Introductory Mathematical Methods for Biologists
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Lecture – 03
Equations as Graphs

Hai, welcome to this lecture, we will continue learning mathematics in the context of biology or and how to use mathematics as a language to express the phenomena or to study the phenomena that we see in biology and express our findings in a quantitative way. So, we learned some simple functions in the previous lecture we learned a linear function where it was Y is equal to m X.

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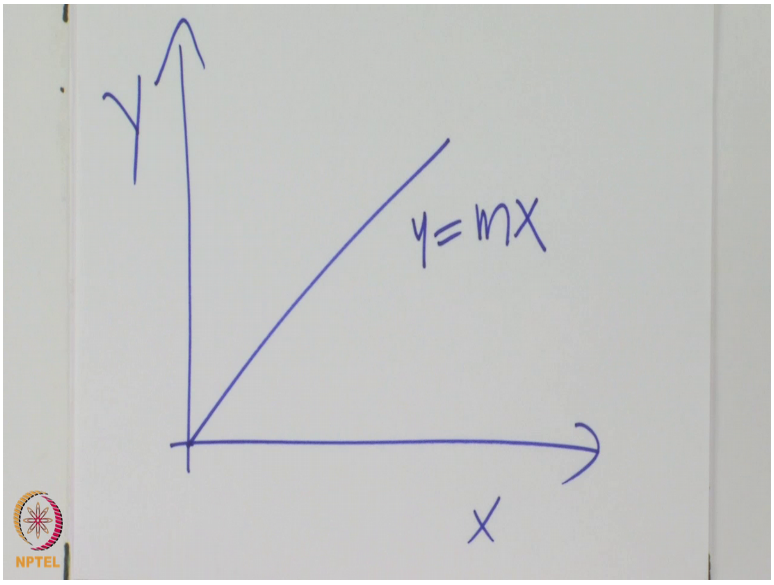

For example; the Y and X it was a linear function the line and we said we learned an equation Y is equal to m X. So, if we want experiment if we do for example; there will be various situations as we knew as we discussed where y is increasing with respect to X for example; concentration would increase or the position would increase with respect to time concentration with respect to preposition so and so forth. Now this is a linear function and if you do other experiments you might find that the quantity of your interest is increasing not as a linear function not as a line but many other ways.

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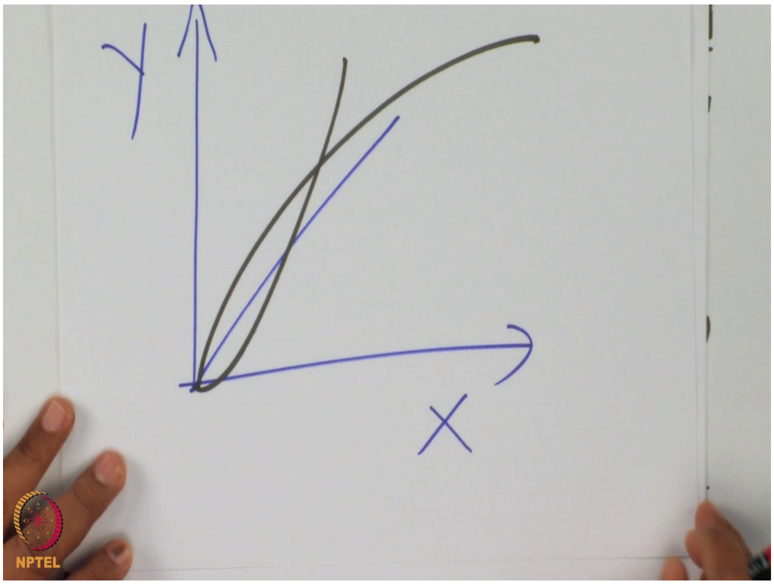

So we would want to learn many function or we might get an experiment many different types of graphs, as when we plot Y is equal to Y and X we had just linear graph but there could be many other graphs you could think of some graph which would look something like this or something some other graph which might look something like this or many different ways Y can change with respect to X and how do we write equations for this how do we mathematically tell quantitatively tell or convey that Y is increasing with respect to X but in this particular way.

So, if you want to precisely tell how Y is changing with respect to X we would have to learn more many more functions just like in a language we have to learn many words we have to have a great vocabulary to say various things that you want to convey here to we would want to use and learn many functions. So, today we will learn few more functions where which we can use to convey the things that you see in experiments or in general in nature around us.

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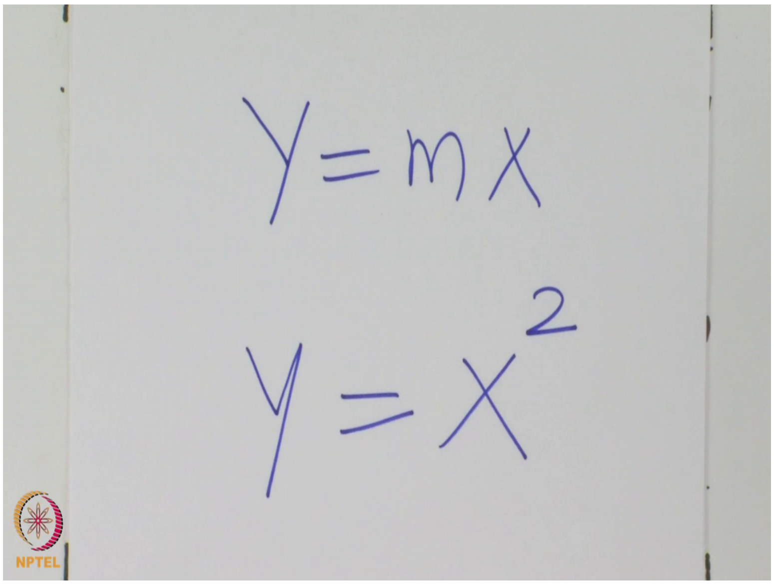

So, we will go beyond the linear equations. So, if we go beyond linear equations what is next? We had Y is equal to X and Y is equal to 2 X and Y is equal to 3 X and in general Y is equal to m X, if we want to go beyond this we could say Y is equal to X square so this is going this is a non-linear function where there is a square X power 2 right.

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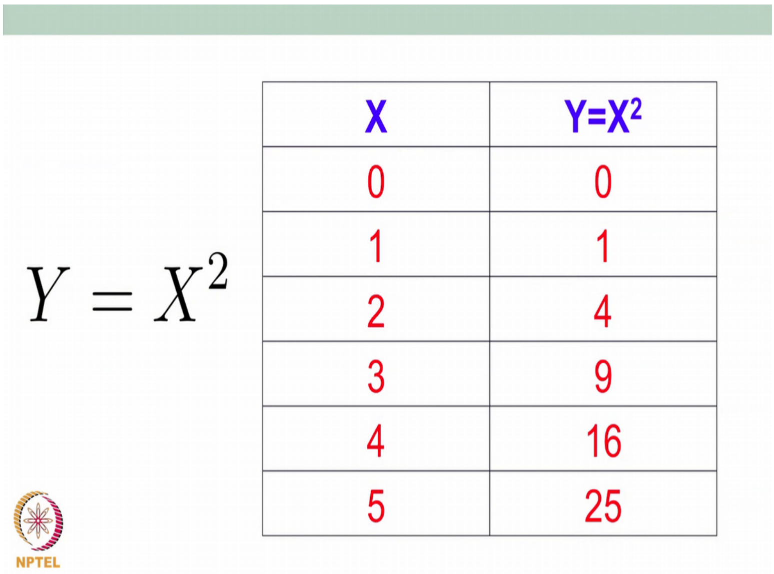

So, how do we plot it many of you would know this, so here is Y is equal to X square. So, just like we did earlier we can make a table anybody can make a table if you have a calculator for any X value you can write the Y value and the Y value is square of X, 0 square is 0, 1 square is 1, 2 square is 4, 3 square is 9, 4 square is 16. So, corresponding to Y is equal to X square we can make a table X in the first column Y in the second column and Y is equal to X square is the function and we write the square values in the second column and we can plot X versus Y.

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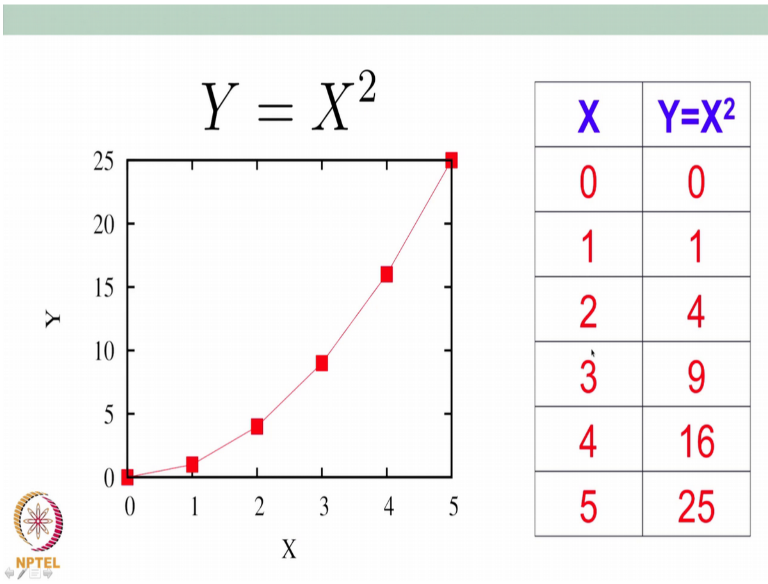

And the plot would look like this, where we have in the X axis Y value is plotted, which is in the second column in the sorry in the Y axis we have the second column and the X axis this is the first column here. So, in X axis we have 0 1 2 3 4 5 and corresponding to 0 Y is 0, 1 square is 1, 2 square is 4 which is just below 5, 3 square is 9 which is close to 10, 4 square is 16 just above 15 and 5 square is 25 and if I plot this you will get a curve which looks like this, so this is how Y is equal to X square would look like.

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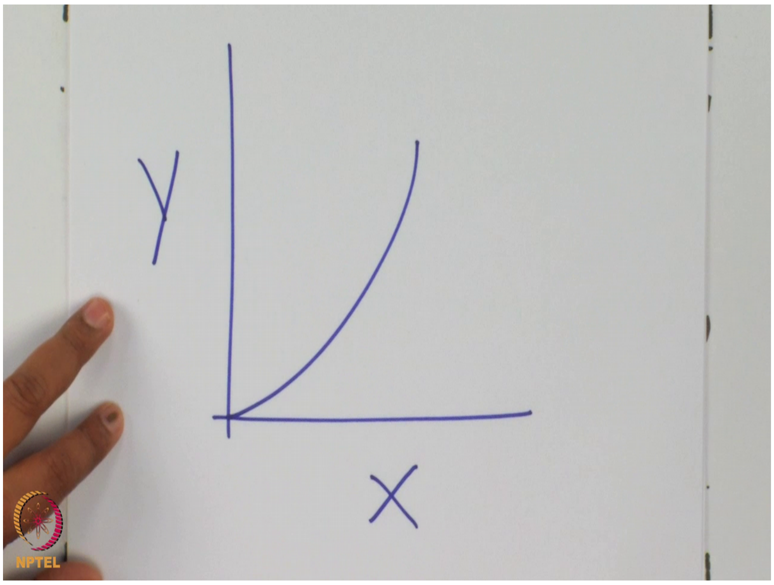

So, whenever you here Y is equal to X square the first thing that would come to your mind or the first thing that should come to your mind is; it is a function which is a curve like this, this is Y and this is X which is a curve like this unlike a straight line that we drew last line which is Y is equal to X which was a straight line.

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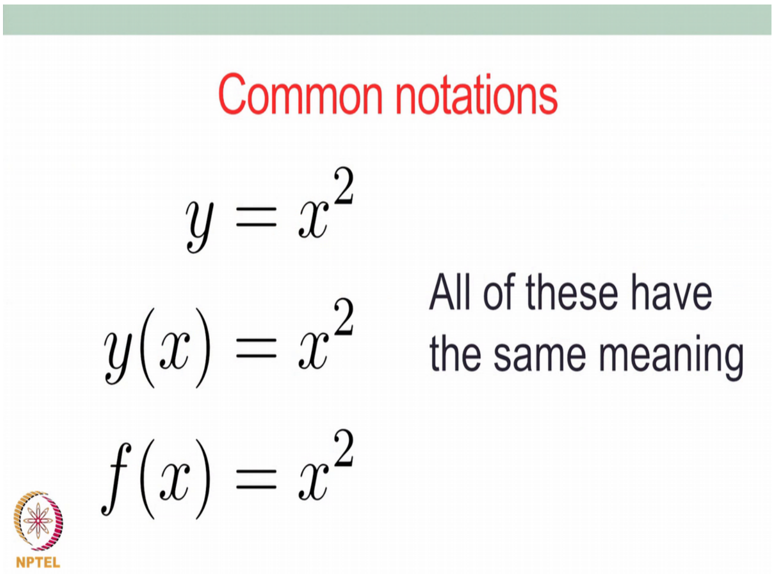

We will compare Y is equal to X and Y is equal to X square ah. So, just like we did last time to say there are different ways of representing this in people write would write y is equal to x square in some textbooks somewhere you might see y of x is equal to x square, in some other context you might see f of x equal to x square and all of this have the same meaning as we said.

Now, we saw here only one part of Y is equal to X square, so in this graph that we have it here we have Y is equal to X square where we had only the positive values of X, but we might also want to plot the negative values of X that is you can be started from 0, so what if X is minus 1 right.

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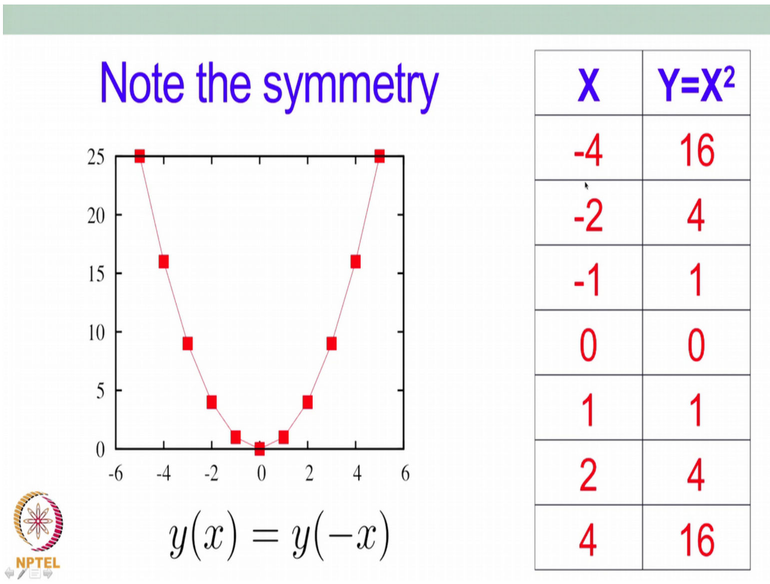

So, if X is minus 1, minus 2, minus 3 we can examine that. So, minus X is minus 1, minus 1 square is plus 1, minus 2 square is 4, minus 4 square is 16, so we have minus 4 to plus 4 and if we take that an plot that here. So, minus 4 square is 16, minus 2 square is 4, minus 3 square is 9 here, minus 1 square is 1 here again 1, 2 square is 4, 3 square 9, 4 square 16, 5 square 25. So, the interesting thing is one should note the symmetry, so what is the symmetry here? So, let me plot here.

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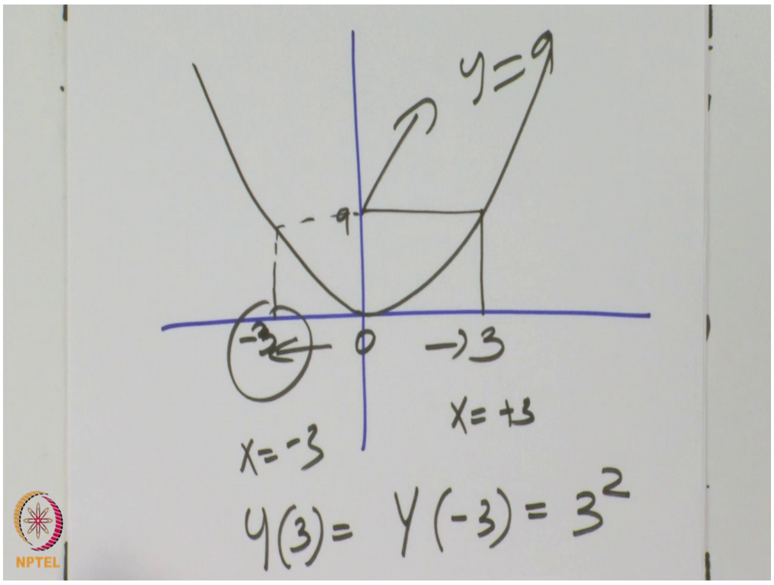

So, what we have we are plotting the both minus side and plus side of this. So, this is 0 and this is negative and this is positive and it looks somewhat symmetric what does it mean whatever be the X value if X value is 3 there will be 9, so if this is 3 this is 9, the corresponding X value for minus if 3 if X is minus 3 the Y is also 9, so if X is minus 3, so this X is equal to minus 3, this X is equal to plus 3, Y in the both in both cases Y is 9.

So, Y for X is equal to Y of 3 is same as Y of minus 3 which is 3 square. So, this is the symmetry when X is put as minus X the function is exactly the same that you can see just by examining the function and I want you to understand this symmetry thing little bit carefully because very often Y is equal to X square that is; whatever the X value you put we are finding the square of this.

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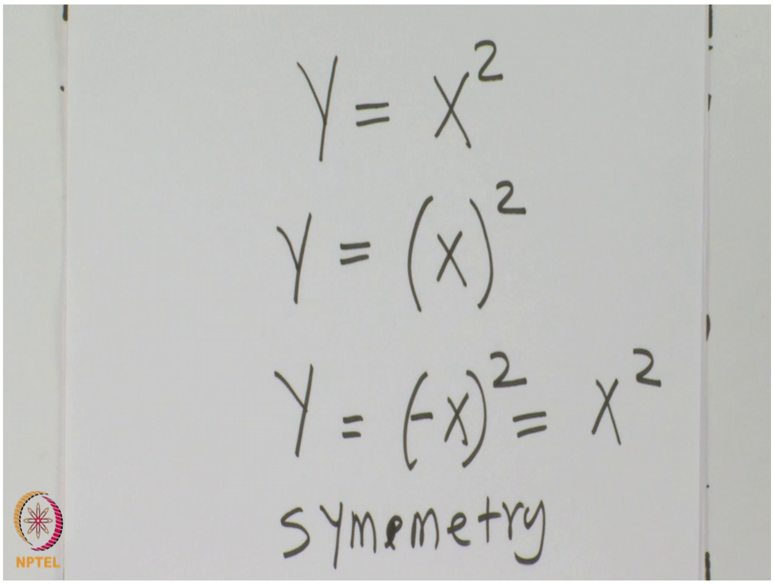

Now, instead of X if we put minus X instead X if you put the minus X we will also get the X square. So, minus X square is also X square, X square is also X square, so the point here is that whether you put X or minus X Y value is exactly the same so this is a symmetry and here because of the symmetry in the curve that we have in this computer this will look symmetric on either side, so it is looks like nicely symmetric whether you take plus 2 or minus 2 it is the same value way value plus 4 or minus 4 is exactly same value, so there is a symmetry around 0, so the symmetry is Y if X is equal to Y of minus 6.

This is important to understand the symmetry and that is why I am stressing because in future there will be many function for which the symmetry is important and these rest on the symmetry because symmetric functions will have some properties which we will have which we will make use of in the future. So, I want all of you to carefully understand the symmetry that Y of X is Y of minus X in this case when function is X square.

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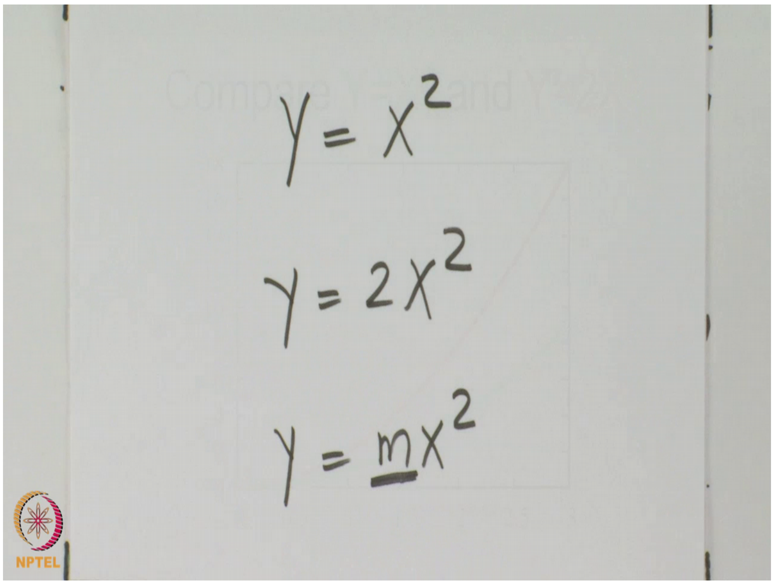

Ah now let us compare so we had now we understood Y is equal to X square. Now, we can think of just like we had earlier we can have Y is equal to 2 X square. So, we had Y is equal to X square and we can think of Y is equal to 2 X square or 3 X square in general Y is equal to m X square or any number here and X square. So, let us compare here Y is equal to X square and Y is equal to 2 X square.

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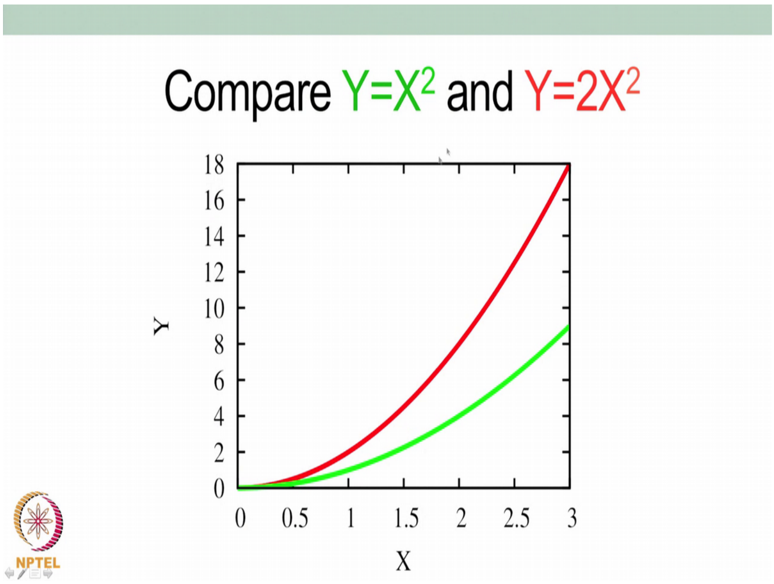

So, the green curve here is Y is equal to X square and the red curve is Y is equal to 2 X square. So, as you can see when X is 0 both are same, it does not matter what the constant is; Y is same when X is 0, 2 X square X square when Y is 1 when X is 1, Y is 1 here it is 2 when it is 2 this is 4 while this is 8. Similarly so Y is equal to 2 X square will be above Y is equal to X square, so this will go above this curve and this how it look like and you can think of if it is Y is equal to 3 X square it will go further above.

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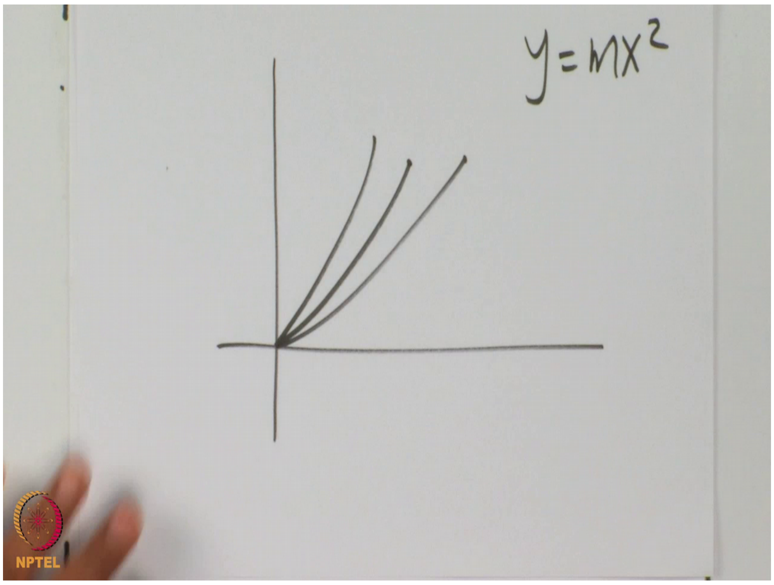

So, you can imagine that Y is equal to X square if it looks like this 2 X square will go above this 3 X square will further go above this and so on and so forth. So, you can think of more fun in general Y is equal to m X square the shape will be the same the curve will look roughly the shape. Now we know Y is equal to X and Y is equal to X square how will that compare? If you plot Y is equal to X and Y is equal X square the linear and the curve, so here is Y is equal to X which is the blue line, Y is equal X square is the red curve you can see that 0 at 0 at X equal to 0 both are same. When X is 1 here they are the same because X square this is 1 square and 1 are the same they have the same Y value so that is why the same.

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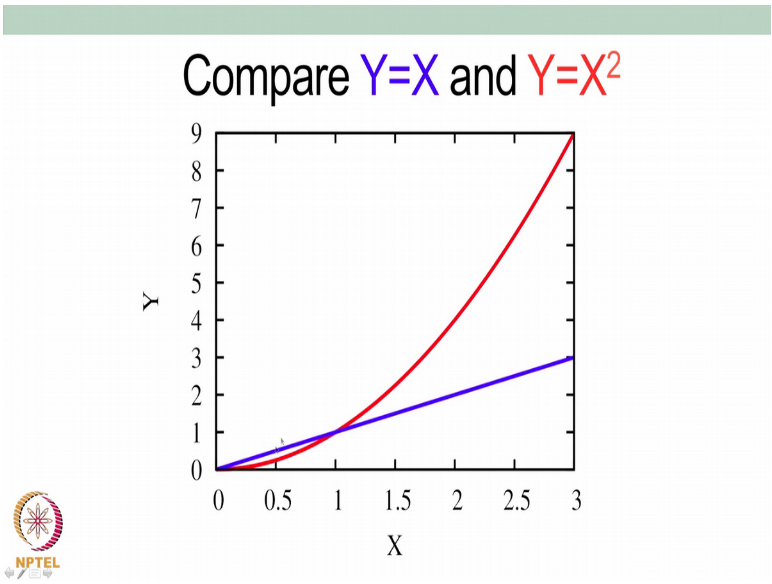

But above 1, X square is much larger than X as X goes to 2, 3, 4 and so on and so forth X square will have much larger the value compared to X. But when X is .5, .5 square is .25 which is smaller than .5, so this is important to note that Y is equal to X square is smaller when X is less than 1 which you can appreciate.

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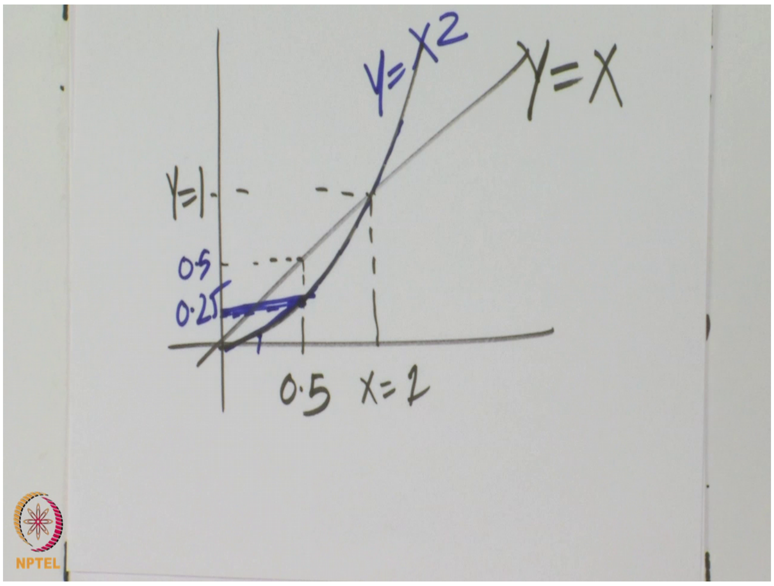

When so you have Y is equal to X and Y is equal to X square, at 1 so this is X is equal to 1 and this is Y is equal to 1. But when X is equal to half .5 Y in this straight line this is Y is equal to X, while this is Y is equal to X square, so in this X square curve which is this curve, this value this is this has to be roughly .25. Since I drew the axis values may not be very nicely scale but you can imagine the this is less than .5 this was below .5, similarly here .1 square and .2 square and so form you can think about it. So, the point here is that when X is when X is less than 1 the Y value is below this blue curve and when X is larger than 1 the X square curve is above the blue curve. So, this is the thing that you should understand.

Now, the such functions for example; the Y is equal to X square this is called a quadratic function because the second power is called the quadratic function and there are many cases where that we can think of this quadratic function coming for example; if you have a spring like molecule and the energy of a spring we write as half k x square.

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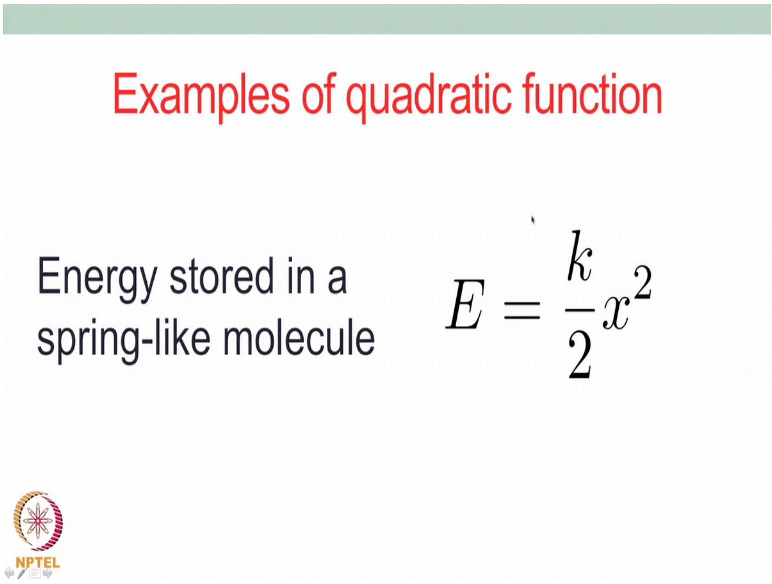

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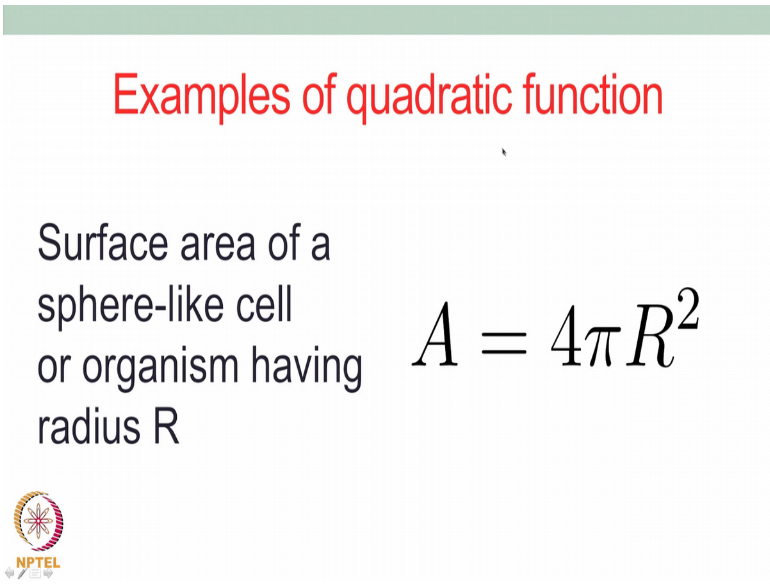

So, energy stored in a spring like molecule is half k x square which is an example of quadratic function. And another example would be surface area of a sphere like cell or an organism having a sphere like shape and if it has a radius R the area is 4 pi R square, so this is also a quadratic function. So if you plot A versus R this will look like a constant times R square and this will look like the function that we look like. So, if we want to plot the area versus the radius of a cell, even if it is approximately assumed to be spherical, so if you want area versus R and we know that the area is 4 pi R square and as you can imagine it look like this some function like which increases with respect to R and this will look like this.

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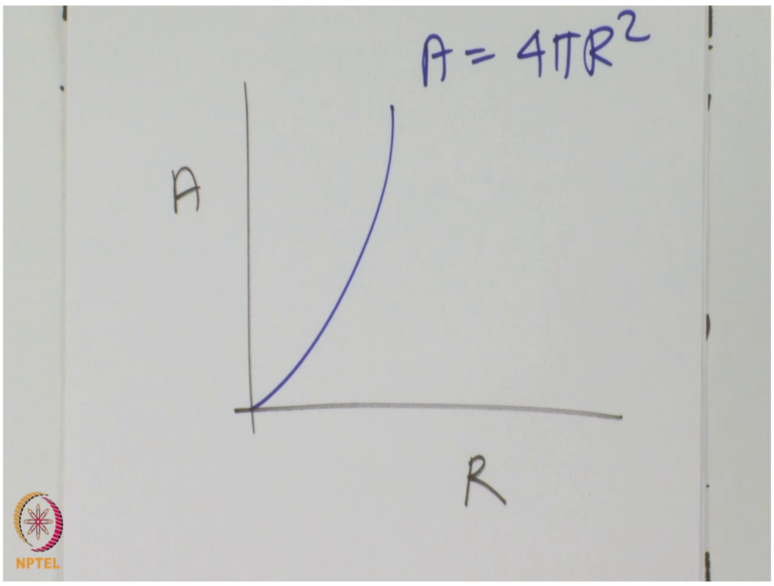

So, that is the quadratic function. Now if we go beyond of course we have cubic function, so what is cubic function? Y is equal to X cubed. So, let me write that here the cubic function is Y is equal to X cubed.

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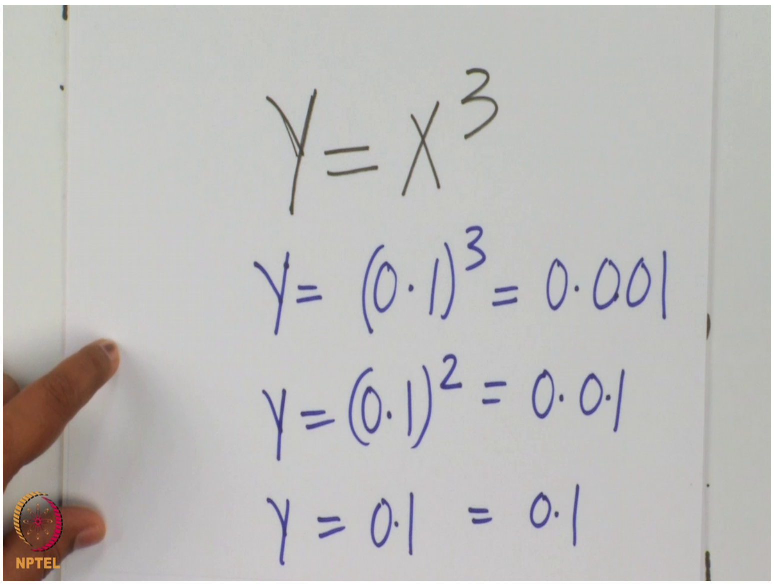

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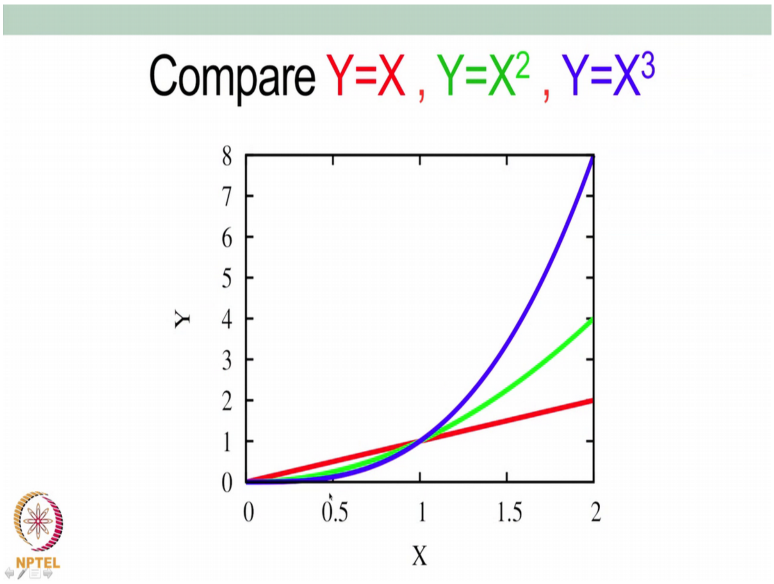

And how will that look like. So, we are going to show that in comparison with X and X square, so we had Y is equal to X which is the red curve, Y is equal to X square which is the green curve, now Y is equal to X cubed which is this blue curve. And as you can see when X is larger than 1, X cubed values much larger than X square or X you can imagine

2 square 2 square is only 4, 2 cubed is 8. So, for any X value larger than 1 the cubic curve will go above the quadratic curve or the linear curve. The shape of it is a curve like this but it goes much about this values of value will be much above this, so but when X is less than 1, X cubed is much smaller than other quantities which I hope you will appreciate.

So, when Y is equal to .1 cubed right this is 10 power minus 3 which is .001 10 power minus 3, so now if you have Y is equal to .1 square which is 0 .01 and Y is equal to .1. So, you can see which is .1 itself so this is smaller than this which is smaller than this, so that is what this curve indicates the blue curve is much below the red and green implies this. So therefore, what we want to the curve look like this and I urge you to familiarize with this plot various functions I urge to plot X, X cube, X power 4, X power 5 and so on and so forth I want you to plot.

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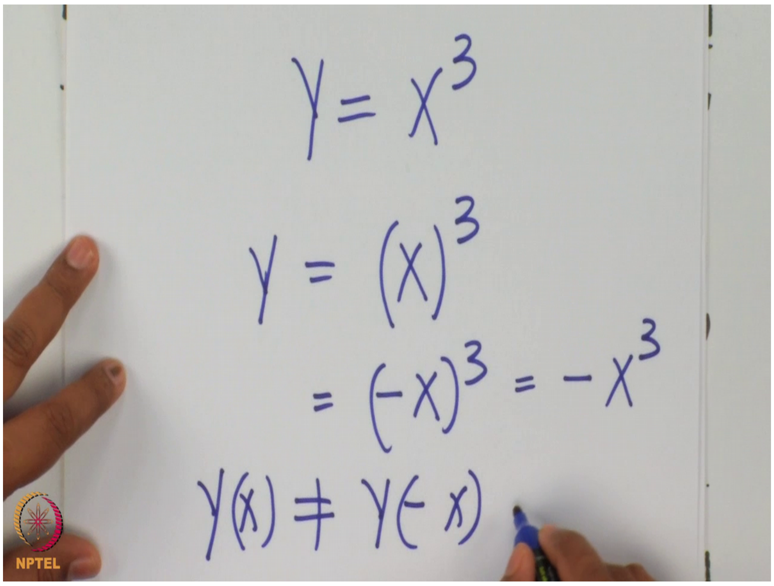

And I want you to also note one more thing that, if you plot Y is equal to X cube what happens when a X goes to minus x? So, as we did earlier so this is X cubed if I just put X is equal to minus X, this is minus X cubed are they equal? We had for the square quadratic function they were equal, but this is minus of X cubed so they are not equal. So, Y of X is not equal to Y of minus X here, this is not equal to Y of minus X here, when X is Y is X cubed rather interestingly what you can see here is that this is minus X so this is X cubed, so this is minus X cubed so Y of minus X is minus Y of X. So, I want you to try I want to write this I want to I want you to see this and let me write this thing here little bit carefully.

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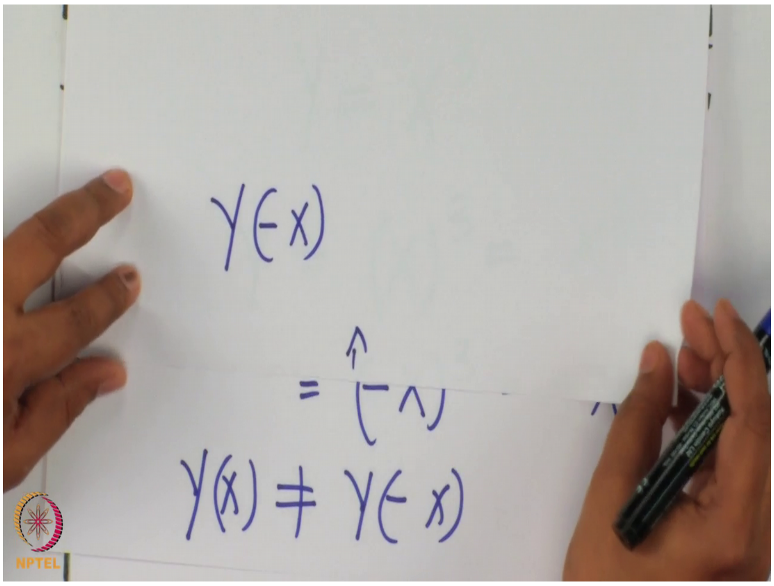

So, I want you to I want to write right here one thing what I am going to write is Y of minus X. So, let me write this again, so we had Y is equal to X cubed and you want to Y so our function is X cubed. Now we want to write Y of minus X and we saw that this is minus X cubed and what we want to say is that Y of minus X is equal to minus of Y of X I want you to think about it and get used to this kind of language.

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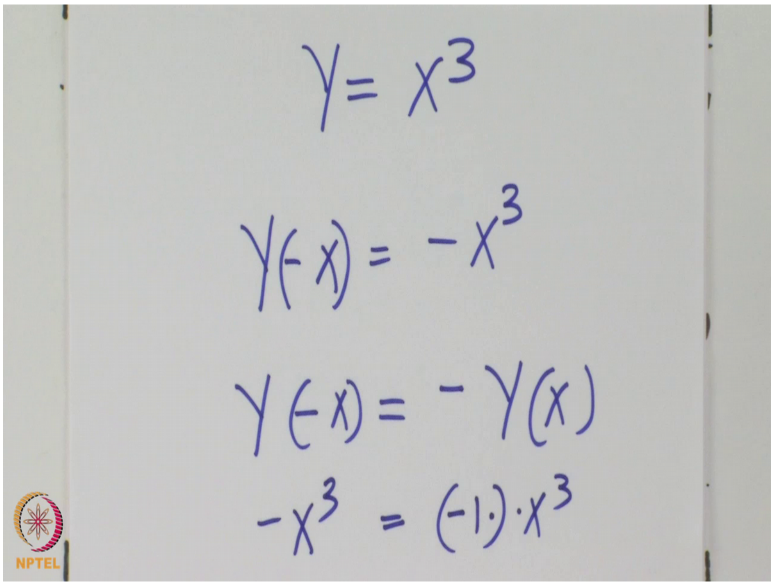

What I am trying to say is that minus X cubed is nothing but put a minus sign and multi 1 times X cubed which we can see here, think about it only by thinking about it you will learn this familiarize yourself with this kind of language that Y of minus X is minus Y of X which means that minus X cubed, which is Y of minus X is minus 1 times X cube right minus 1 times X cubed that is what it essentially we are saying. And therefore this is the here the we can that symmetry of Y is equal to X square is not here but there is a different kind of symmetry or the anti symmetric if you wish and we will come and discuss that later. So, there is a inverse the there is that different symmetric here which we will discuss later, but I want you to think about it a little bit ok.

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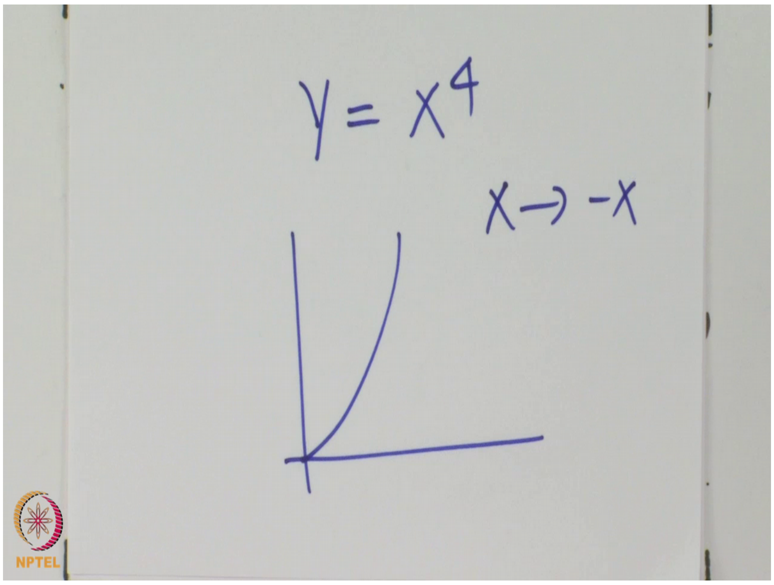

So, what I want to say next, so we have we can have other functions now Y is equal to X power 4, so the next function could be Y is equal to X power 4. So, this is also we will look like a function which increasing which will increase much fast faster or much above X square and X cube. And this so I have that symmetry that if you good X 2 minus X this will look symmetric, therefore it will have it is own property and I want you to think about this. So we can have various functions Y is equal to X square X per cube and various combinations of X square X cube and all that.

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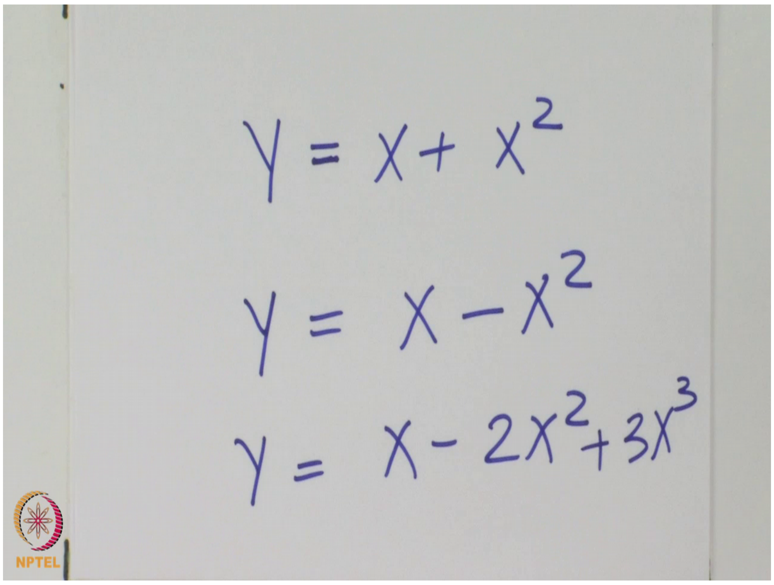

Now, what happens if you sum 1 or 2 such functions for example; you can think of Y instead of just taking X or X square, you can think of X plus X square or X is equal to X minus X square or Y is equal to X minus 2 X square various functions one can think of which are some combinations of this things or you could be Y equal to X minus X square plus 3 X cubed.

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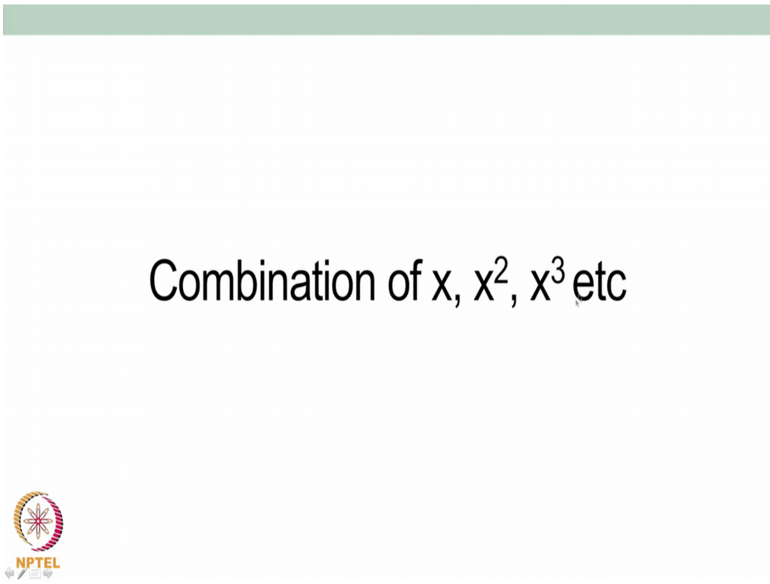

So, one can think of combinations of X, X square, X cubed etcetera, so this is what we will explore next. So, thus this are more functions which essentially can be written as some combinations of X, X square, X cube, X square 4 etcetera, which we know how to plot now we know how to plot X, X square, X cube etcetera now what how will these combinations will look like.

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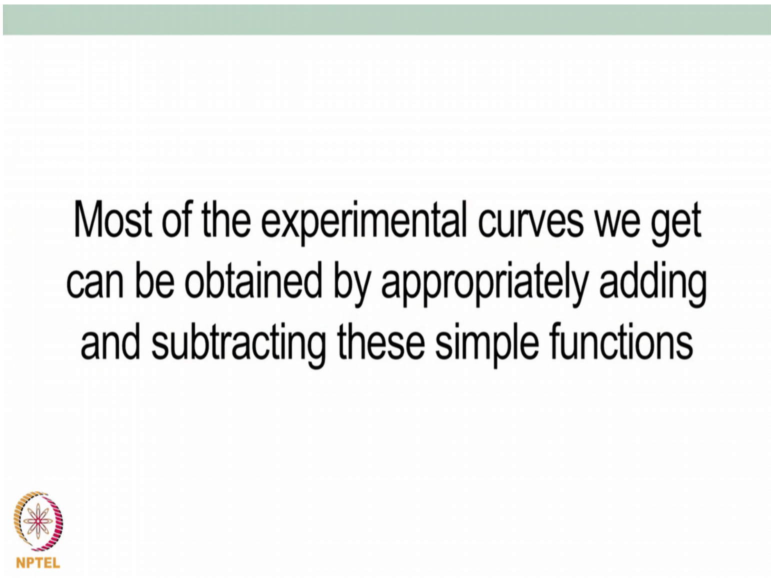

It turns out that most of the experimental curves that we see in earlier you do an experiment and you get some curve and most of the experimental curves that we get can be obtained by appropriately adding and subtracting this simple functions, if you add and subtract we can get some curves which would represent typical experimental curves. Or in other words you get some experimental curve it can be written as some combinations of X, X square, X cube etcetera and we will see how this can be done and we will that is what we will discuss now. So, one primary one important function that we will learn today is called the exponential function ah.

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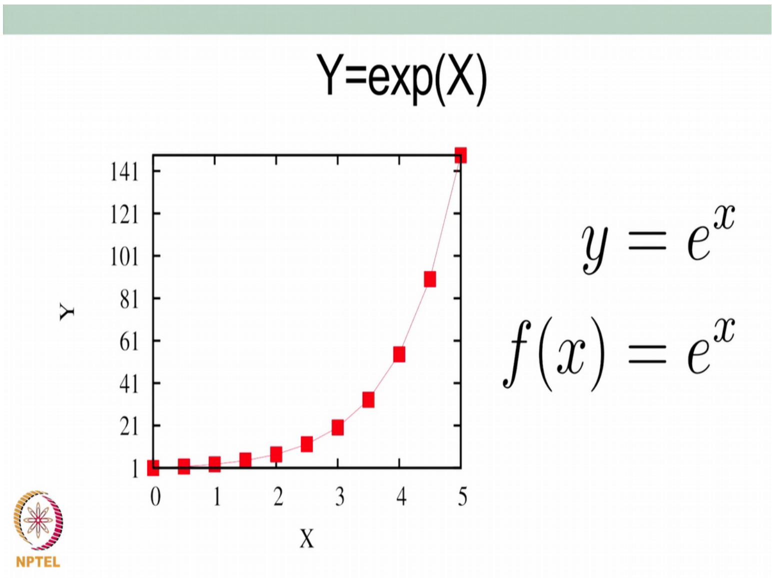

You might have all have seen on calculated this function X exponential e power. So, Y is equal to e power x, so you all can take a calculator and take an x value let us say 0 and put on my calculator e power 0 and make a table; e power 0, e power 1, e power 2 and then plot that values on a table and then you will get the curve like this.

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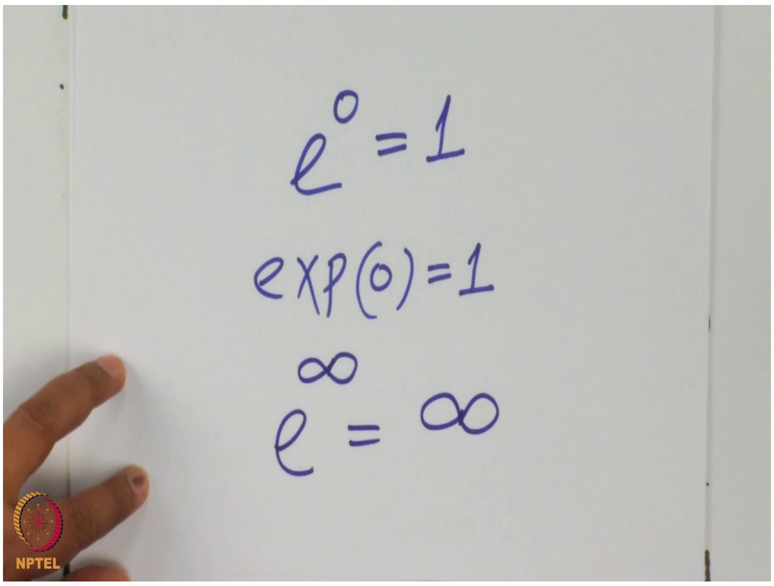

Interestingly note that e power 0 is 1, so I want all of you to understand. This is very important to understand e power 0 is 1, this is the you know it cannot be written as x power exponential of 0 is 1. So that is why we started when here the Y axis in the curve

Y axis I would start with 1, so in the graph in the computer we will have Y axis starting with 1. So, when X is 0, Y is 1 when X is 1, Y would have some value and so on and so forth. So, this curve you should reproduce by taking a calculator and plot it yourself make a by making a table like we did earlier and to plot this yourself in a graph sheet and this often either representing Y f Y is equal e power x or f of x equal e for x.

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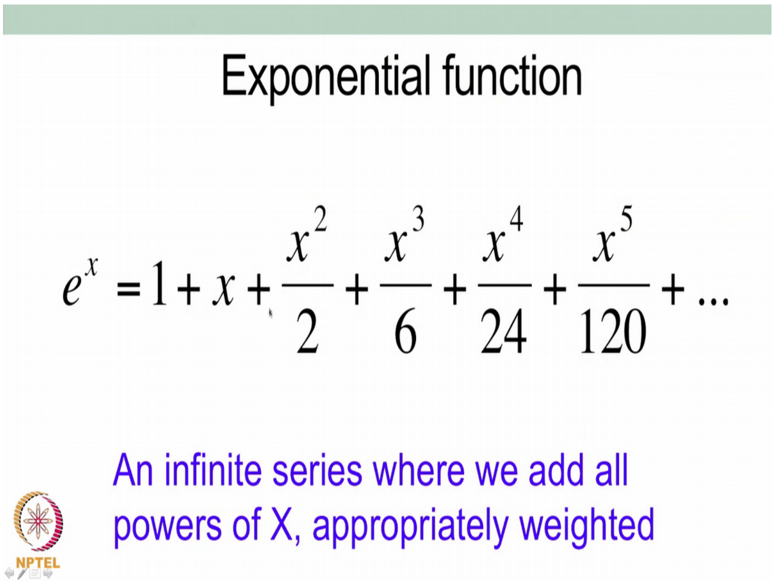

And it turns out, that this function can be written as a linear combination of all this 1 plus some combination of X square and all that, it turns out that that linear combination is 1 plus, x plus, x square plus, x cubed plus, x power 4 plus, x power 5 x sorry let me repeat e power x can be written as 1, plus x, plus x square by 2, plus x cube by 6, plus x power 4 by 24, plus x power 5 plus 120, plus an infinite series. So, this is an infinite series where we add all powers of x appropriately weighted by factors like half 1 by 6, 1 by 24 and 1 by 120, it turns out that if we write such a infinite series one will get e power x.

In other words e power x can be written as an infinite series like this, how and why? We will discuss that later. But at the moment is sufficient to understand that one can write e power x as an infinite series like this, so we learned so on one more thing which I want to say is that if you look at for very large value of X this is increasing and going to infinity. So, e power infinity is infinity this is another thing which I want you to learn, when X in this curve goes to infinity, this function also will go to infinity it is increasing and it will reach infinity so e power I infinity is infinity.

So, these things I want you to understand. We will come and learn again in the next lecture how this exponential function can be written as 1 plus, X plus, X square and what does it mean, how does the series what is the meaning of this infinite series etcetera we will learn in the next lecture. So, in this lecture we will summarize with the following thing.

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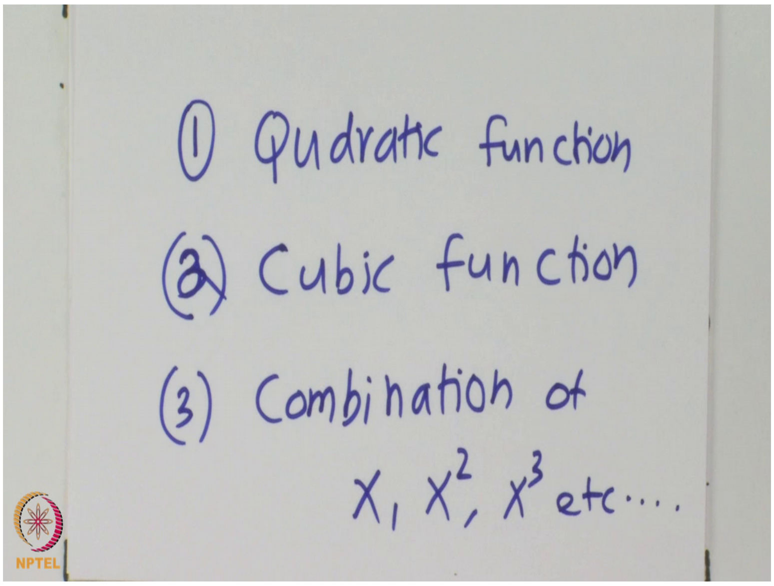

So, what will we learnt? So, we learned quadratic function. So, this is our summary, we learn cubic function, second thing we learn this cubic function and we learned combination of we briefly talked about combination of X square, X cube etcetera. And we will continue along this line and we said that e power x can be written as a combination of this which we briefly mentioned we will expand on this in the next lecture and we will see how we can learn about exponential function using this expansion and how we can use other functions as a combination of X square, X cube etcetera which we know now how to plot. So, we will continue in the next lecture bye.