Introductory Mathematical Methods for Biologists
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Lecture – 33
Fourier Series: Part II

Hi. Welcome to this lecture on introductory mathematical methods. In this lecture, we will discuss Fourier series.

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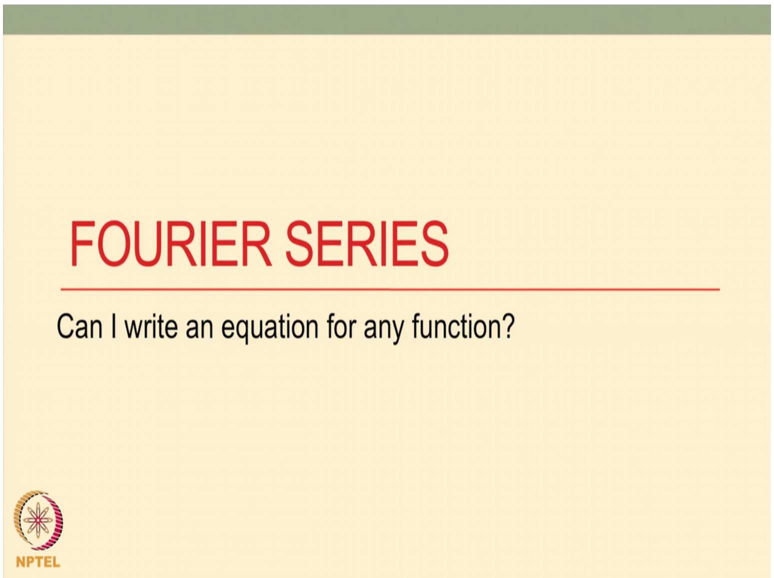

So, title is Fourier series can I write an equation for any function that is what we will continue discussing what we said is that if you have a periodic function something that is repeating in space or in time essentially they whatever be the function x or t if it is periodic you could write an equation for it use in terms of sines and cosines; what we will do is to if we have a periodic function f of x.

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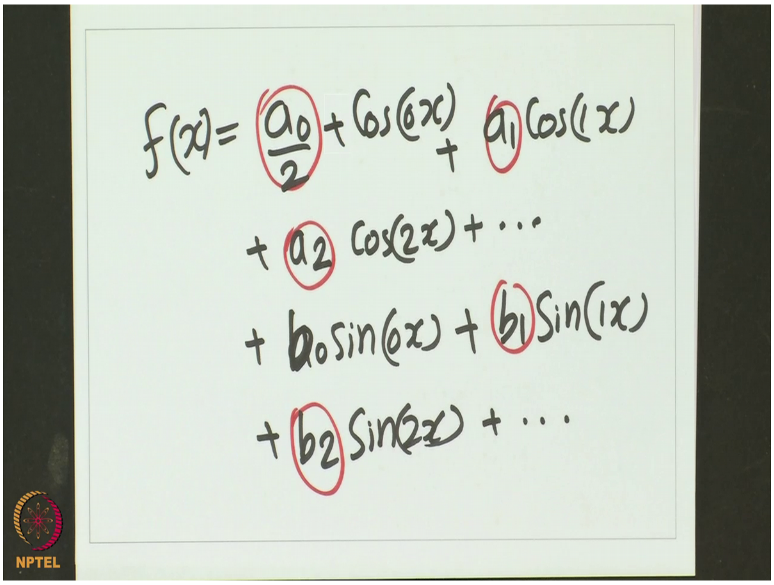

I could write f of x is equal to a 0 by 2 cos 0 x. Since cos 0 is 1, this will be just a 0 by 2 plus a 1 cos 1 x plus a 2 cos 2 x plus dot, dot, dot infinite series plus a 0 sin 0 x.

Since sin 0 is sorry; b 0; I would write a different coefficient b 0; b 0 sin 0 x since 0 is sin 0 0 b 0 this term is irrelevant plus b 1 sin one x plus b 2 sin 2 x plus dot, dot, dot infinite series. So, if I specify this coefficient; for example, a i's a 1, a 2, etcetera and b i's b 1, b 2, etcetera, I get this function f of x. So, if I specify all this coefficients, I can write down this function as suppose to this if I know f of x, I can also calculate this coefficients and there is a procedure which we discussed, I multiply if I want to calculate f a 1; I multiply this f of x with cos one x and integrate.

If I want to calculate b 2, I take f of x multiply with sin 2 x and integrate. So, if I do like this I would get any a i's a n's; I want or a i's I want.

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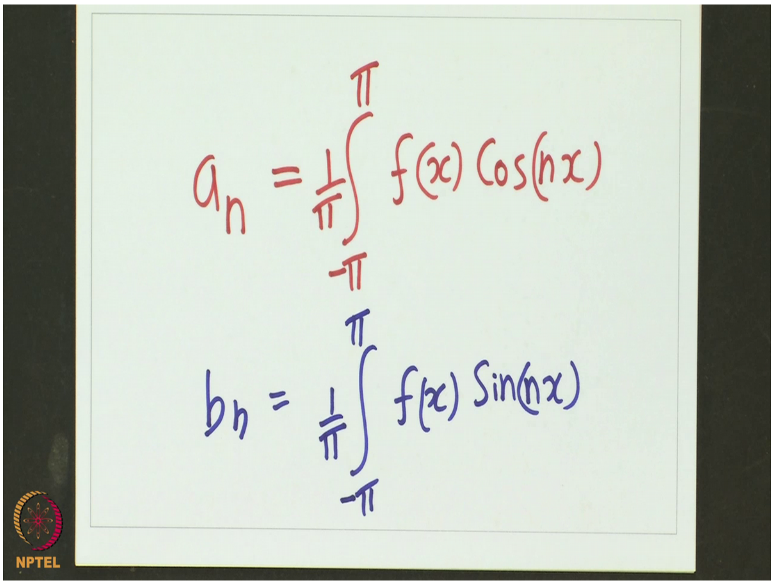

So, if I; in general, if I want to get a; whatever be the n to do that; I can have to integrate I have to take f of x integrate as are coefficients of cos. So, I would take cos nx and integrate typically the way it is written minus pi to pi and I would define this one by pi this is 1 by pi is just a matter of definition there are different ways of defining it you can look at in the book.

Similarly, b n would be integral take our f of x since b n's are coefficients of sin I would multiply sin nx and integrate minus pi to pi it is a matter of definition again and I would get the like this. So, this is the way one would get this coefficients if I know f of x, let us quickly look at an example imagine that now you have a protein or you have a periodic function which has the following shape which is like this which is like an inverted l if I wish.

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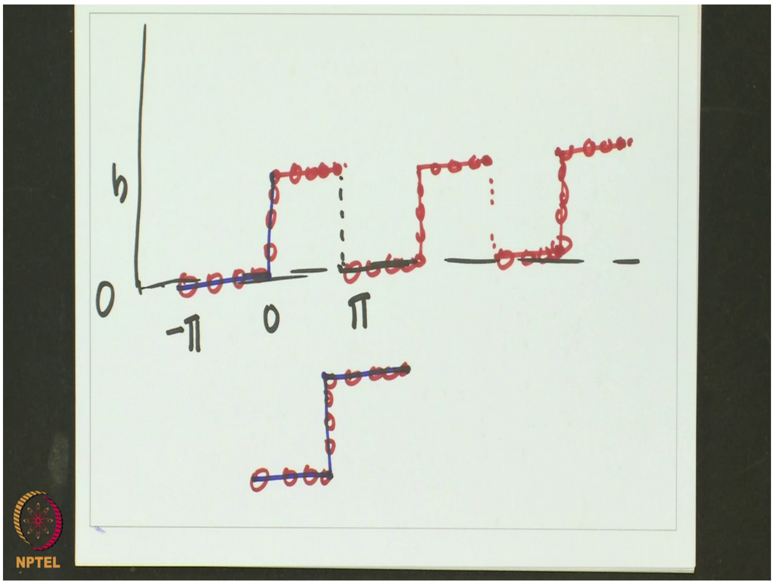

So, basically you have atoms which is like this and like this. So, there is also like this. So, beyond up to some value this just let us let us call this 0. So, this is my I would call this as the origin of my graph 0. So, I would call this if I have a graph I would call this 0. So, I have 0 here and it is 0 from some point to some point. So, let us say this is minus pi to 0, their function value is 0 and 0 to pi, this has some value, let us call it h some height h and then this will repeat again many times.

So, I would again repeat from here to here; it would repeat again. So, this is where pi it would repeat again. So, I have values here and then it will go like this then again it would repeat. So, this is my function which would look like this if I look at it, it looks like a square wave; if you wish; if I specific; specify this height and this and all then it will looks like a rectangular square wave depending on how you plot it. This looks like a wave front is like repeating any repeating thing would look like a wave.

So, but this is basically as we saw it is repetition of a function like this. So, I have a function or I have a protein which has a shape like this and I crystallize it I make repeats of this in one d along this x axis many times then it would look like this. Now what is the equation for this is the question and to understand the equation for this; we could use the Fourier series what is the; so, this is my example another example of Fourier series my function is 0 from minus pi to 0.

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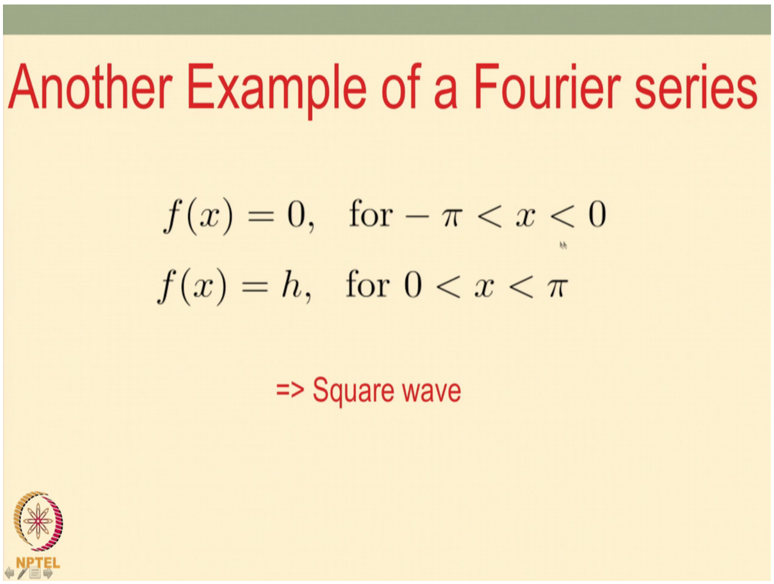

So, this is what the first part is minus pi to 0 the function is 0. So, here it is 0; 0 to pi 0 to pi the function has a value h. So, 0 to pi the function has a value h there is some y value.

And again repeats 0 to in this region minus pi to some a region of a distance of pi it has value 0 then it has a constant value 0 constant value. So, this is the repetition of this that would look like a square or rectangular wave depending on the value of x.

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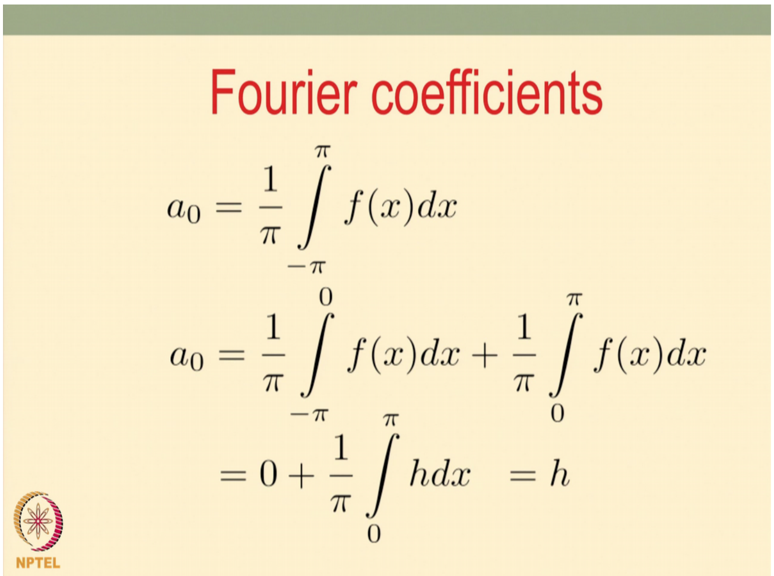

Now, we can get the coefficients a n and b n by the definition of integral that we wrote. So, you could get a 0 by this integral f of x dx y our f of x is a constant which is x it is minus pi to pi I can do and which basically, I can write it is minus pi to pi and 0 to pi. So, the first part is 0 minus pi to 0 the function is 0 0 to pi the function is just h. So, here f of x I would put h.

So, this is integral hdx which one with one over pi it will be just h here itself. So, you have this value of h. So, I would get a 0 equal to h.

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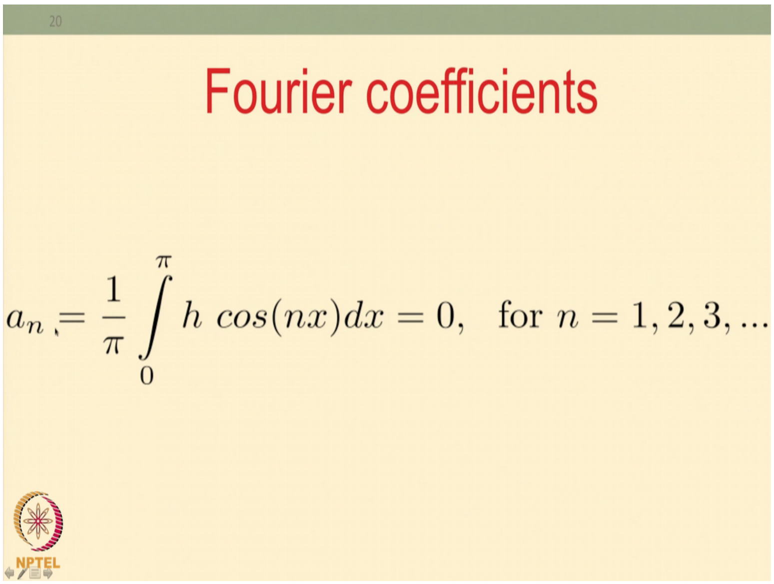

What is a n which is a 1, a 2, etcetera; I have to multiply h which is my function cos nx and integrate for various values of n. So, whatever value I take n is equal to one h is a constant. So, integral of cos would turn out to be 0. So, this is this function would turn out to be 0, I urge you to integrate this and check this yourself you can check 0 to pi cos one x which is cos x would be 0 there will be a plus part and a minus part they will cancel area under this curve signed area would be 0, right.

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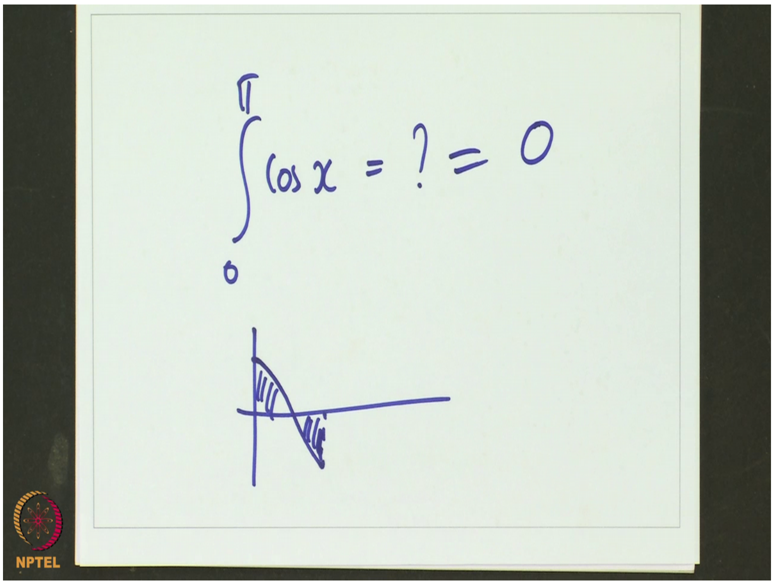

Similarly, so what does; what are we saying? We are saying that integral 0 to pi cos x, this you calculate and you would get an answer which would be 0, if I have a able to plot the function would look like this. So, there is a positive part and a negative part right. So, there is a positive or negative part they will cancel each other and you would get the answer to be 0.

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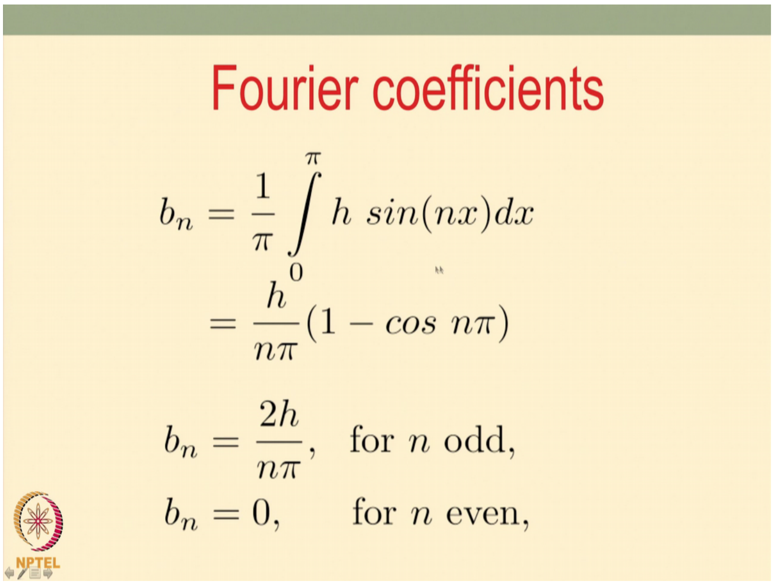

So, that is what the reason one would get 0 for any value of n what is the b values bn; b n again this integral h sin nx dx if I do this integral of course, integral of sin is cos and if I apply the limit 0 to pi. So, integral of sin is cos if I apply limit 0 to 2 pi 0 to pi I would get one minus cos n pi. So, please do this carefully. So, for when n is odd there is n is 1, 3, 5, etcetera. This would be 2 h by n pi when n is even this would become 1 minus 1 0. So, you would get answer to be 0. So, when an n is even bn, there is b 2, b 4, b 6, etcetera, 0, b 1, b 3, b 5, b 7, etcetera are 2 h by n pi; what does it mean b n is 2 h by n pi what does that mean.

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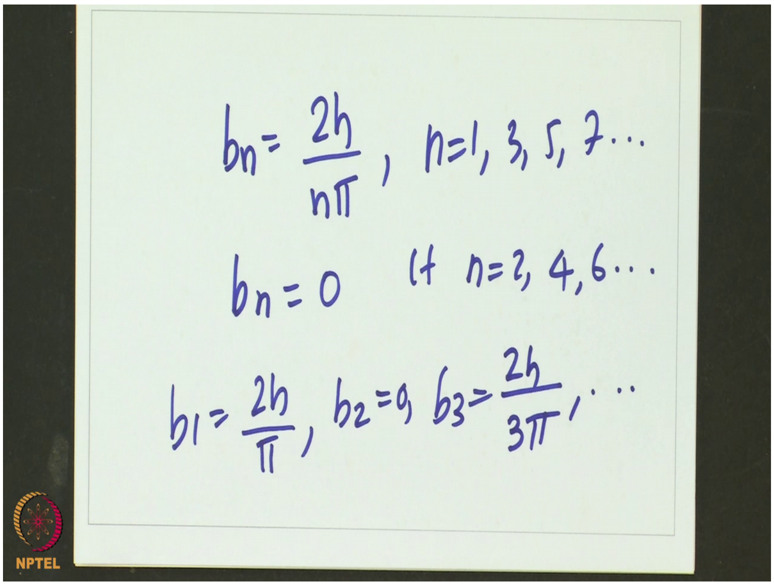

So, b n is 2 h by n pi for all the odd values that is n is equal to 1, 3, 5, 7, etcetera and b n is equal to 0 if n is 2, 4, 6, all these even values. So, this means b 1 is 2 h by 1 pi b 2; b 2 is 0 b 3 is 2 h by 3 pi and so on and so forth. So, we can get all the b values and a values and if you know the all the a values and b values.

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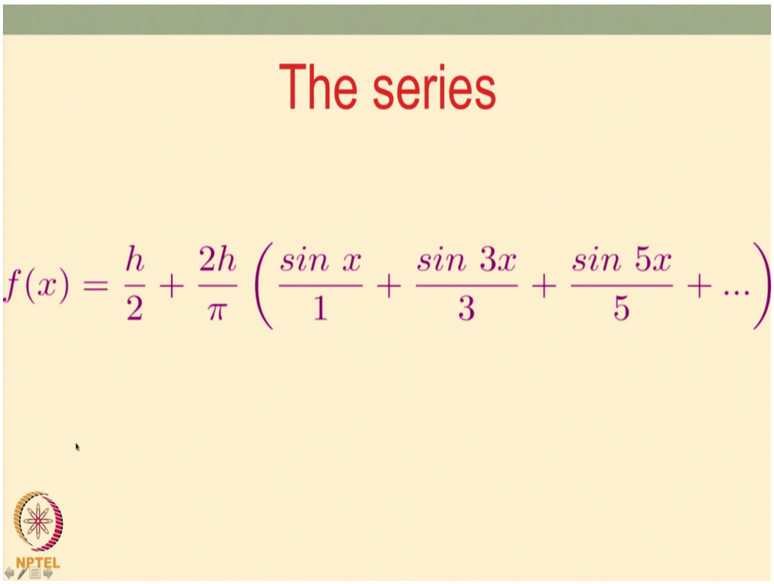

We can write the series f of x as h by 2 which is the first term next term is 2 h by pi sin x.

The next term is even terms are 0. So, the next term is 2 h by pi times sin 3 x divided by 3 the other next term is 2 h by pi sin 5 x divided by 5 plus dot, dot, dot; this is an infinite series. So, we have an equation for the protein for such a repetition of this kind of a protein that we saw. So, what did we start with we started with a protein which is like this we started with a protein like this and this was repeating if you if I have such a repeating protein which I want to write like this.

So, if I have a protein which or any molecule which has this shape.

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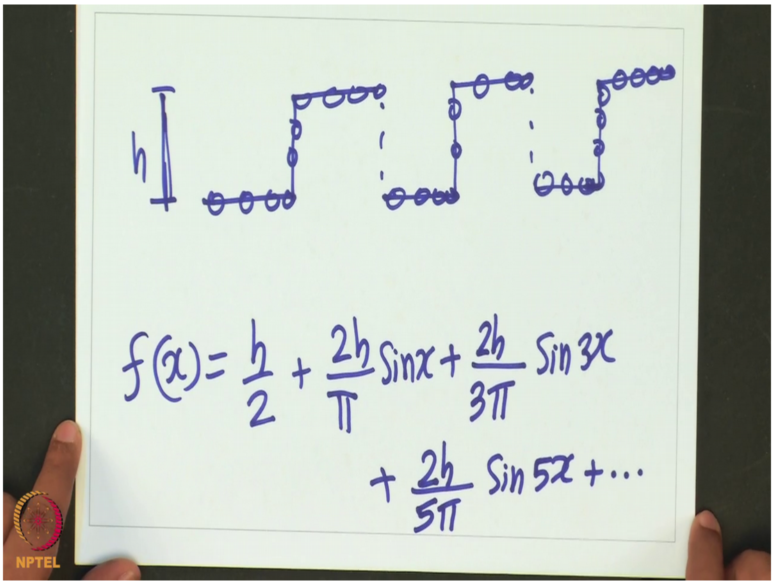

So, these are and if I repeat this many many times again I repeat again I repeat. So, if I make a periodic array of this protein I can write an equation for this and that equation is what is written here which is let me write quickly here the equation is f of x is h by 2 where h is the height this height is h this height is h.

So, h by 2 plus 2 h by pi times sin x plus 2 h by 3 pi sin 3 x plus 2 h by 5 pi sin 5 x plus dot, dot, dot. So, I have a infinite series like this which would be an equation for such a repeating protein. So, I can write down an equation for such a repeating protein which is nothing, but a series like this why did I use; I just used a simple idea that any periodic function can be written as a combination of sines and cosines.

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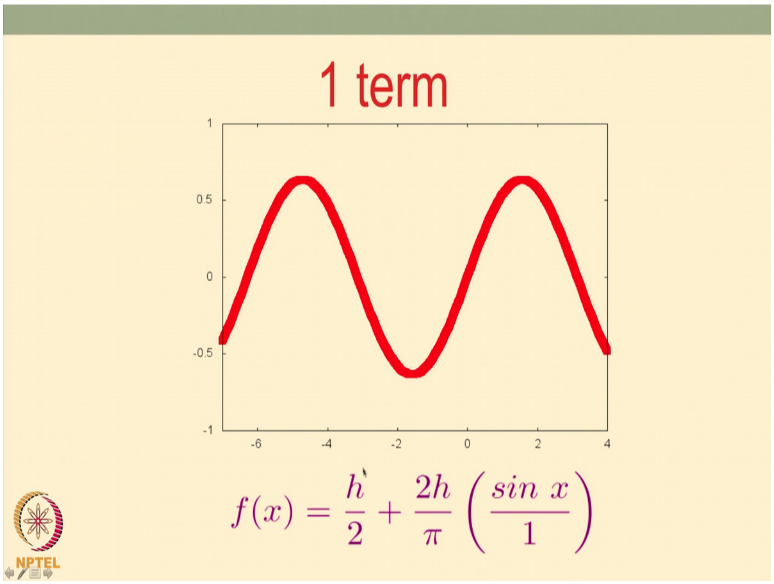

Now let us plot this and see let us lets plot this and see. So, if I plot just these 2 terms if I h by 2 is just a constant if I add one more term first sine term you will get 2 h by pi sine x. So, this is like a sin x.

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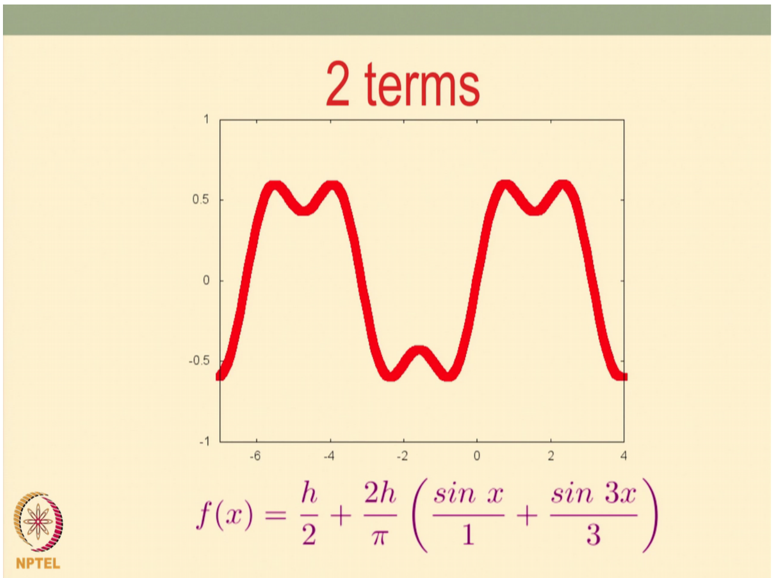

And if I plot it would look like this if I plot 2 terms which is 2 h by pi sin x plus 2 h by 3 pi sin 3 x if I add these 2 terms with h by 2, I would get a shape like this.

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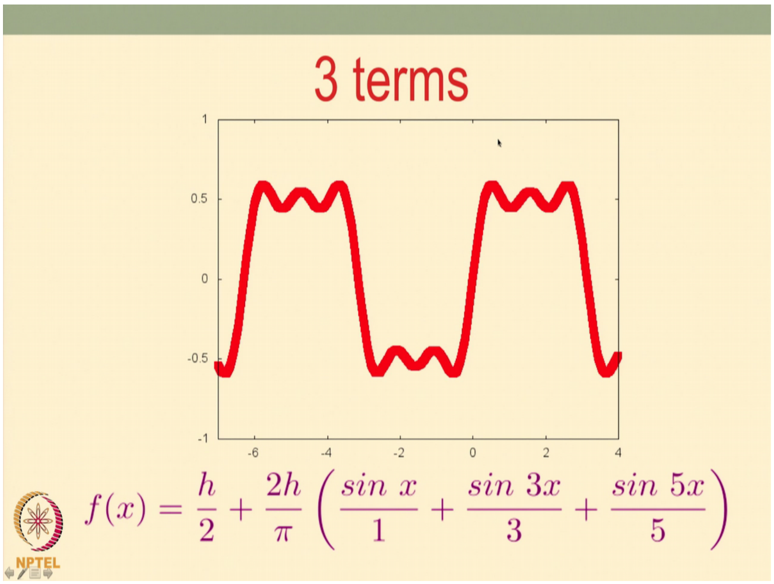

If I add one more term it would look like this, if I add one more term it would further look like this.

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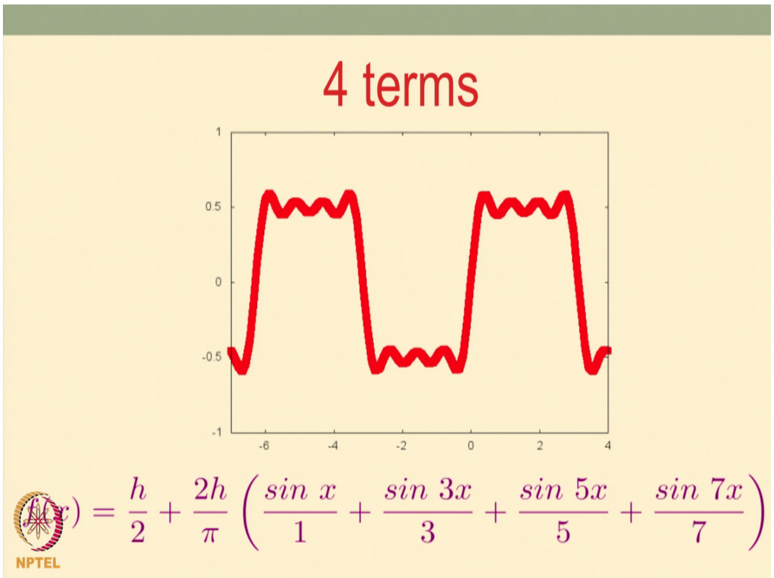

So, now it is coming closer to a wave which is like a rectangular or square shape. So, I have now up to sin 7 x by 7.

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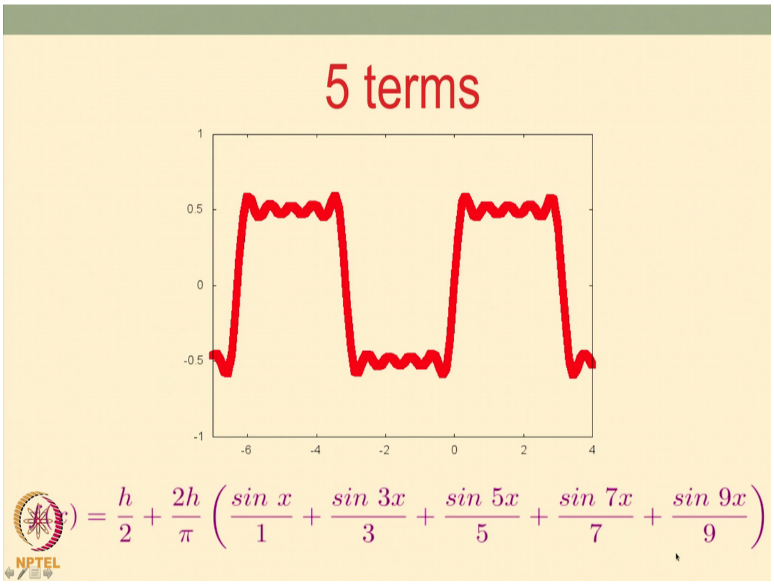

If I add one more term up to sin nine x by nine I would get something, I can approximate to the shape that we have here. So, I can approximate to this shape that I have we except some wiggliness here except there is some wiggliness here and some wiggliness here and so on and so forth; apart from this wiggliness, I can equate this pretty much like this. So, I could essentially write an equation here which is like a repetition of a protein which is this this part, right.

If I just take this part this equation is essentially given by this. So, this is the idea; we can if I have a repeating function if you have a crystal you could imagine that is like a repeating function and you can write an equation for this now it turns out that whenever you do a scattering experiment you will have sent some kind of light wave X ray waves are also some kind of light is not visible light, but some electromagnetic waves they scatter they scatter on a periodic function or a crystal or lattice or and when you collect the light coming out of after scattering all the optical processes will happen and you collect the light.

It turns out that; from that light one could back calculate the shape of this protein by doing some mathematical operation which is related to understanding of this Fourier series. So, once we know the Fourier series we can understand how that back calculation is that is the interesting thing I how this is going to become useful just to think of it let us just suffice to understand in a simple terms.

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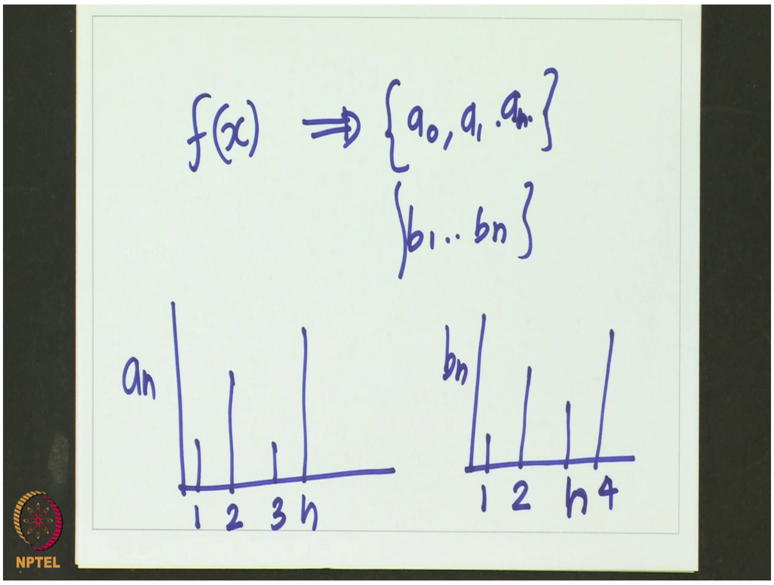

I have this f of x; I have this f of x which is my periodic function if I specify all this a 0 a 1 etcetera and b 1 b n; all this b n values and an values if I specify this I can get f of x as I said if I can plot an versus n and b n versus n for each n value I could plot the values of a n’s and b n.

So, what does it mean n is equal to 1 the a value is this much when. So, this is one n is equal to 2 the a value is this much when n is equal to 3 the a value is this much when n is equal to 4 the a value is this much and. So, on and. So, forth similarly here also when n is equal to one the b n value is this when n is equal to 2 the corresponding b n value is this height n is equal to 3 the corresponding b n value is this when n is equal to 4 the corresponding b n value is here.

So, by drawing a graph of a n’s and b n’s which looks like some peaks. So, if I specify some peaks that is equal under specifying this function in simple terms you could imagine that when you do a scattering experiment you might get just this a n’s and b n’s. So, this if I do an experiment, if I can get a n’s and b n’s which are some peaks like this here if I get these peaks a n’s and b n which are peaks which I can if I can get this a n’s and b n’s from my experiment I can use that in my equation back calculate this f of x this kind of idea, I am simplifying the idea little bit, but some version of this idea some advanced mathematical version of this idea is essentially what is being used in many of this scattering experiments including x ray and many scattering experiments.

What does it mean when we do experiments we get equivalence of a n’s and b n’s; you can combine an and b n and write a new function called c n, then you would get just c n. So, let us quickly see that. So, what do I have what am I saying.

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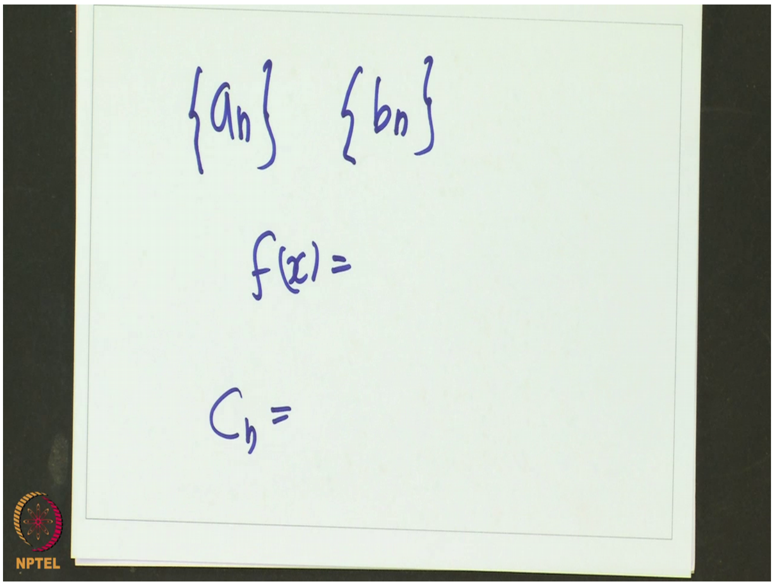

If I know an and b n, if I know the values of all values of an and all values of b n, if I know this from experiment I can get my f of x if I do an experiment I can get this an and b n it; I can also define some function C n, which is some combination of an and b n and experiments might just give this and it is also enough to get the c n and that is also you say enough to get this f of x.

How this is not very important, but I just want to tell you the idea let me quickly tell you how knowing how to get define a c n which is sometime useful it turns out that.

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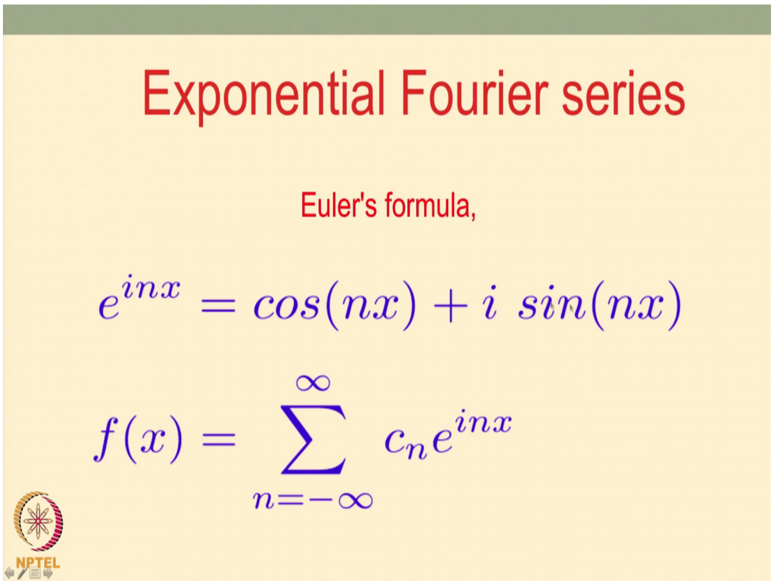

If I use cos nx plus i sin nx this i is root of minus one this i is called imaginary number root of minus 1; I can define cos nx plus i sin nx as e power inx, then I can write f of x as just c n e power inx.

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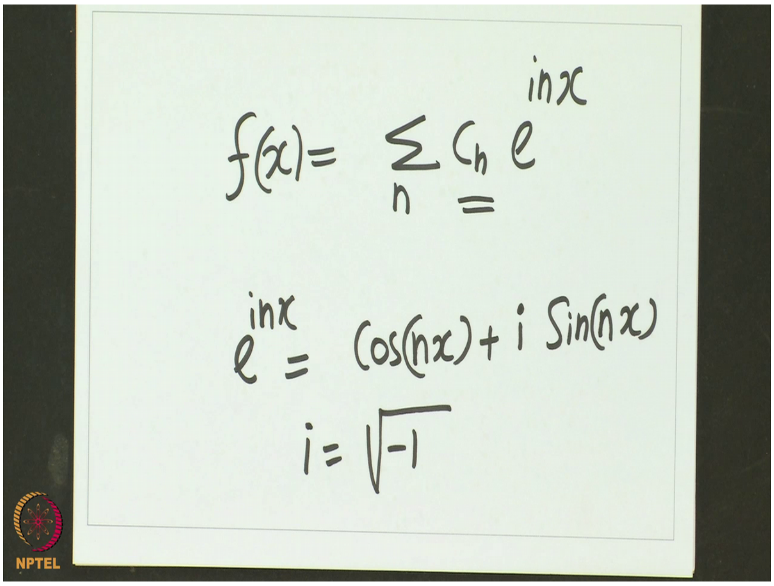

So, instead of writing this whole cos and sin I could just write f of x as sum over n c n e power inx, this is possible if I define c n appropriately.

But e power inx is nothing, but cos n x plus i sin nx what is i? I is nothing, but root of minus one which is call the imaginary number. So, if I have; I can simplify the a n and b n in some particular manner it is just a mathematical trick is nothing physical about it; it is just if I use this mathematical trick; I can define this and I can get e power inx written in this particular fashion, if I do this I can get c n appropriately by doing integral of f of x multiply with e power inx. So, that is if I want to get a coefficient I can multiply e power inx with f of x and integrate and I would appropriately get the values of c n.

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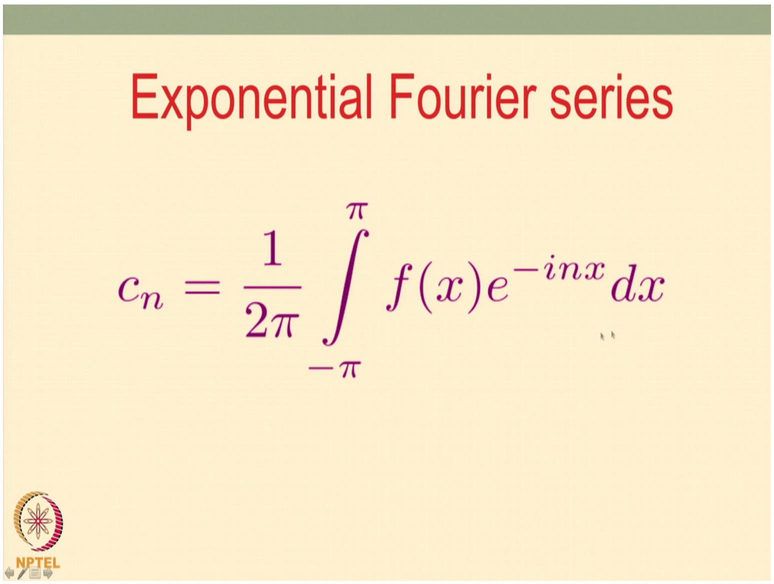

So, for example, C n; I can get as 1 by 2 pi minus pi to pi f of x e power minus inx dx this would give me C n; now how do I define C n.

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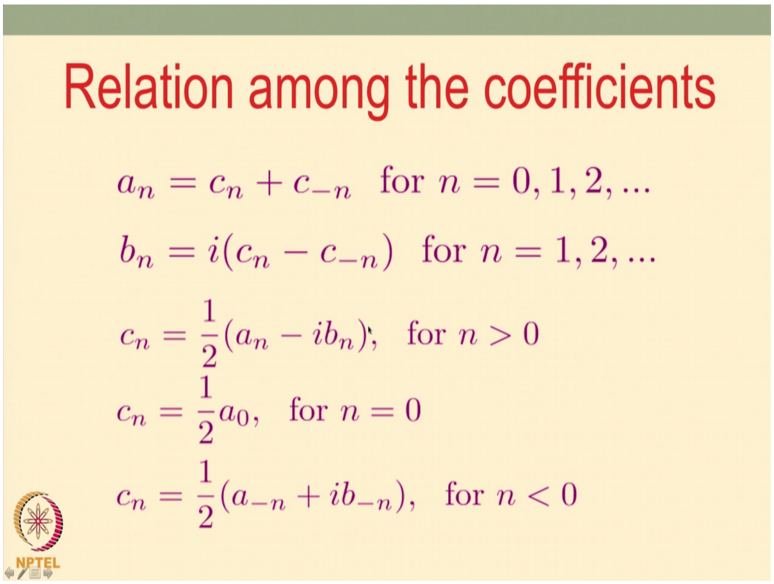

So, there are various interesting ways. So, a relation among the coefficients can be designed in a slightly complicated way it is not very important to understand everything of this in detail, but this is just to show you that we can define an b n and we can combine them to get c n and once.

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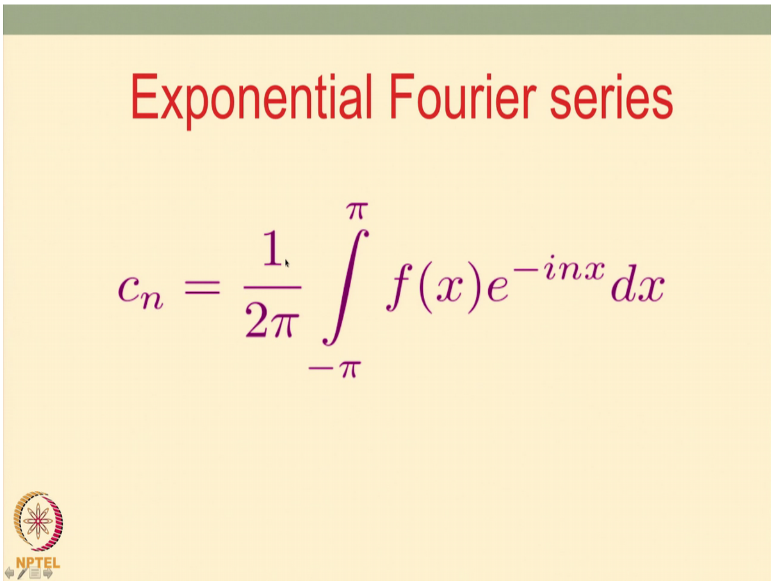

We have C n; I can write f of x as some coefficient of we can write f of x as an infinite series in this particular manner and if I have this infinite series I could get f of x.

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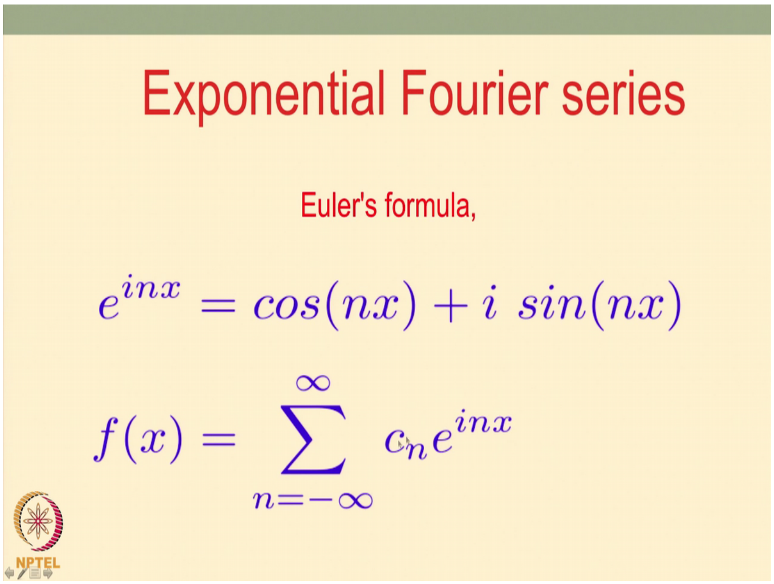

If I know specific c n’s; I could get the values of f of x. So, in other words what would it also mean? So, what am I; what are we saying if I specify this coefficients.

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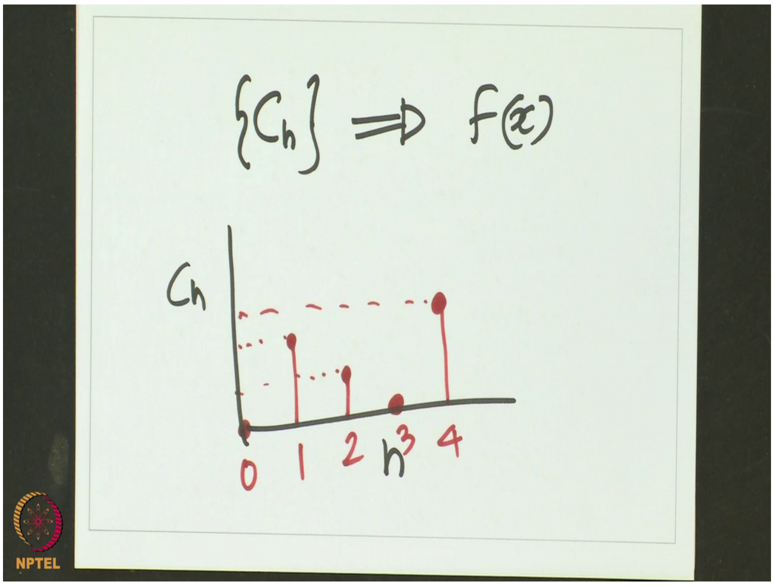

Cn values; I can get the corresponding f of x this also means that if I have a protein or a crystal or a periodic function of our interest I can do an experiment I can get some c n values. So, if I can plot c n versus n and get some values, let us say. So, for some value c n is 0 some other value c n has some value some other value c n has some other value for some value c n is 0 again.

So, some other value c n has some other value if I get some peaks like this c n has a function of n some numbers like this. So, these all corresponding y value, I have a corresponding y value here I have a corresponding y value here; I have a corresponding y value here. So, if I know the corresponding y value for a given n. So, this is 0, 1, 2, 3, 4 and so on and so forth. I can reconstruct the f of x and one can for simplicity; let us imagine; let us think about in the following way we are doing a scattering experiment and from the experimental data we are getting c n values and then we can reconstruct f of x that is the idea.

And many experiments can give these coefficients c n’s and little bit about more about this we will learn in the coming lecture and we will try to understand this in detail little bit more and how to play around with this and do something more in the next lecture. So, with this I would; I want to summarize by saying that any periodic function can be written as some combination of science and cosines, which it input terms can be written as e power inx some exponential function e power inx, which if you wish it is like a wave sine and cosine like wave. So, if I do a scattering experiments with some wave like light for example, from that experiment I can get the coefficients a n's and b n's or c n's and from that we can back calculate this function f of x. So, that is one way of thinking about it in the next lecture we would little bit more discuss important things related to this.

With this I will stop this lecture and continue later. Bye.