Introductory Mathematical Methods for Biologists
Prof. Ranjith Padinhateeri
Department of Biosciences & Bioengineering
Indian Institute of Technology, Bombay

Lecture – 31
Nernst Equation

Hi. Welcome to this lecture on mathematical methods for biologists. We have been discussing diffusion and applications of basic calculus in understanding diffusion. In this lecture, we will use the ideas that we learned in calculus and diffusion to study how ions diffuse. So, when in biology we know that there is so many ions na plus C l minus k plus.

So, how does this charged ions diffuse across for example, a membrane channel we would do a toy model to understand this of course, we will not be able to study the details of diffusion across a membrane channel, but we would do the simplest model to understand this we would discuss that and the simple mathematics that we learned so far will be useful in deriving something which is very important which is called the Nernst equation.

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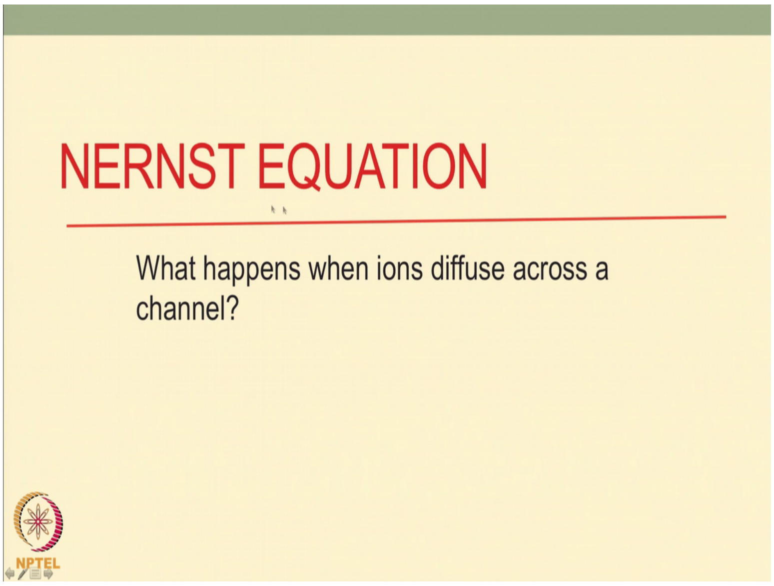

So, today's topic is Nernst equation and what we will discuss is what happens when ions diffused across a channel this is the question that we will answer we will answer the question what happens when ions diffuse across a channeled course because we would do a toy model. So, the topic of today's lecture is ners not equation and we will discuss the Nernst equation. So, what we have been discussing so far is diffusion and what we know is that if you have ion concentration difference there will be a flow.

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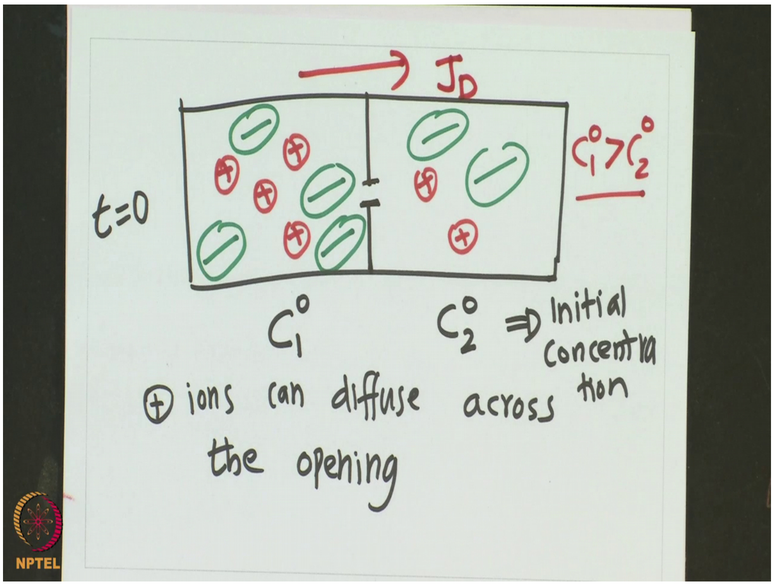

So, now let us consider this case the following case for ions. So, think of 2 separated chambers like this is you can think of this as 2 parts of a cell like this inside the cell outside the cell. So, you have 2 closed chamber like thing and one of this have ions. So, let us say they have kcl ions we have k plus and C l minus in water of course. So, let us draw k plus. So, some concentration of k plus ions and some same concentration of C l minus ions, I am drawing C l minus. So, I drew plus ions small and minus ions big because typically that is how it would be and some other concentration of plus ions here and minus ions here.

So, just to clarify at the beginning, this is a time is equal to 0 you have a chamber in which you have these 2 contain these 2 this box separated by a small membrane like a channel where you have a channel also and there is a concentration of ion like lets C 1 0 with a concentration in the first part and C 2 0 be the concentration in the second part. So, the 0 indicates it is a initial concentration. So, this is a t equal to 0 these are initial concentrations. So, these are initial this is important to remember there is a final concentration; there is an equilibrium concentration which is different from this. So, C 1 0 and C 2 0 this could be like.

Let for example, 10 micro molar and 5 micro molar. So, there could be 2 concentrations C 1 0 in this side C 2 0 in this side. Now there is a small opening here through which only the plus ion can diffuse. So, the minus ions are too big for this opening to diffuse. So, plus ions can diffuse across this opening. So, plus ions can diffuse across the opening. So, only plus ions can diffuse. So, plus ions will diffuse. So, since there is a higher concentration on the left side compare to the higher concentration.

Yes we know from this that C 1 0 is larger than C 2 0 of course, I have drawn more number here than here which is to imply that C 1 is larger than C 2 0. So, since C 1 0 is larger than C 2 0 there will be a flow from this to this. So, this flow is dual diffusion. So, let us call as JD we know as of now from the discussion we had the diffusion the flow due to diffusion JD.

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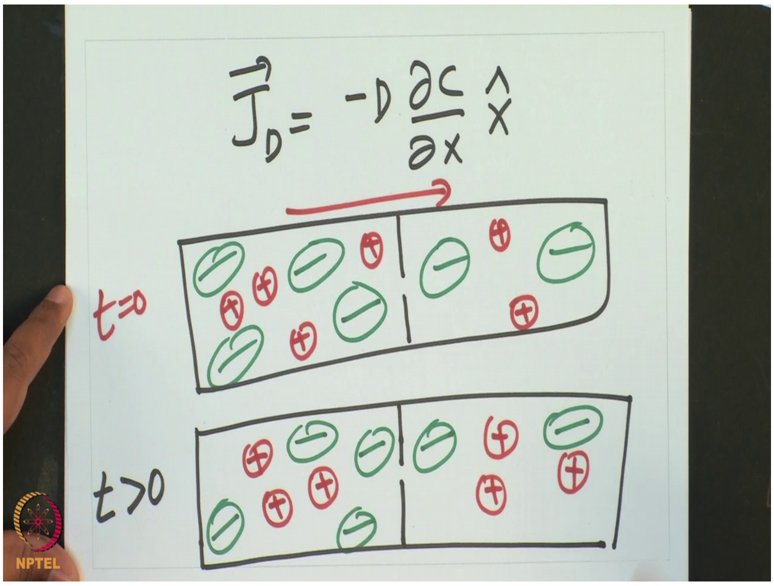

Let us write it the amount of this is. So, I would write minus D del C by del x x cap. So, this is the quantity if you quantify; this would be the equation for this diffusion current what we have is this chamber with plus and minus. So, we have some amount of plus ions and some amount of minus ions on both sides.

Now, a flow will happen of plus ions from here to here this is a flow. So, and this is what happens at t equal to 0 now after some other after some time let us say t is much lesser than 0. So, after some time what would happen is the following. So, I want to indicate here t greater than 0 things will diffuse after the diffusion some plus charge would be on this side. So, you would have let us say different number of plus charge here and different number of plus charges here and on the other hand minus cannot diffuse. So, that would remain the same.

So, just remember that at t equal 0, this side was electrically neutral because there is equal number of plus ions and equal number of minus ions those left side equal number of plus ions and equal number of minus ions on the right side. So, at t equal to 0, these 2 sides separately were electrically neutral after the diffusion of the plus ions from higher concentration to lower concentration, you reach this situation where you have 3 in this picture you have 3 plus ions here and 2 minus ions here. In other words since some plus ions moved from here to here this, there is a net positive charge here and there is a net negative charge here because there is more negative charge and less positive charge here.

Because of that there will be a charge difference. So, there will be an accumulation of negative charge here accumulation of plus charge here this is the situation that would be at t greater than 0. Now if there is more minus charge here and more plus charge here an electrostatic potential will be developed an electrostatic potential; potential difference what does that mean this minus charge will have some electric field which would try to bring back this plus ions back. So, this minus charge would want the plus ion to bring back.

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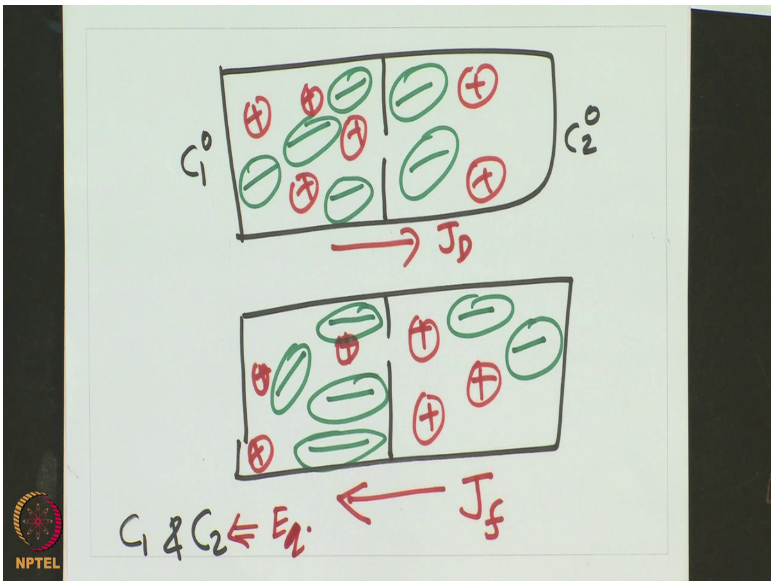

So, let me redraw this just for us to understand. So, you have. So, this is situation as t equal to 0, I would draw situation t greater than 0, I would draw here and I would there were ions like this here I would draw. So, I would draw the following way. So, there are negative is of course, much larger. So, there will be a. So, because of this there will be an accumulation of charges these more negative charges here and more than the plus charge therefore, these plus ions will be pulled back. So, there will be a flow there will be a force there will be a pull. So, it will be electrostatic attraction of due to these negative charges on this plus charge and this plus charge will try to flow back.

So, there will be a flow J f in this direction there was a JD in this direction in this case there will be a J f which is the current due to the force electrostatic attraction force this way where this plus charge will be pulled into this side and essentially you would reach some kind of flow this way and that way and this flows will continue whenever there is more plus charge here, they will flow this way back and whenever this charge difference happens there will be a pull back in this direction what is the if we quantify this J f. So, let us quantify this J f.

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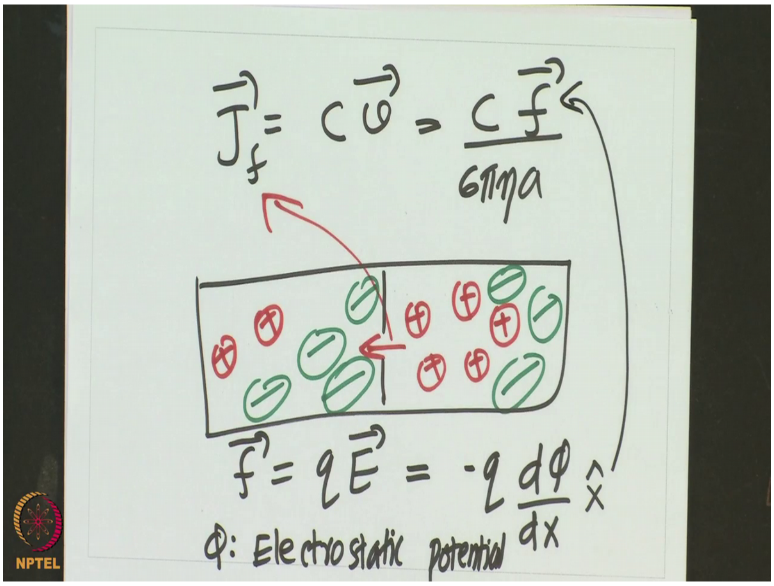

So, J f is the current due to the electrostatic attraction and which we know that C times v where v is the velocity and this is related to the force velocity is always related to force the C f by 6 pi eta if you have particles spherical particles. So, we ions we assume to be spherical. So, if you think of this situation where you have more plus ions here.

So, I am drawing some different number here this number as such does not indicate anything this number only indicates some concentration and there will be some other number here and so which only indicates that there are more positive charges more negative charges on this side and more positive charges in this side and this positive ions will be pulled in this direction positive ions will be pulled in this direction and that flow that flow is J f this flow will be C times that is the concentration at the side times the velocity and the velocity will depend on the force and what is this force depend on.

So, now how much would be the force due to this minus charge on this plus charge, the force f in electrostatic we know that force is q times E where is the electric field and this e can be written as derivative of potential. So, it will be minus q del phi by del x which is called the gradient and with the appropriate sign and so on and so forth. So, D phi by dx the change in where phi is the potential phi is electrostatic potential. So, I use the symbol phi for electrostatic potential if phi is a electrostatic potential d phi by dx is proportional to electric field and there will be an appropriate direction and so on and so forth.

Which can be fixed and this formula is basically the force. So, I can substitute f here which is plus q del d phi by dx with appropriate direction x cap or minus x cap depending on the direction you have the charge difference. So, I am going to substitute this here and so, you have 2. So, I can substitute this for the force here.

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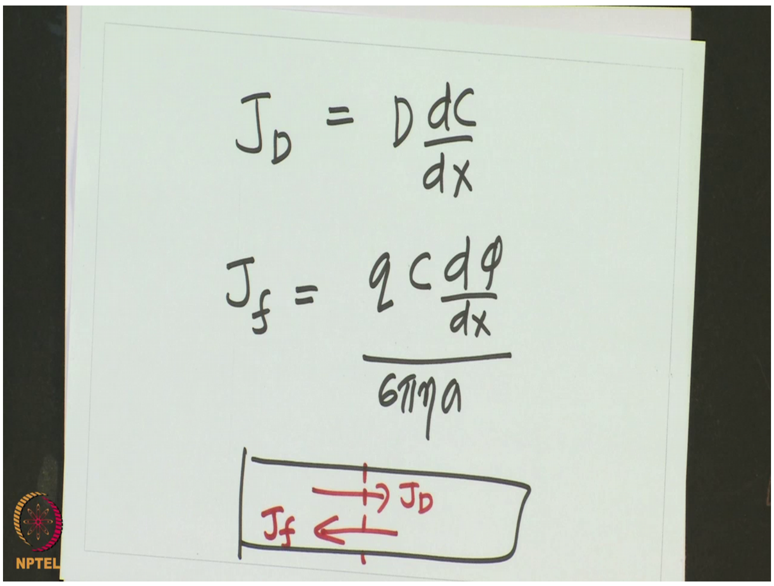

So, I have 2 currents one is diffusional current JD, I am going to write only the magnitude of this 2 currents because the direction we know is opposite. So, d dc by dx I write ordinary derivative because this is a partial derivative because that is this is independent of time I want to discuss everything in equilibrium. So, this is going to be q C d phi by dx divided by 6 pi eta a.

So, I substituted wherever there was a C f; I instead of f I substituted the formula that we had written earlier. So, we have 2 curve currents the diffusional current and the current due to the force and one will flow one would the diffusion of current would want things to be flowing; this way while the J f will be in this opposite direction and this difference these 2 flows these 2 currents when they are equal and of course, they are in opposite direction. So, when they are equal and opposite direction they are equal in magnitude and opposite in direction then the net flow will be 0 ions will flow one way due to diffusion and the other way due to electrostatic attraction and when these 2 balance the system will reach equilibrium.

So, what we want we want the magnitude of these 2 currents to be equal and the direction to be opposite. So, the direction is opposite which we already saw now we want to equate this magnitude of this 2 currents and see that what we get. So, if we equate these 2 currents we would get an equilibrium quantity and something will emerge by equating this, but the idea is simple that there are 2 kinds of flows when there is more number of positive ions on this side and more number of less number of positive ions on this side positive ions will flow from here to here.

When there is more number of negative ions on this side and positive ions are more on this side there is an electrostatic potential difference. So, this plus would be pulled back and that flow is J f. So, these 2 flows will balance and their magnitudes are this. So, we will equate these 2. So, let us equate; these 2 magnitude.

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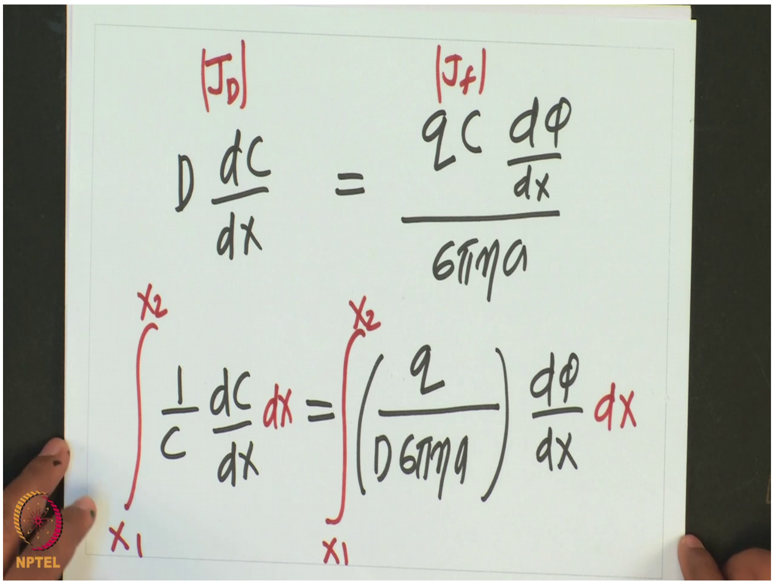

So, what do we want we want D dc by dx is equal to q c d phi by dx divided by 6 pi eta a. So, what we want is to equate D dc by dx with q c d phi by dx 6 pi eta a. So, this is our this is our diffusional current with the magnitude of the diffusional current and this is the magnitude of the flow due to external force and if you equate these 2; what we get is this.

Now, you would rearrange this a little bit and integrate this little bit. So, I would take C to this side. So, let me rewrite this let me take C to this side. So, I would write one over C dc by dx C equal to I would take everything to the other side q divided by d 6 pi eta a these are constants d phi by dx. So, I have derivative on both sides I would integrate this; now what do I mean by integrate this integrate this from x 1 to x 2.

So, let me just I can put in integrate this from x 1 to x 2; x 1 and x 2 are dx integrate x 1 x 2 dx. So, this integration if I do this here I have dc by dx dx. So, this is basically dc by c. So, the we have an integration over c. So, one over c dc is the what you would get here and here you would get integral of d phi which is phi itself. So, integral of D phi by dx dx. So, this is a constant.

So, let me rewrite this what do we get essentially at the end of the day.

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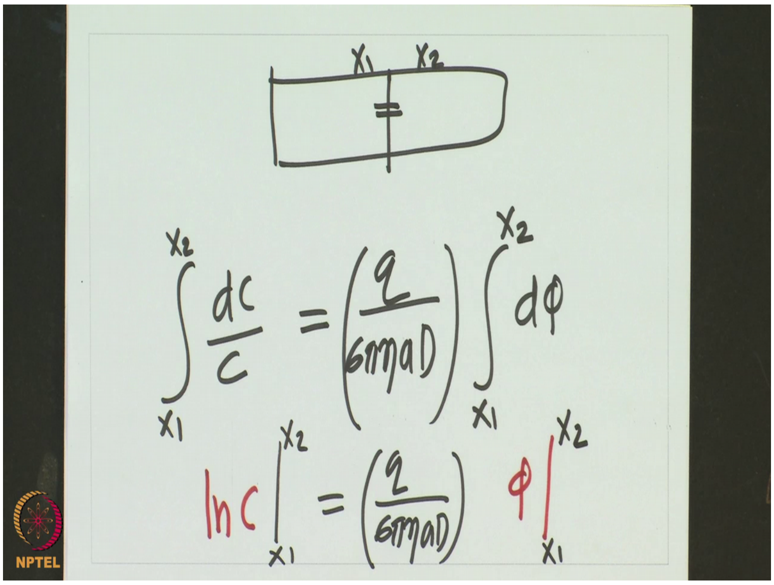

So, first of all just I want to understand that x 1 and x 2 are this side and this; this is x 1 and this is x 2 this is across the channel both sides. Now if I rewrite this what do I get is dc by C x 1 to x 2 the C is a function of x 1 and x 2 at x 1 there is a concentration at x 1 and there is a concentration at x 2 is equal to q by 6 pi eta ad integral d phi x 1 to x 2.

So, if I integrate this one over C integral is log C. So, this is log C in the limit in the limit x 1 to x 2 C equal to q by 6 pi eta ad as it is and this integral is just phi in the limit x 1 to x 2 . So, if I apply this limits log C at x 2 minus log C at x 1 that is what I would get. So, if I apply this limit what do I get?

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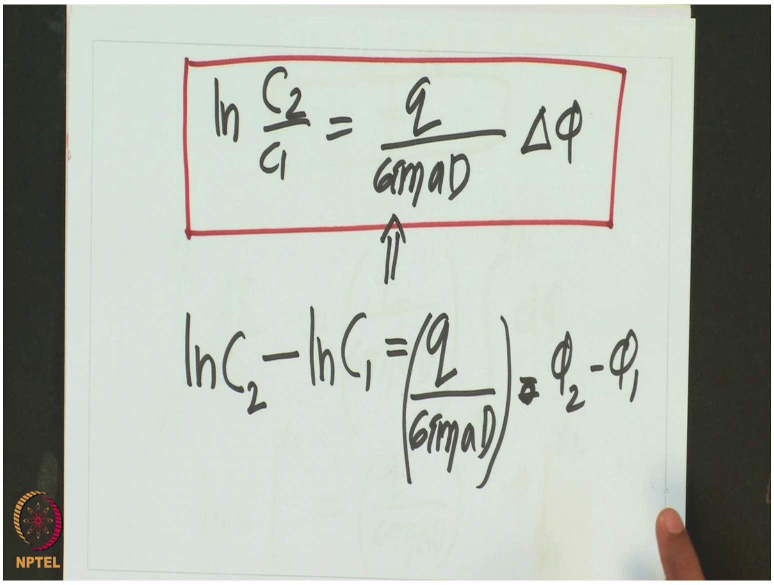

If I apply this limit I would get basically log C at x 2 which is a let me call log C 2 minus log C at x 1 which me call log C 1 is equal to q by 6 pi eta ad times phi at x 2 minus phi at x 2 minus phi at x 1. So, this is what I would get log C 2 minus C 1. So, this same thing, I can write the same thing, I can write as log C 2 by C 1 is equal to q by 6 pi eta ad delta phi which is the change in potential. So, this equation is call the Nernst equation.

Let us again rewrite this little bit more. So, we already know. So, this is called Nernst equation and we can rewrite this in the way we would understand it or typically one would see this. So, I would again rewrite this little bit differently, but this is the relation between the concentration in either size with delta phi which is the potential difference.

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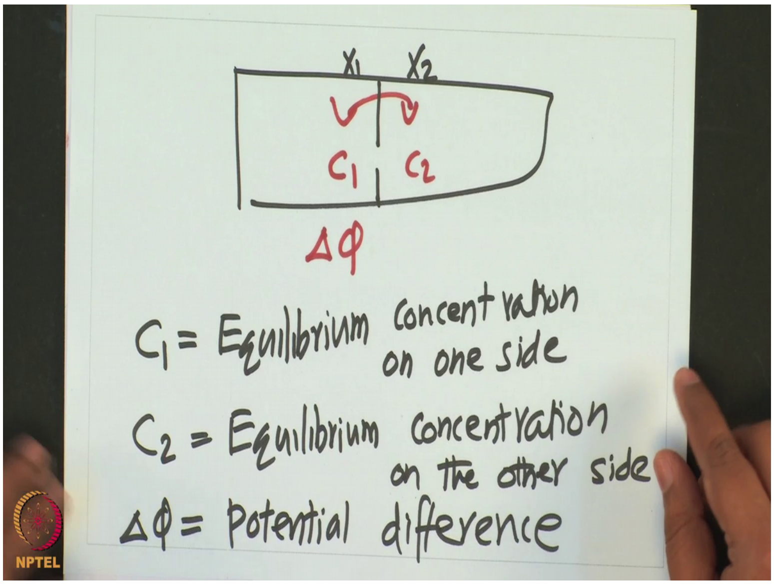

So, you have 2 sides of the chamber. So, we have 2 sides of the chamber, if I call this x 1 and x 2 the and there are ions on this, the potential difference across this. So, this potential difference this potential difference is delta phi and the concentration difference C 1 and C 2.

Now, you would remember the C 1 and C 2 are the final concentration after the current like when the system is in equilibrium. So, just remember it is important to remember C 1 is equilibrium concentration this is different from C 1 0 equilibrium concentration on the first side on one side C 2 is equilibrium that is after long time when both the flows balance the concentration after the flows balance the equilibrium concentration on the other side and delta phi is a potential electrostatic potential difference.

So, these are the quantities of interest delta phi is C 2 and C 1 and this is the relation with these quantities. So, let us write I can write this this way.

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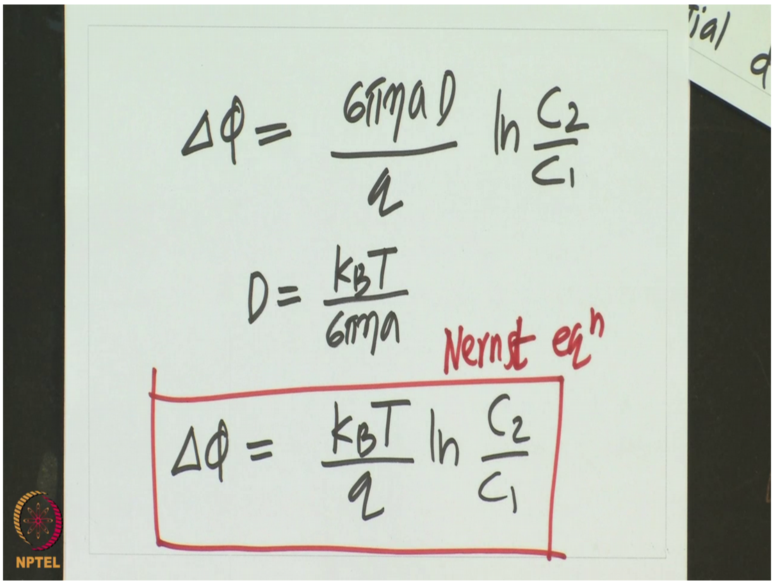

That delta phi is equal to 6 pi eta ad by q times log C 2 by C 1 I took with the other side and we already know that from Einstein's relation that d is equal to KBT divided by 6 pi eta a. So, if I substitute D is equal to KBT by 6 pi eta a I can rewrite this further as delta phi, yes, I am substituting diffusion coefficient d as KBT by 6 pi eta a. So, 6 pi eta a 6 pi eta a will cancel and I will have KBT by q and the log of C 2 by C 1. So, this is another form in which Nernst equation will be taught or familiar where q is a charge and delta phi is the potential difference C 2 and C 1 are the equilibrium concentration. So, this is called the Nernst equation. So, this is called the Nernst equation Nernst equation. So, this is very famous Nernst equation.

Now, what does it take away what do we understand let me just let us go back to the picture that we had before few slides and what we had essentially is the flow of ions on either side. So, what we have is a following at t equal to 0, we have some concentration of ions which we call C 1 0 C 2 0. So, we have C 1 0 on one side and C 2 0 on the other side.

So, here C 1 0; C 2 0 this is at t equal to 0 where more plus ions here than here. So, there will be of diffusion of plus ions and since the plus ions will increase here and minus ions more here they will be pull back because there is more negative charge here this negative charge pull this there will be a flow back these 2 flows balance and when they balance you will reach a final concentration C 1 and C two. So, C 1 and C 2 are equilibrium concentrations when these 2 flows balance.

So, these are the equilibrium concentration this is the equilibrium concentration equilibrium at equilibrium there will be C 1 and C 2 are the final concentration on either side and the relation. So, there will be a potential difference and when equilibrium. So, this flow; this stuff will always this flows will keep going and this would lead to a develop this would lead to a potential difference would develop a potential difference this potential difference is what the delta phi that we have.

So, this is the potential difference that we will get delta phi because it will and it will not be electrically neutral there will be a potential difference and this potential difference will be given by the final concentration of C 2 and C 1 the equilibrium concentration where the flows what is the concentration at which the flows balance that is C 2 and C 1 and of course, the charge temperature and of course, kb is the Boltzmann constant.

So, the final equilibrium concentration C 1 and C 2 will dictate the potential difference across this channel which is the simplest problem that we can discuss which is similar to a membraned ion channel of course, there are other activities going on in real cell which we would not discuss, here we would not be able to discuss here, but this is the basis by using which you can build and the bottom line is that we can use simple mathematical ideas of calculus that we learned to think about problems that is relevant in biology.

So, with this we will stop this lecture and continue later. Bye.