Introductory Mathematical Methods for Biologists
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Lecture - 26
Gradient, Forces and Flows : Part II

Hi. Welcome to this lecture on Mathematical Methods for Biologists. We have been discussing vectors, and we learned about a gradient, gradient of concentration. And, we said briefly that the flow of proteins the diffusive flow is proportional to the gradient of concentration. And in this lecture we will continue to learn about gradients. So, the title is again Gradient, Forces and Flows the second part of it.

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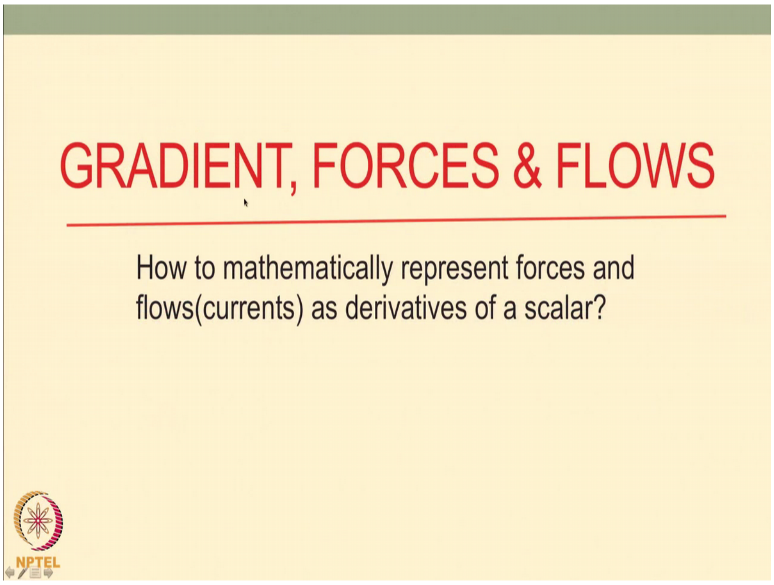

And we would again discuss how to mathematically represent forces and flows as derivatives for scalar. So, we just learned about concentration gradient, and let us understand this a little bit more in the context of diffusion.

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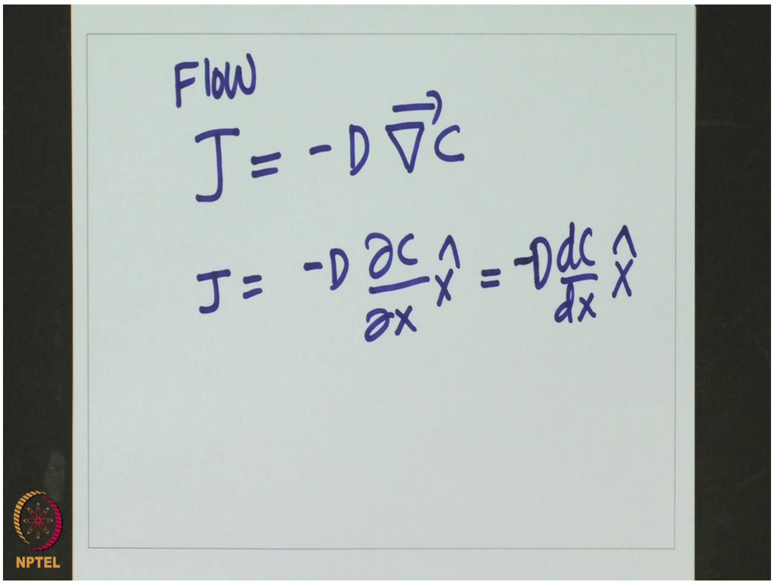

So, what we stated is that the flow. So, let me write flow of proteins given is equal to minus D del C. You know if in 1-dimension this would mean that is equal to minus D del C by del x x cap. This is just; if I could also write this as dc by dx I could write this also as minus D dc by dx x cap; since I have only x here. This is something that we learnt.

Now, let us understand what does this actually mean, and what is the flow, what is the dimension of this, what exactly we mean, and why this negative sign and so on and so forth. So, let us think of a two situations where you have two things here.

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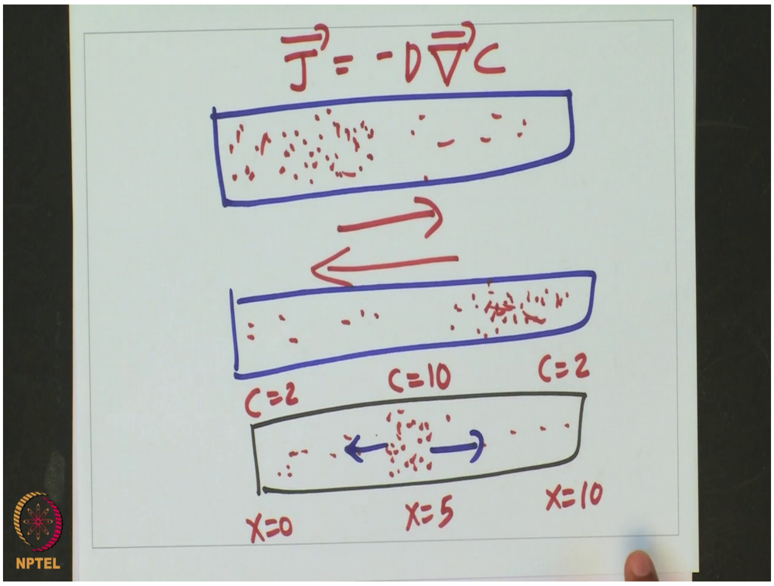

So, if I have a situation like this, where you have higher concentration here at the middle; so a higher concentration. First let us think of this higher concentration is left side and lower concentration on the right side. So, higher concentration here, lower concentration here. So, the flow will be in this direction.

If I have a situation which is like this where you have higher concentration here and lower concentration here the flow will be in this direction; from higher concentration to lower concentration will be the flow of protein molecules. So, the protein molecules will flow in this direction, here the protein molecules will flow in this direction. If I have a situation which is in between, where I have in the middle I have a lot of molecules and very few you know either sides; let us say this is x equal to 0 this is x equal to 5 and x equal to 10 all in same unit let us say micrometers. If I have this, and I have a concentration here, here c is equal to 10 micromolar; this is the unit in concentration units and let us say here it c is equal to 2 and here is also c is equal to 2; so a high concentration at the middle low concentration on either side.

Even here the flow will happen to both sides it will flow this way and this way from the center it will flow to either side, all of this situation these three different situations should be describable by just this one equation which we just said which is the flow with a particular direction is minus D del C. This one equation should be able to describe all these three conditions. Where, here the flow is this way here the flow is this way here the flow of proteins would be to either sides like high concentration of the middle on the left and on the right there are the concentration is small. So, the flow should be on to either side. And this should also be describable by this equation.

So, let us take this situation first, once we understand this all other situations will be trivial and we can understand. So, let us consider and focus on this situation and try to think about what is the J which is the current in this particular direction. What is let us calculate the J. So, we are considering the case.

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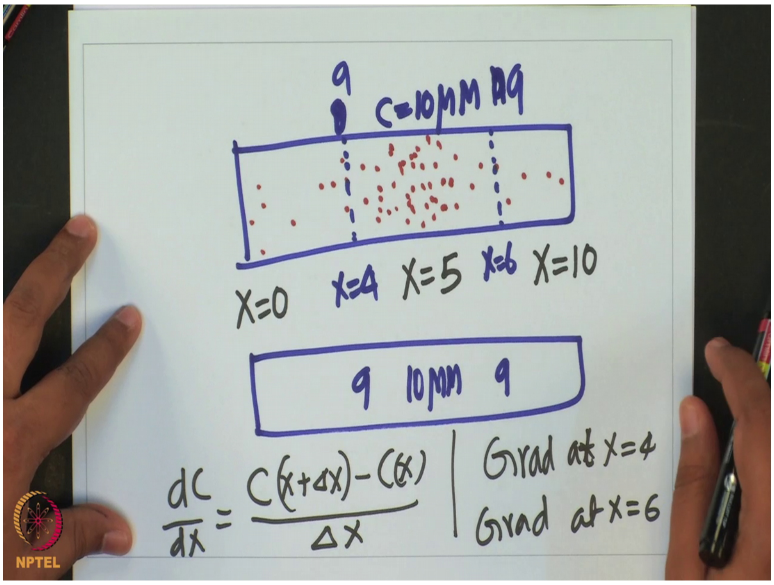

A tube, and here is x equal to 0 is the distance in a unit let us say micrometers, here it is x equal to 10 and let us say somewhere in the middle which is x equal to 5; all in units micrometers. And the concentration of protein is very high in this middle where x equal to 5 very high concentration; and as I go along this it will be lower and lower concentration. And if I have such a situation high concentration with the middle and lower concentration with this as to either side; and the flow should happen to either side.

And let us now compute J. So, let us say here the concentration is 10 micromolar. And if I go to a 4 here, x equal to 4 here let us say this is and x equal. So, here this is x equal 4; so this is x equal to 4 and here this is x equal to 6. So, these two lines are x equal to 6 and x equal to 4 and here the concentrations are let us say respectively 9 and 11; 9 and 11; 1 micromolar change. Here it is; sorry 9 and 9, so it should be 9 here too. So, this is 9 micromolar is the concentration here 9 micromolar is the concentration here. So, concentrations decreases on either side and let us think of this situation.

The situation is we have a tube and here you have 10 micromolar as a concentration, here you have 9 micromolar as the concentration, here is also 9 micromolar as the concentration, and this as I go along this x. Now let us compute in this case dc by dx. So dc by dx; what is dc by dx? Which is c at x plus delta x minus c at x divided by delta x. So, if I compute dc by dx between this and this. So, let us take let us look at first the flow at 4. The flow at 4 would be a concentration at 5 minus concentration at 4. So, let us first you would calculate the gradient that; so what are we going to calculate? We are going to calculate gradient at x equal to 4, then we will calculate the gradient at x equal to and the direction of this flow we will compute.

So, this is what we are going to do; we are going to calculate gradient at x equal to 6 and gradient x equal to 4 and let us see what we would get. So, let us do this, let us redo this, let us understand; to do these properly let us redraw once more.

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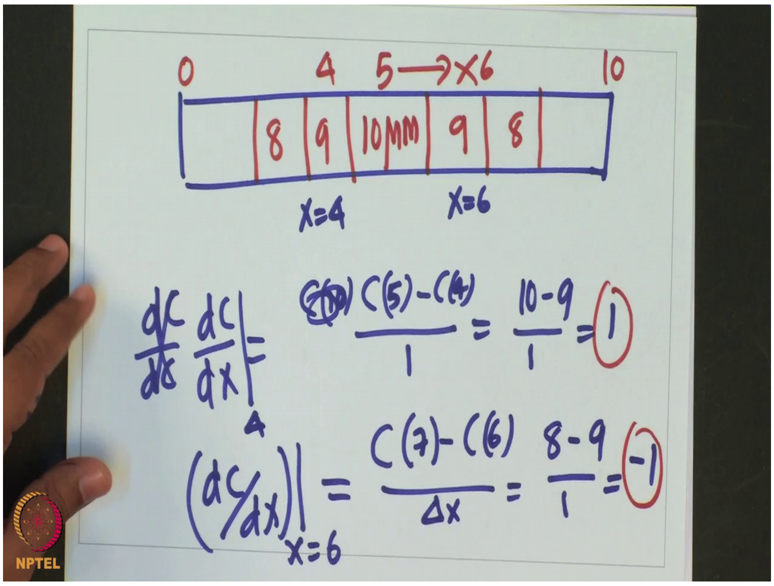

So, we have this long tube and at the middle we have concentration 10 micromolar. And as I go the concentration becomes 9 here, 8 here, and so on and so forth. And here 9 here 8 and the concentration decreases as we go along. So, this is the high concentration to either side the concentration is decreasing as we go along go along the x axis; x is 0 here x is 10 here, 5 here, and 4 here x is 6 here. We would calculate, so this is x equal to 4 and this is x equal to 6 and we will calculate the gradients at these two locations dc by dx. So, c at dc by dx is 4 a c at 10 c at 5 sorry; c at 5 minus c at 4 divided by 1. So, this would mean 10 minus 9 which is 1. So, the dc by dx is one here. So, this is dc by dx calculated at 4.

Now let us calculate dc by dx at x equal to 6. If I calculate here this is 8, the concentration at x equal to 7 minus the concentration of 6 divided by delta x which is 1. And if I do this I would get 8 minus 9 divided by 1 and the answer is minus 1.

So, the concentration at 6 the gradient of the dc by dx here is plus 1. So, the concentration gradient at x equal to 4 is plus 1; the dc by dx at x equal to 6 turns out to be minus 1. Now let us write; so this in the language of gradient. So the gradient, let us calculate the gradient.

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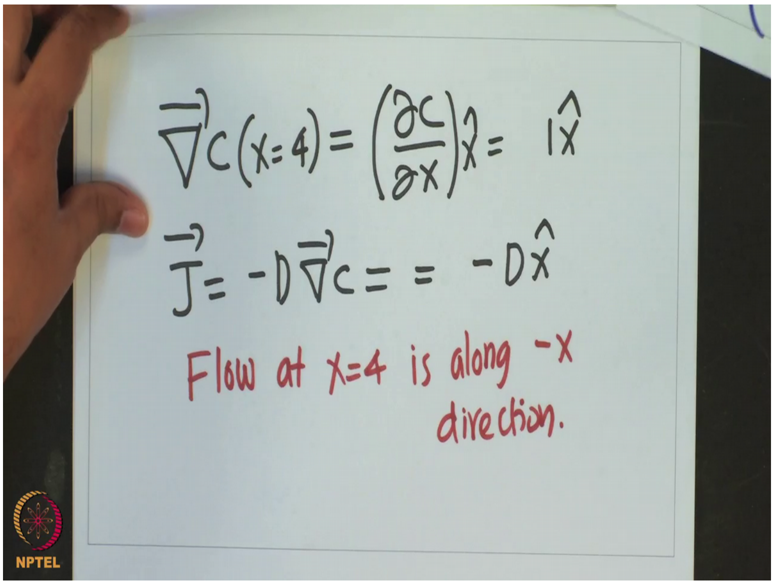

Del C, evaluated at x equal to 4 will gave us del C by del x and the flow; and the del C by del x x cap which was 1 x cap, and the flow is minus D del C which would mean that minus D x cap.

So, this would mean that at x equal to 4 the flow has to be along with the minus x direction. So, let us look at here; at x equal to 4 the flow has to be in this direction, it has to be along the minus x in this direction. And this would say that the D is a constant and the flow at x equal to 4 J is minus D x cap; minus D is the quantity x cap is the direction. This would say is that, diffusive flow at x equal to 4 is along minus x direction which is very much true. At x equal to 4 where the concentration is the flow will be along this minus x direction.

Now let us look at x equal to 6, where the flow has to be along the plus x direction because higher concentration here, lower concentration here. So, the flow has to be along from here to here along the plus x direction. So, if I do the same thing at x equal to 6, we already found that at x equal to 6 the dc by dx is minus 1. Since, the dc by dx at x equal to 6 is minus 1, if we write the same thing; if we write J is equal to minus del C.

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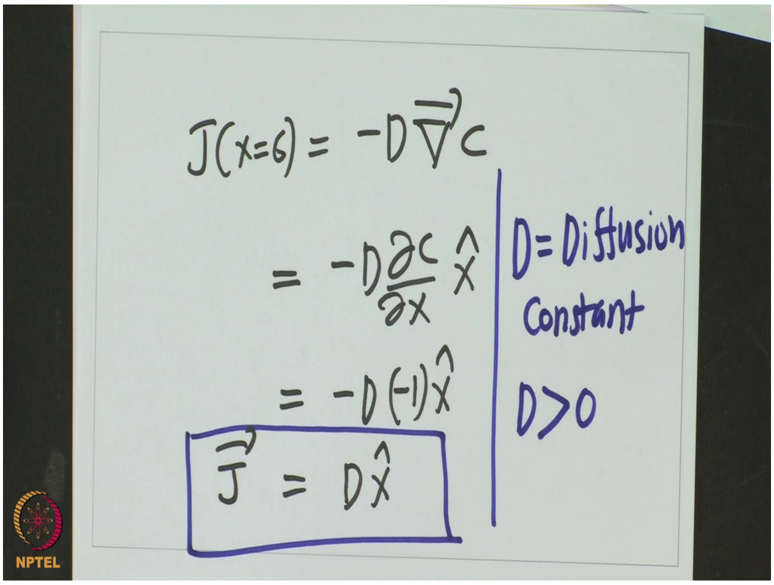

So, let us write J at x equal to 6 is minus D del C which is equal to minus D del C by del x x cap, and this is equal to the dc by dx which is same as delta del C by del x here is minus 1. So, this would mean is D x cap.

So, the flow at x equal to 6 is dx cap where D is a positive constant. And you have to remember D is called the diffusion constant this is some number greater than 0. So, D is a positive number. Once we understand that D is a positive number and we will understand about D later this J is along the x direction at x equal to 6. So, let us relook at this; at x equal to 6 J is along the plus and x equal to 4 the J is along this. So, once we redo this we will understand this that the formula J is equal to minus D del C can describe this flow and this flow with appropriate direction. And that is the key in understanding the direction of flow and this can be now represented as a vector.

So, I urge you to redo this think about this, and understand how this formula that J is equal to minus D del C represents the flow with appropriate direction. There are many other quantities which are also gradient and one of the other famous quantity is force. Something that force that we all know, something is falling down because the earth is pulling the gravity is a force, the spring force which is elastic force; all of these are forces which are gradient of energy. So, it turns out that one can write force as gradient of energy.

So, let us write force as a gradient of a scalar. So, another force is a very well known vector.

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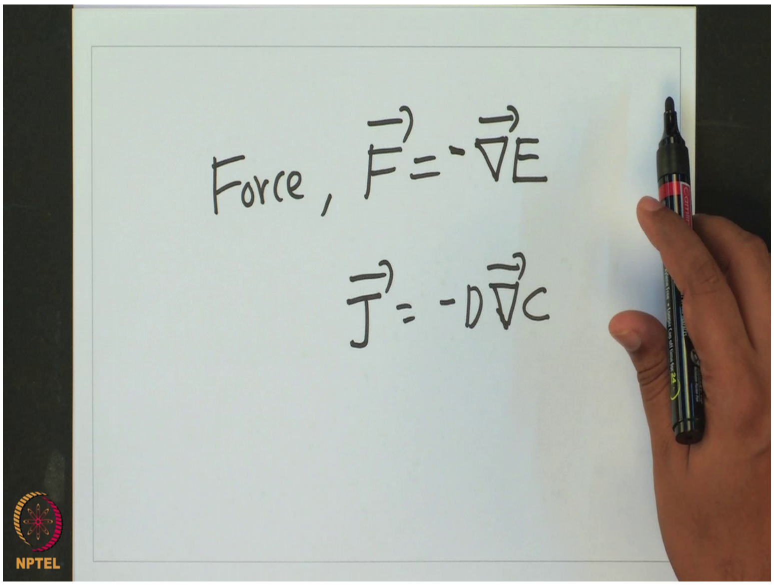

It turns out that force is minus it is a gradient of energy with a minus sign; minus the gradient of energy minus del E. Energy is a scalar so gradient of a scalar concentration of scalar, so J is minus D del C. So, here concentration is scalar, energy is a scalar, the derivative of scalar this quantity del gradient is a vector and a force and flow are vectors having a specific direction.

So, let us now understand something about del E. So, let us think of gravitational energy which is easy for us to understand. So, let us think about the energy.

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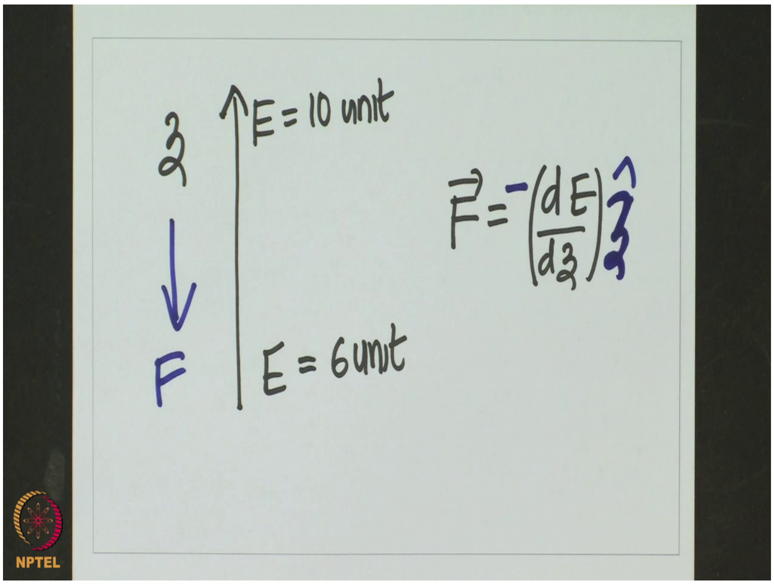

If you think of any energy let us say which is, high here energy is high here. So, let us say E is equal to at this location let us say 10 unit whatever be the unit of energy. Here the energy is 6 unit. So, the energy decreases as we go from here to here. and the force will be; let us this direction if I take this as direction which is let us say z direction or y direction if I want. If I take this as a z direction and this is the energy is 10 unit here and 10 unit here. And the D E by dz how does this energy change along the z; so the change will increase as z increases. And the force direction; so if this will be a positive number, but the force will be along this direction; so the direction of the force will be from high energy to a lower energy, so the force will be with a minus sign and along the z cap.

So, F will be minus de by dz z cap. So, this is higher energy and lower energy, and this would mean the definition of force. Now, let us think of this by taking a simple example of a spring something that we all know. So, many of the biological molecules proteins can be thought of as a spring. So, as far as elastic energy is concerned or many bond energies are concerned.

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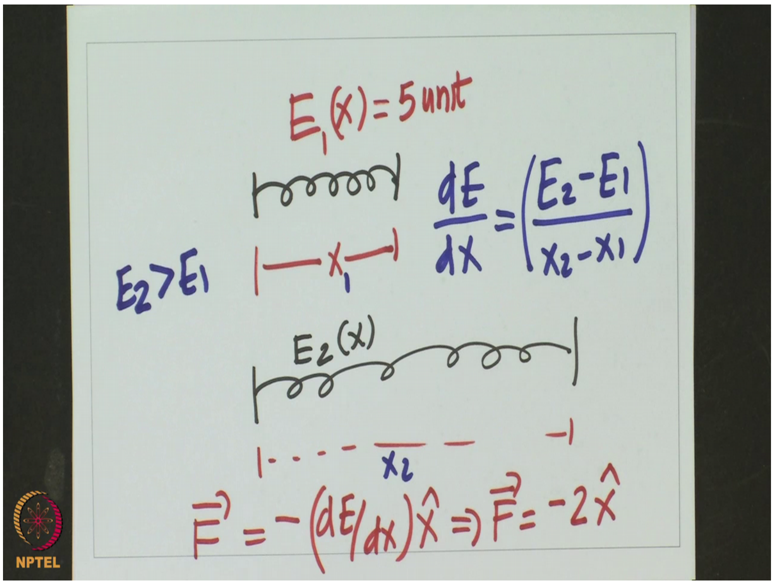

So, let us take a spring. This has some x and there is some energy for this particular x let me call E 1 x. So, let us say this is 5 unit and this unit could be joule whatever be the appropriate unit of energy. Now if I have the spring stretched by a distance, now this is a new x. So, if where x is 1 here it is like 2 or 10 higher x. And here the energy for this x is E 2; for this particular x the E 2. So this minus this, so del E by del x: basically E 2 minus E 1 divided by this x, so let me call this x 1 and this is x 2; x 2 minus x 1. Assume that these changes are very small then we can write this as de by dx.

So, this will be the change in energy. Now we know that once this is the case the force will be in this direction; this will be pulling from higher energy if this is a higher energy and of course you need higher energy to pull longer. So, this of course, E 2 is we know that E 2 is larger than E 1 it. So, this is going to be a positive number, x 2 is larger than x 1 and E 2 is larger than E 1 we know this because we need lot of energy to pull this. This say this thing will be a positive number.

Now, the force here which is the vector is minus, this positive number which is dE by dx along x cap. So, this is going to be the force, this implies as the force is going to be some negative number if it is a minus 2 x cap. That means, the force is going to be along the minus x direction; so minus 2 x cap. So, if I pull from here to here the force is going to be along the minus x direction. So, that is what this would mean- that the force is minus del E by del x x cap.

So, the force is also the gradient of energy. And this is also true for many other things like in general energy is basically potential energy. So, even electrostatic potential they have a higher potential lower potential the electrostatic potential difference; like sometime we will be written as del V. So, the electrostatic force can be written as.

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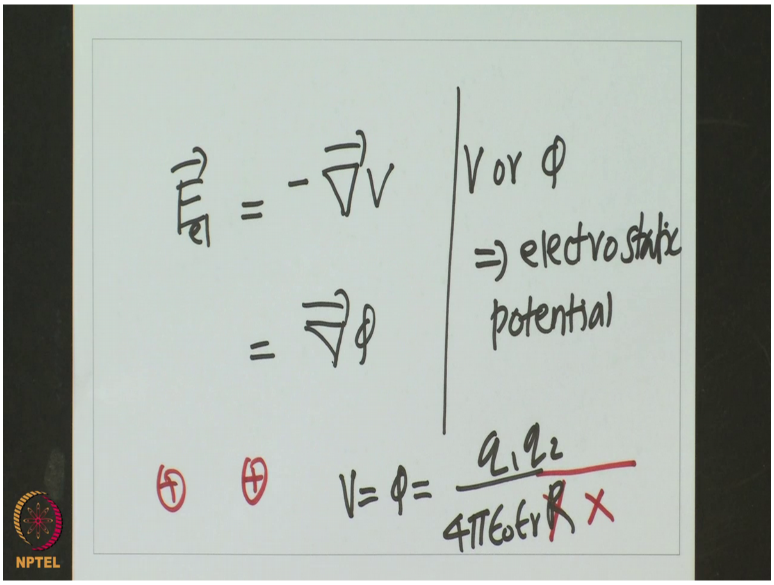

Minus gradient of minus del V, this is basically the electric electrostatic force which we would write; I would write electrostatic or the electric field. So, I would write E sometime. And this is true in many other areas. We will come and learn this thing as we learn something about Nernst equation, where we would use like sometime this could be written as del phi where phi is the electrostatic V or phi; is electrostatic potential.

What is electrostatic potential? V or phi is nothing but q by 4 pi epsilon 0 epsilon r R; q 1 q 2 there are two charges it will be q 1 q 2 we have two charges. The electrostatic potential energy can be written as q 1 q 2 by 4 pi epsilon 0 epsilon r. The derivative of this would with a minus sign, would give us some direction of this from this force and we can learn many things about this. So, think about what is the derivative of this. In this case is just x, so I could write r as x. So, if I have here I would replace this r with x.

So, basically what we are learning now is that the gradient many of the all forces are gradient of energy. So, this is an important thing that all of we should understand: all forces can be written as a gradient of energy.

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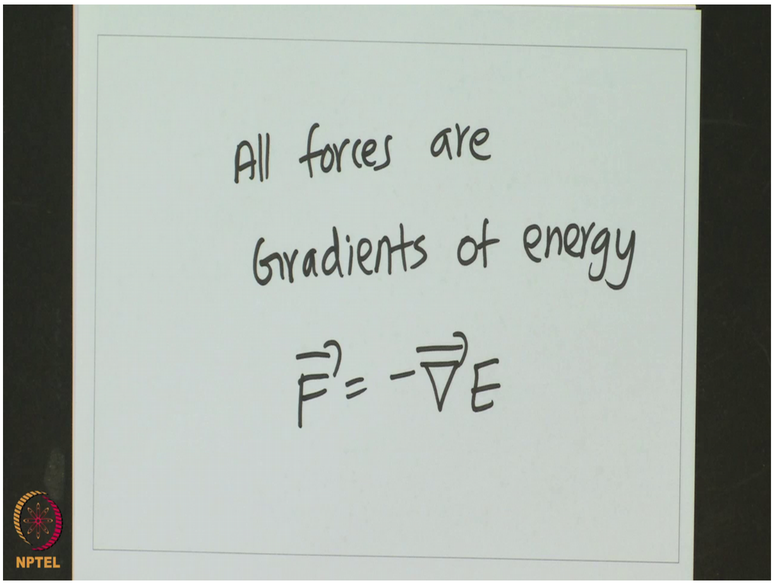

All forces are gradients of energy. It turns out that we can only measure this gradient in energy which is typically called the potential difference. We cannot measure the energy itself absolutely. We typically only measure the change in energy which is basically the force.

Since, the force is minus gradient in energy we only measure the gradient in energy, therefore we can only measure the force which is not the energy itself. I can add any number to energy, but the derivative will remain the same the slope energy will remain the same. So, we can also only measure the change in energy of the slope in energy in any experiment. Therefore, the meaningful quantity is the gradient energy, how energy is changing, what is the potential difference, how concentration is changing, etcetera are a highly meaningful quantities. And measuring this would give us very important information.

And most of the experiments would measure this, change in energy which is the force or change in concentration the gradient, the change in number of particles like how the numbers of proteins change as we go along the cell. And this measure change in these numbers are meaningful and it convey us a lot of interesting information about the phenomena happening let it be diffusion or let it be the pulling force.

There are few other examples or many other examples of gradients, and we will discuss some of them as we go along. But twos of two famous examples are energy, gradient as force, and the concentration gradient as the flow which is the diffusive flow.

With this, we will stop this lecture and continue in the next class. Bye.