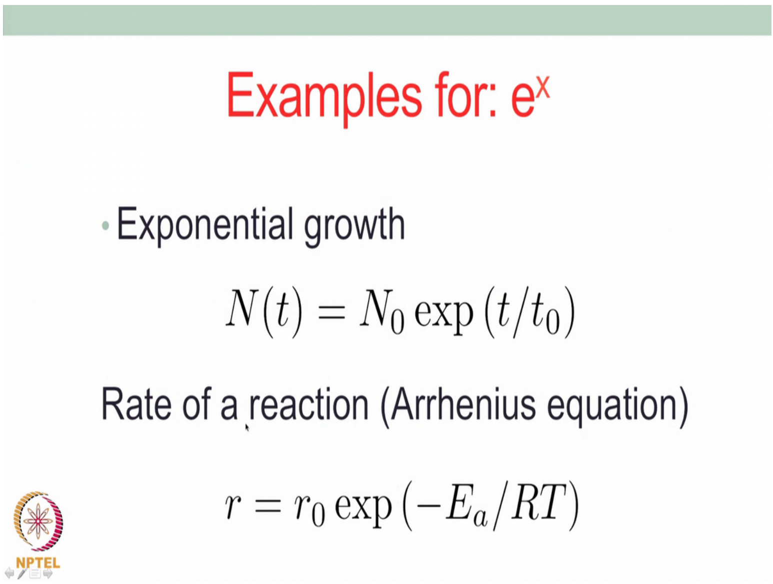

We will learn about exponential growth, but this implies that the number of cells or number of bacteria or number of anything that is growing has this form N is N 0 e power t by t 0. So, with time this increases exponentially. So, this is an one example and we will come back and discuss this.

We can again here see this t by t 0, so dimensionless quantity. Rate of a reaction we might you might have heard of this thing called Arrhenius equation where rate of a particular reaction can be written as r 0 exponential E power minus Ea by RT. So, this is an e power minus x where x is Ea by RT. So, Ea is some energy activation energy. RT is also some term which has a dimension of energy you can it is having unit in joules and this is also in joules. So, this Ea by RT is our x which is dimensionless, here t by t 0 is our x. So, this is e power x and this is e power minus x. So, these are two examples where such functions will come and as we go along we learn more and more examples.

Now, when it comes to e power x and e power minus x things will become slightly complicated as we go along to plot it by making a table, there are many ways of plotting it there are many software available to plot it I urge you to go and look at internet and figure out some software some standard software to plot.

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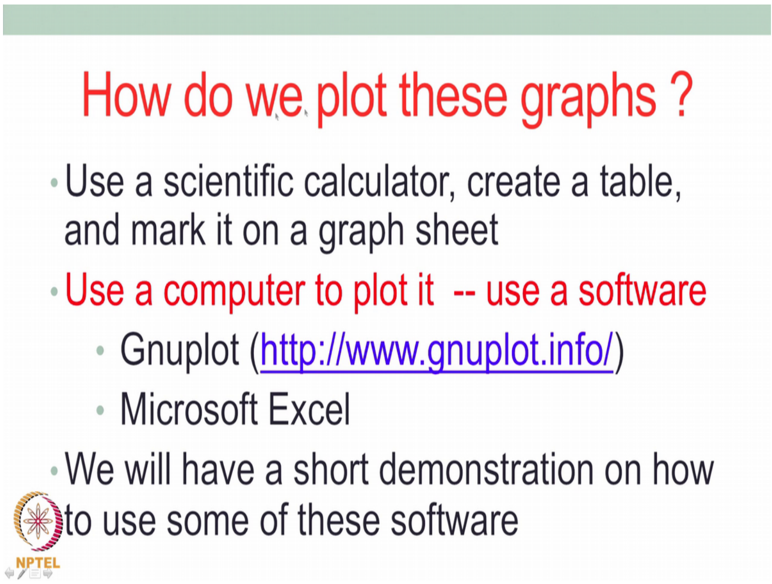

So, how do we plot this graphs? You can use a scientific calculator create a table and mark it on a graph sheet this is the simplest thing that anybody can do, but you can use a computer to plot it you can use a software, one free software is called gnuplot. So, you can go to this website or you can Google search for gnuplot and download gnuplot its a freely available software it is a freeware, it is a open source software which anybody can download and use or you can use Microsoft excel if that is available with you which one will have to pay I think. So, you can use either of the software and if you Google search, Google will search will give you some tutorials which will teach you how to use this software to make plots. So, this softwares will help you to plot e power x all functions and I urge you to play around with this one way to learn mathematics is to plot all functions I urge you to play around with this software and plot all kinds of functions that we will discuss.

So, if you have a computer do download this either of the software and learn how to plot some function some equations using this either of the software or you could use any other software you wish MATLAB or various other free software available that would be available. But you should use whatever be the software should use it and learn how to plot some functions. We will have a short demonstration at some point on how to use some of these software. We will do that and put it up that we will have the demonstration in one of the lectures. So, this is will help you to plot some functions.

Now, we will go to another kind of function that would be oscillating. So, we had e power x increasing e power minus x that is decreasing, what is the examples of function that oscillating.

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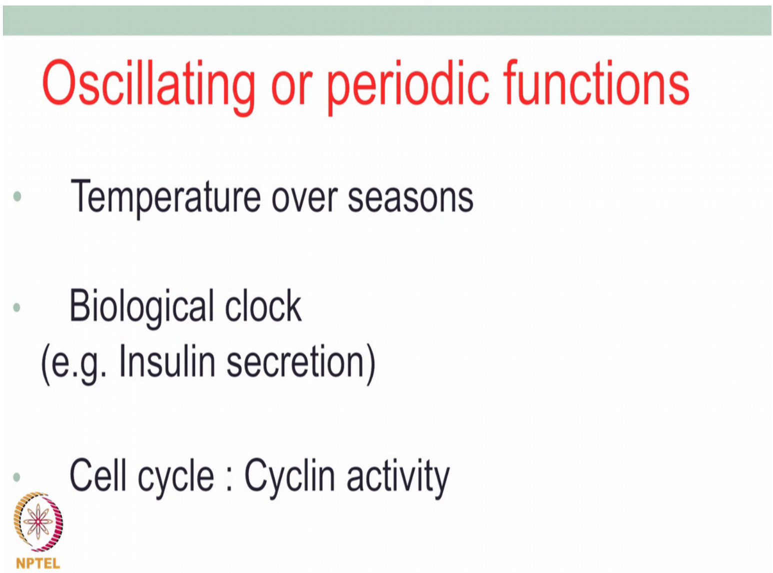

So, there are many phenomena that is oscillating for example, if you think of temperature over seasons. So, you will look at if we begin the lecture by saying that we would want to think of events around us as experiments that is, if you look at if you make a table of the average temperature of every month or every week in around an year you will see that the we haves summer winter and all that. So, the temperature will increase and decrease. So, if you plot temperature as a function of time seasons you will surely see some interesting curves. So, I urge you to do this. So, plot all of you should plot.

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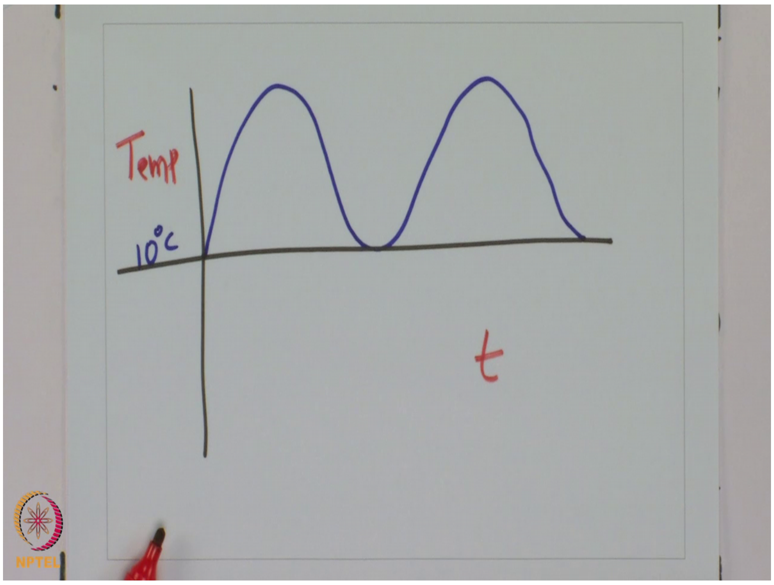

So, if we plot here temperature in the y axis and time in the x axis in months you see that at winter you will have small temperature and then it will increase and then again decrease and so on and so forth. So, you will see that temperature increases then decreases increases decreases as a years go by. So, this is not 0 this is some finite value. So, in India this would be like 10 degree Celsius or depending on the place this value will be somewhere some small number depending on the place you live in the up north this could be 0 or even negative. So, this is how the temperature might look like.

So, this is some guess plot I urge you to take some data and plot temperature versus time and see how it look like. So, this temperature is one example where we will have things oscillating with time.

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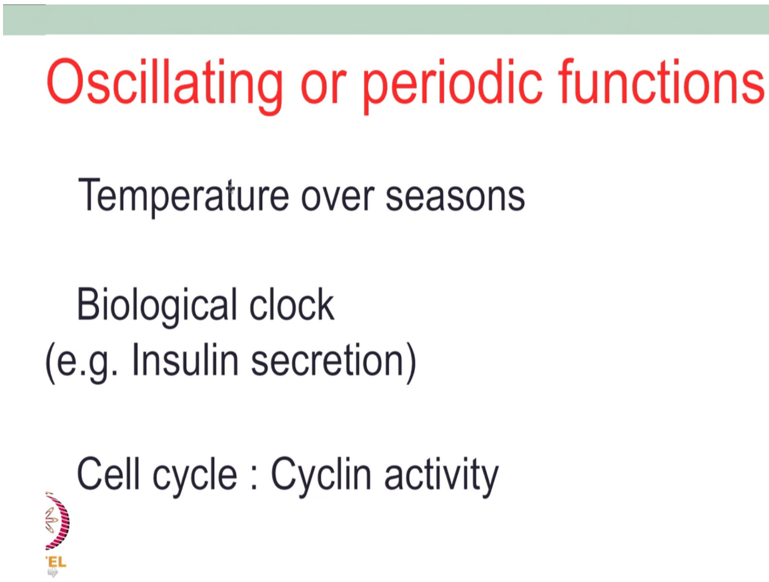

Another example is biological clock. So, as in a day like we have day and night you can even as the day and night proceeds, we will have various proteins like insulin secretion etcetera we will have some oscillatory pattern. In the cell cycle we will have as the cells divide some proteins will have some activity which will also be increasing and decreasing.

Now, the question is what is the mathematical function that we would use to represent this cyclic activity increasing and decreasing activity and one simple function is sin x.

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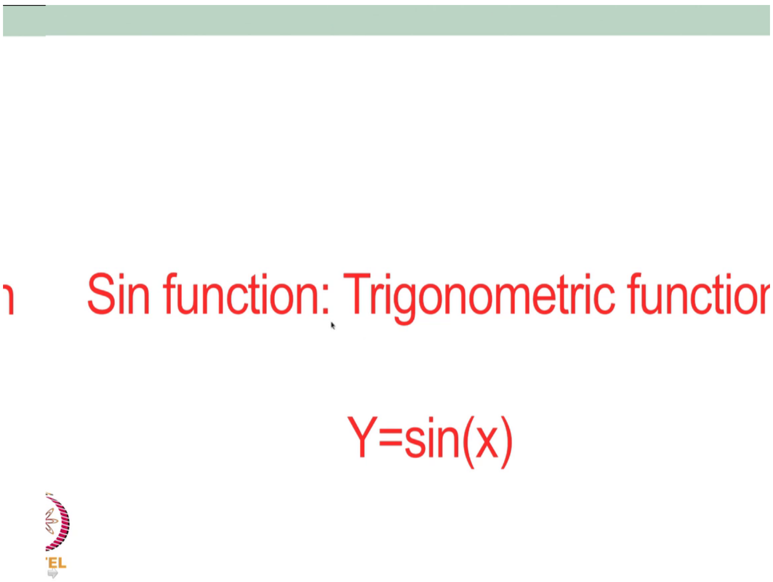

So, sin is the function the trigonometric function. So, you have this function Y is equal to sin x, how do we if we plot this we want to familiarize with this.

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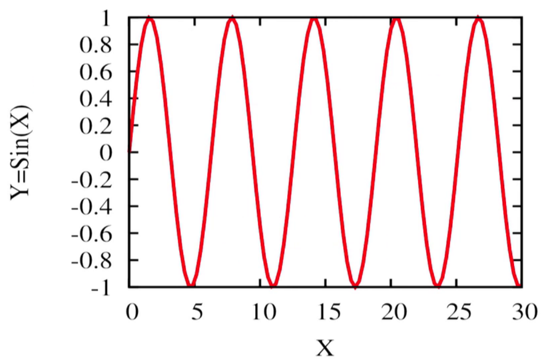
$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \ldots$$



So, if we turns out the sin x can also be written as a combination of x, x of various powers of x. So, if you expand sin x you will get x minus x power 3 divided by 6 plus x power 5 divided by 120 minus x power 7 divided by 5040 plus dot dot dot. So, this is again an infinite series you only have odd powers of x, x power 1, x power 3, x power 5, x power 7 and so on and so forth.

So, you can see there is a minus and plus alternating. How is this expansion coming? I am not discussing that now I will discuss that later. At this moment I just want to tell you that sin x can be written as a combination of x, x power 3, x power 5, x power 7, x power 9 etcetera and it turns out that this is the combination. How this is coming we will discuss later if we plot this, this many terms and sum this and plot it you will get this oscillatory function like this. So, when x is 0 you can see that if 0s all terms will be 0. So, when x is 0 the function is 0 and then it increase and reaches a maximum value of one then it decreases a minimum value of minus 1 then it increases reaches 0 then a maximum value of plus 1 then decreases. So, it oscillates between minus 1 and plus 1.

So, sin x is a function that would oscillate between minus 1 and plus 1.

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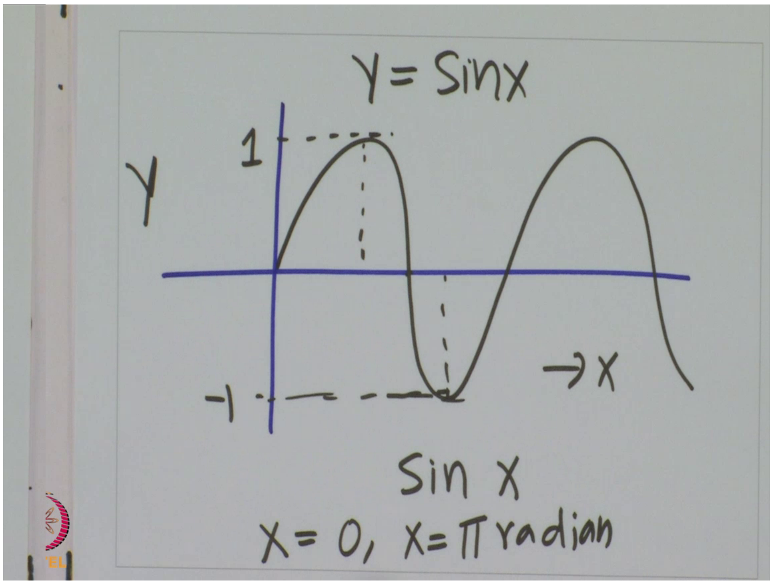

So, if we have; it would oscillate between, this is plus 1 this is minus 1 and sin x will and x is 0. So, this is your x and this is y is equal to sin x and when x, x is 0 this is 0 and then at some point it will become 1 and at some point this will become 0 again at some point it will become minus 1 again x.

So, let us say typically as you know in trigonometric function x in angle. So, x can be represented in radiant. So, when x is 0 radiant or x is equal to pi radiant. So, will you when x is equal to pi radiant that is when x is equal to 3.14 pi you know pi is a number which is 3.14 it is a irrational number. So, when x has that value you will get some function when x is pi by 2 when x is pi by 4, pi 2 pi and so on and so forth you will have various values here. So, one can take various values of x and plot this and you will get this oscillatory function.

So, today we discussed e power x, how e power x can be approximated as 1 plus x plus x square or 1 plus x for small values of x, and then we learned e power minus x, we learned sin x which is an oscillatory function.

So, to summarize we learned, let me summarize first what we learn today.

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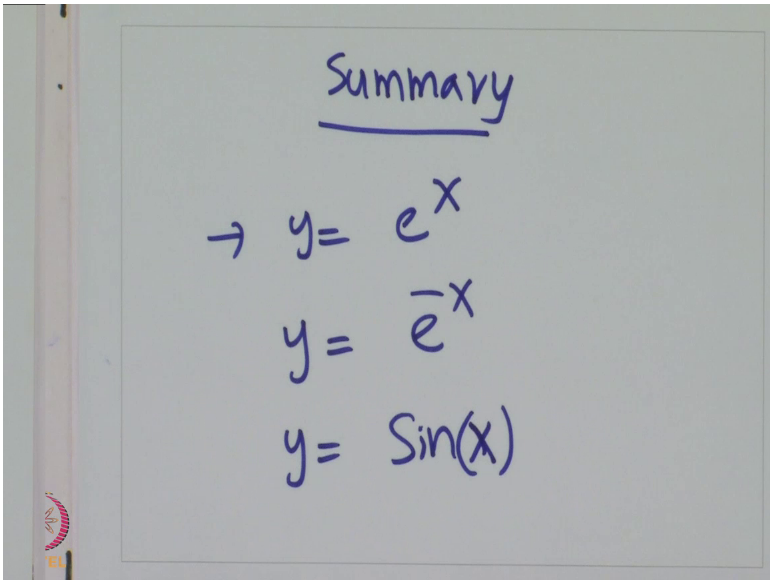

Summary we learned y is equal to e power x is expansion, and then we learned y is equal to e power minus x, then we learned y is equal to sin x which is an oscillating function and we learn a few more functions in the next lecture. And with that we will stop the various functions and we will learn more things in the coming lectures. With this I will stop today's class, see you in the next class.

Bye.