Introductory Mathematical Methods for Biologists
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Lecture – 27
Understanding Diffusion

Hi, welcome to this lecture on mathematical methods for biologists. We have been discussing about diffusion and we will continue discuss diffusion a little bit more in trying to understand; how do we apply mathematical methods and we learned to describe phenomenon like diffusion that is one of the examples that we are taking to describe; how to apply the calculus, we learned to understand this phenomenon.

So, we discussed that whenever there is a concentration difference the concentration difference will lead to a flow a current of proteins so, and this current from high concentration low concentration which is called diffusion.

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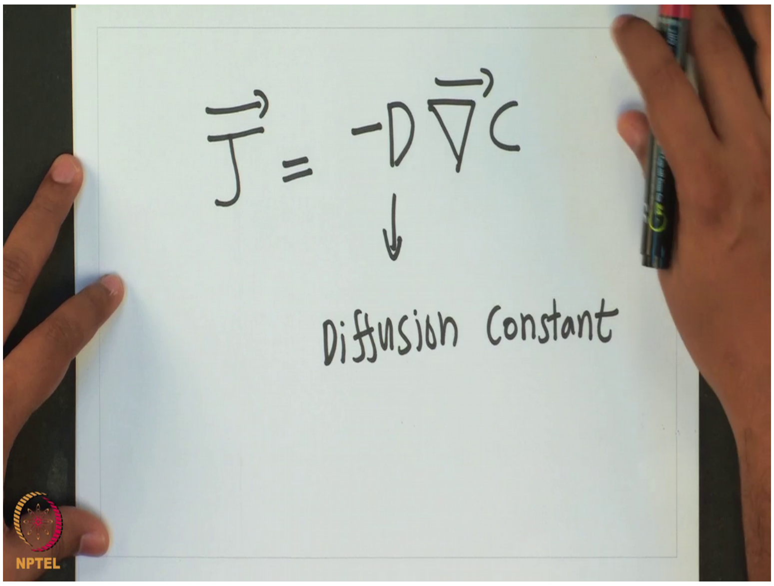

And this diffusive current diffusive flow is its quantified this amount of this flow the current we wrote as minus D gradient of C and we said that this gradient would mean derivative with respect to x, y and z and this is the flow or the current and this is the concentration which is the number per volume.

So, let us understand this and there is this thing D which is called the diffusion constant which we called as the diffusion constant this D is a constant. So, positive number which is a diffusion constant first question we will try to understand is; what is the dimension of J and what is the dimension of D the diffusion constant? So, that are the 2 there first 2 questions we will answer is what is the dimension of J what is the dimension of diffusion constant D.

So, let us first think about the dimension of the J the current. So, if I write this in one dimension.

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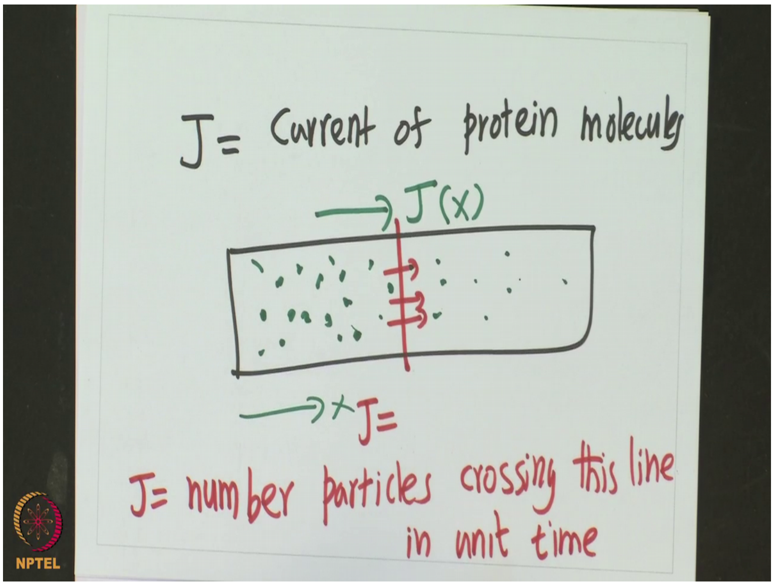

So, what is J? J is current of protein molecules or any particles or flow of protein molecules that is if I have a cell and if I have a concentration gradient high concentration here and low concentration here the particle will flow from high concentration to lower; low concentration and the flow is what we call J. So, we first thing to understand is J is a function of x where this x is this distance along this is x.

So, here there will be higher flow because there is a depending on the concentration difference there will be a flow difference. So, now, let us to understand this flow let us take one point here x. So, this is the particular point x of our interest and now what is the flow here flow would mean the number of particles crossing this line right. So, in this 2 dimensional description J would be what is J the flow or current would mean that number of particles number of protein molecules.

So, when I say particle; I mean protein molecule here number of protein molecule protein particles protein molecules crossing this line crossing this line in one second in unit time in every second in unit time, if I take a unit time how many particles are crossing this that is the that would be a definition that would be the definition of J; if I consider this instead of this a tube think about a 3 dimensional tube like this; like if I have a 3 dimensional tube like this things will be flowing out of this pipe like if it is a pipe things will flow out of a particular point.

So, how many molecules will flow out of this. So, here what you have to consider is the area. So, if I have this area how many particles will flow out in every second from this area? So, if I have this same thing in a 3 dimensional tube, like a situation the same thing J.

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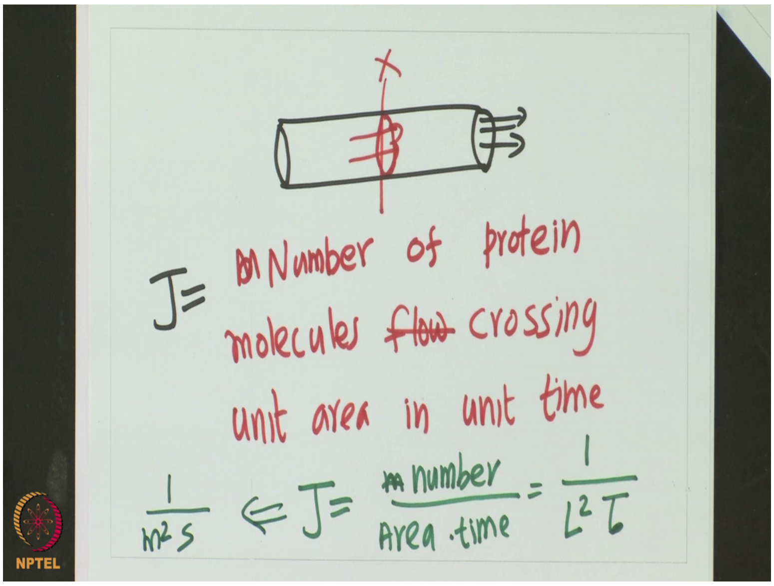

If I have to describe in the context of a in the context of a 3 D tube like this there will be particles flowing out from this and the question is J can be defined as number of particles number of protein molecules number of protein molecules flowing across this area that is if I have this particular area here across this area how many will flow.

So, flowing number of particles crossing, I would write crossing unit area at unit area at x in unit time in unit time. If I take a unit area in at this location called x, how many particles are crossing is the J. So, J will have a unit. So, let us think about what is the unit of J. So, J will have a unit number of particles which is a dimensionless quantity. So, number divided by area divided by time. So, this will have a dimension of one divided by area has L square and this has time has second; L square t time has t L square one over L square t this will be the dimension of J and this will have a unit which is one over meter square second.

So, this will have a unit the corresponding unit the corresponding unit for J will be one over meter square second one over meter square per second that is meter 2 meter minus 2 s minus 1 m minus to s minus 1 will be the unit of J that is the flow which is basically number of protein molecules crossing across unit; per unit crossing unit area in unit; if I take a unit area number of protein molecules crossing this area in unit time is this J.

So, if 100 molecules are; so, if I have a one micro one meter square pipe and if there are 1000 molecules crossing 1 meter square pipe in 1 second that will be flow will the J will be 1000. Now if I just think of a cell typically dimension of the cell will be micrometers. So, if I just take this area as micro meter square and if there are 1000 molecules or 100 molecules crossing this cell area a cross section of a cell which is a micro meter square area in 1 second and the J would be. So, let us calculate the J.

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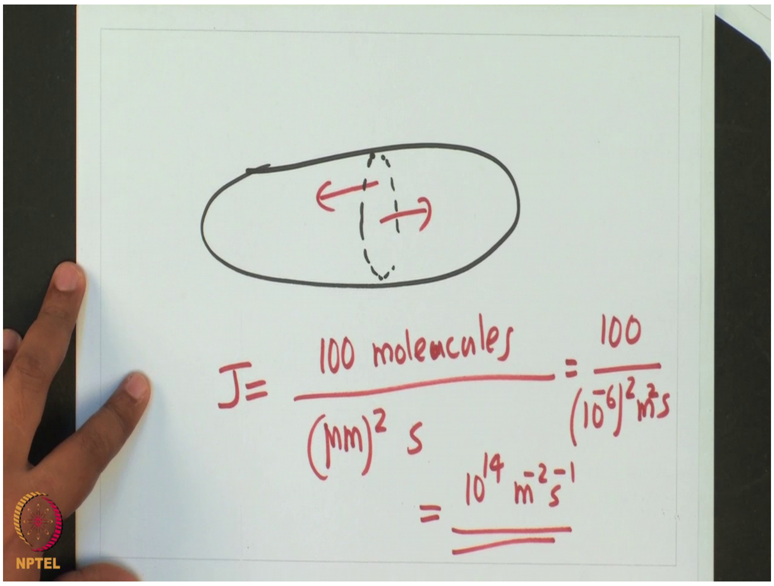

So, if I have a bacterial cell or a typical cell. So, if I am drawing some kind of a cylindrical cell here and I can take a cross section here this will have typical dimension of micrometers. So, the J would be let us say a 100 particles. So, let us say 100 molecules in area will be micro meter square second. So, this will be if I this in si units micro meter, I can convert to 10 power minus 6 meter. So, this will be 100 divided by 10 power minus 6 meter square second square second.

So, this would be a big number. So, this will be 100 divided by 10 power 12. So, this will be 100; 10 power minus 12 will become 10 power 12 and this will be 10 power 14 meter minus 2 s minus 1. This could be the typical number associated in si units the J in other words, it could be it could be some 100 or 1000 per micrometer square per second in si unit it could be 10 power 14 per meter square per second.

So, something like this could be the typical number that one would be interested in. So, so this would be the J the magnitude of the J and the direction of the J would be either they along this or along this depending on the, this there will be a dimension and magnitude would be something like this now that we understand this J. Let us also think about the dimension of D the diffusion constant.

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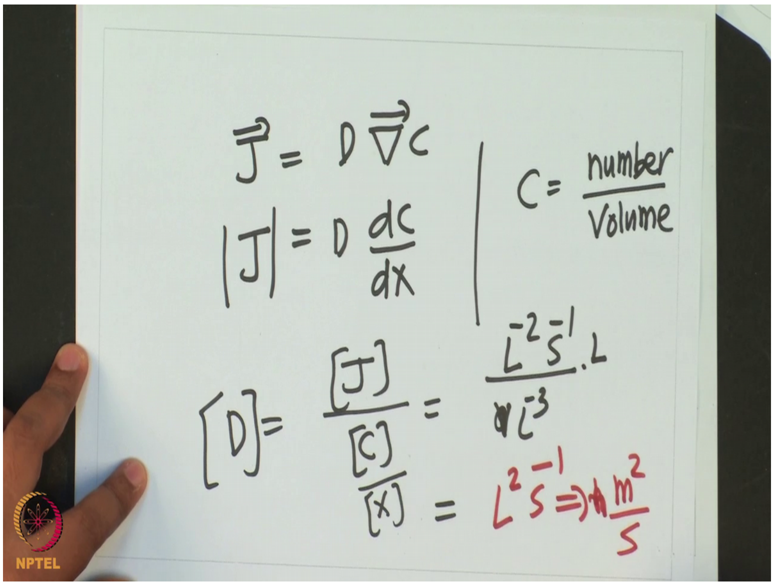

So, what we know is that J is D del C which would mean D dc by dx. So, the magnitude of J is J dc by dx them this would be the magnitude and there will be an x cap if I want to put a direction, but this is the magnitude now let us what is the dimension of D. So, what is the first of all let us understand dc by dxc what is C? C is number by volume number of this is a number concentration. So, let us take C as number by volume. So, this is a number constant C is the number concentration number by volume x as of course, the dimension of length. So, this J has a dimension we just saw.

So, if I just take everything to this other side J divided by the C dimension of C divided by dimension of L. So, that will be dimension of c. So, this dimension of J the dimension of D would be dimension of J divided by dimension of C divided by dimension of x. So, which would be J we just saw that L power minus 2 s power minus 1 divided by concentration is one by volume. So, which is L power minus 3 and x is L. So, divided by that is like multiplied with L. So, this would give us. So, if I just do this L; L power minus 3. So, this would be L power minus 1. So, this would give us L square s power minus 1.

So, you know the unit would be meter square per second the corresponding unit would be meter square per second. So, the dimension of diffusion constant is L square s inverse and this unit is meter square per second. So, that is the unit of D; D is in meter square per second now. So, that is the basic understanding of the flow equation that we learn we know; what is J; which is the flow we also know; what is the D which is the diffusion constant.

Now, the question is; is this diffusive flow is the only flow the answer is no there are many other kinds of flows you could think of 2 two very products phenomena that you all know one is sedimentation if you put some heavy molecules in a test tube they will all sediment to the bottom right, either you could they could sediment in the gravitational force or if you apply, if you rotate it like due to centrifugal force, they could also sediment, then you could also come to the bottom interpret typical situation, there are some other situations where you have charged molecules, then you apply electric field and under the influence of this electric field this molecules would come to this bottom of this tube.

So, if we consider 2 things one is this you have an external field which either could be the gravitational field or it could be the force due to electrostatic force electric field. So, if you consider flow under an external force. So, this is an; if whenever you apply an external force particles can flow.

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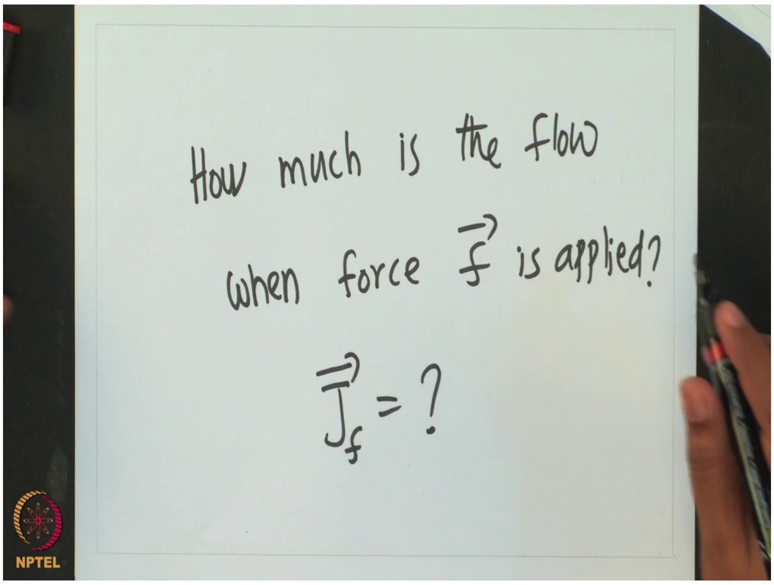

So, something that we will discuss now is how much is the flow how much is the flow when force f is applied. So, this is the question that we will quickly want to see and it turns out that this flow. Let us call J f the flow due to this the current due to this external force what is this how much is it a function of f.

So, let us think of think of some situation where you have an electric field and things are flowing. So, this flow could be in any direction. So, depending on it could be in the direction of the electric field or the force that you apply so.

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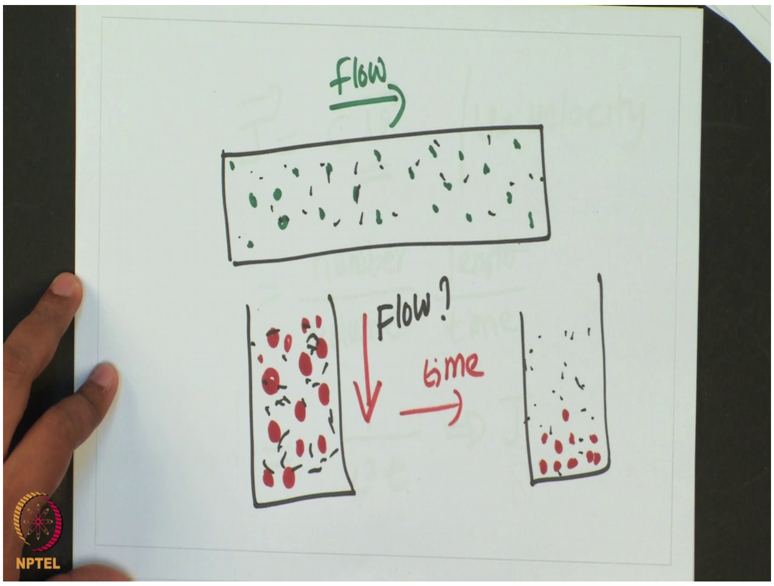

So, let us say; think of low concentration here. So, some let us think of a uniform concentration if there is particles some uniform concentration everywhere. So, there is a uniform concentration C of particles.

Now, these are charged particles let us say and I apply an electric field. So, if I apply an electric field depending on the charge there will be a particular direction you to apply electric field, but the flow will be in this direction. So, apply electric field appropriately depending on the charge. So, that the flow it will flow in the direction appropriate to the electric field.

So, let us say this is the direction or I could apply a force such that all this particle will be forced to flow in this direction the equivalent situation would be like sedimentation gravity. So, you could if you have an heavy particle. So, now, this is all in the medium. So, of course, there is water everywhere there is water; water molecules everywhere. So, this is also a medium here also this is the medium. So, there is water everywhere in this water let us say we are putting heavy molecules which can come down under gravity.

So, we are putting relatively heavy molecules which can come down under gravity. So, there will be a flow from top to bottom all will sediment and you would expect after sometime after sometime you would expect all of them sedimented most of them something like this most of them at the bottom some of them were like this and of course, there is water. So, this is what you would expect after some time everything will come down and sediment at the bottom.

Now, how much is this flow can I can we quantify this flow and it turns out that this flow J is proportional to of course, the concentration and is also proportional to the velocity of this flow. So, the simplest it can be shown that the flow J in this case.

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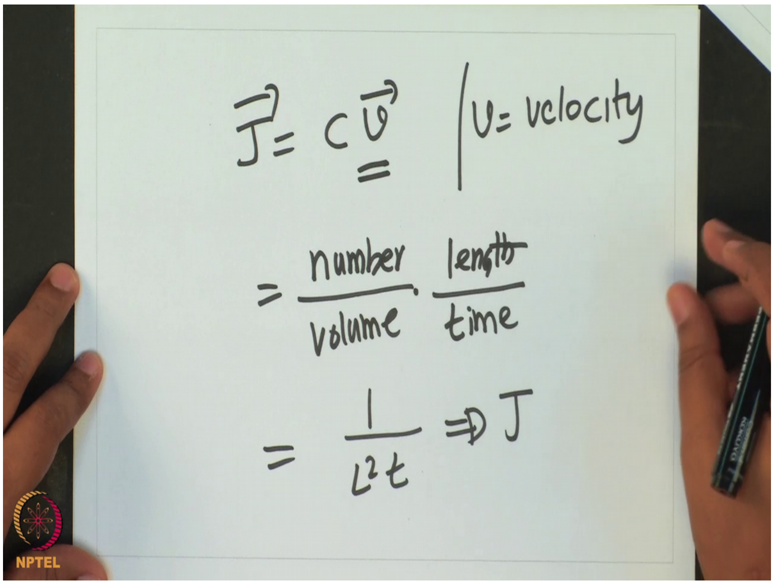

The flow due to external force J is the concentration times the velocity where v is velocity why is there the velocity; velocity is there because there is the force when there is a force when you drive things there will be a velocity and that velocity is v.

Now, let us look at the dimension of this a concentration has number by volume and velocity has length divided by time. So, number is the dimensional as thing. So, this would length and volume the volume is length cube. So, this will give us 1 by L square t. So, this is the dimension of current of course, something that we learned.

So, dimensionally this is indeed correct now the question is what is this v how can we know can we know what is this v to understand this we will go back to what we learned in school that is if something is moving in a viscous medium. So, if I take this situation look at this protein molecules, they are going to flow in viscous medium could be water or it could be any medium it could be the cytoplasm whatever be there; there is a viscous environment and your things are moving in a viscous environment.

So, if there is viscosity the viscous environment the force would be if I apply a force it will be there will be viscosity there will be a drag viscous drag and things will reach some kind of a terminal velocity which we know from the school that what we have learned is that a force f would lead to a velocity.

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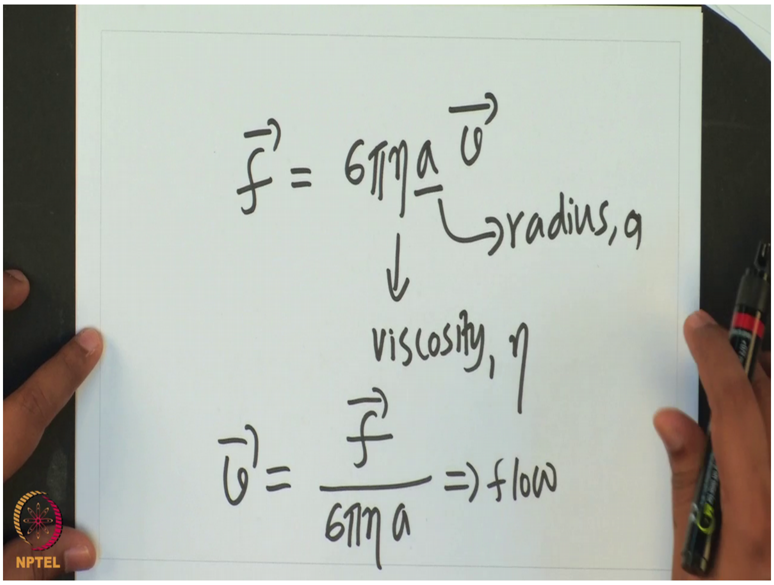

The relation between force and velocity in a viscous medium was that if where spherical particles of radius a; the viscosity eta and radius a. So, this is viscosity and this is radius.

So, you have viscosity and radius viscosity eta and given these 2 parameters the force f would lead to a velocity v eventually the velocity would be v in other words v would be for a spherical particle of radius. So, the assumption is protein is as like a sphere of radius a f by 6 pi eta a. This would be the typical velocity that one would get. So, this is the velocity. So, there is a; remember the f is a vector velocity is a vector.

So, in the direction of force there will be a velocity this is important to understand the direction of the force and the direction of velocity will be the same and there will be a flow which is essentially driven by this velocity. So, the velocity is what is leading to flow. So, this was lead to a flow and that flow is proportional to the velocity the more the velocity the more will be the flow and their proportionality that constant would be the local concentration.

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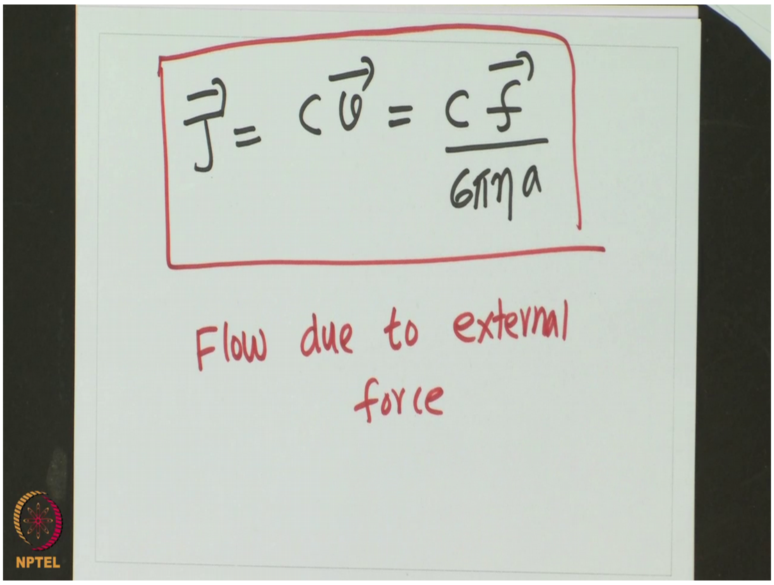

So, it turns out that we can see that the flow J is equal to C times v which is equal to C times f by 6 pi eta a. So, for a spherical particle; so, this is the flow due to external force this is the flow due to external force. So, this is flow due to external force this does not depend on diffusion constant of course, it has nothing to do with diffusion it has what it has to do is with this external force.

It is important to remember the vector is the direction of the force you apply will be the direction of the flow and it will depend on the concentration which is a scalar and viscosity a and all that the precise form could change. So, here the assumption is the spherical particle of size radius a if the radius if this not a sphere this particular form might change a little bit, but it would be some function like this and this is the simplest thing one can think of.

So, we can think of 2 types of flow; flow due to diffusion and flow due to external force the flow due to diffusion will depend on the concentration gradient while the flow due to external force will depend on the force and the local concentration.

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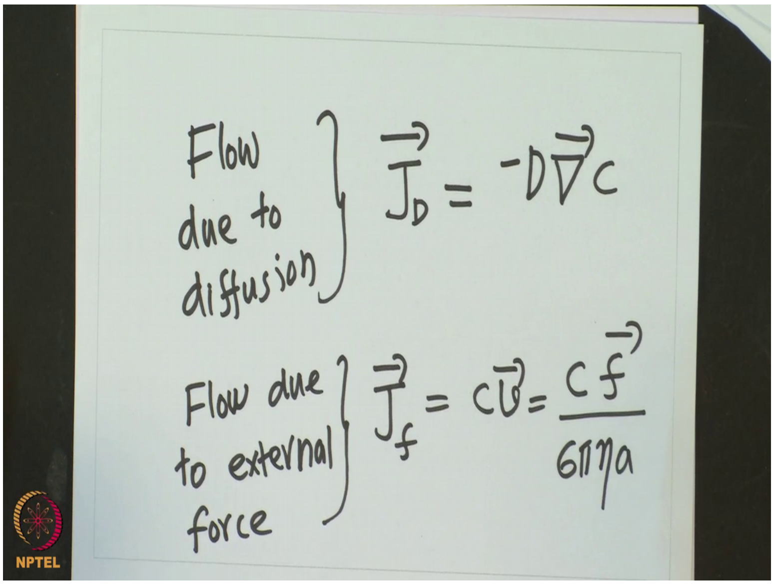

So, let us write these 2 things the flow due to diffusion let us call this jd diffusive flow which is minus D del C which is the vector notation. So, it depends on the change in concentration and as the direction the flow will pick up and flow due to external force it could be electric field force due to electric field it could be force due to gravitational force this is like sedimentation let us call this J f and this will be cv and which we can write C f by 6 pi eta a. So, these are the 2 things flow due to diffusion constant and flow due to external force.

So, the summary of this lecture is there are 2 types of flow we at least we can think of this is when is diffusive flow because of the concentration gradient which we know the dimension we know the dimension of J both the currents which is one over L square one over L square t there is L square minus 2 t minus 1 and we also know the diffusion constant dimension and units.

So, its L squared divided by t which is meter square per second and typical numbers, I urge you to think about typical numbers that would be relevant for a biological context guess the numbers it may be wrong, but still make a an educated guess a of numbers of all this what is the magnitude of force in a cell what is the magnitude of velocity what would is magnitude of all this quantities to think about this so that we will get a physical perspective.

With this I will stop this lecture and continue in the next lecture. Bye.