Introductory Mathematical Methods for Biologists
Prof. Ranjith Padinhateeri
Department of Biosciences & Bioengineering
Indian Institute of Technology, Bombay

Lecture - 23
Vectors : Position and Movement in 2D

Hi. Welcome to this lecture on Mathematical Methods. We will discuss more about quantities that have a particular direction, and how do we describe things in biology which needs to proceed in a particular direction. So, the title is Vectors Position Movement in 2D.

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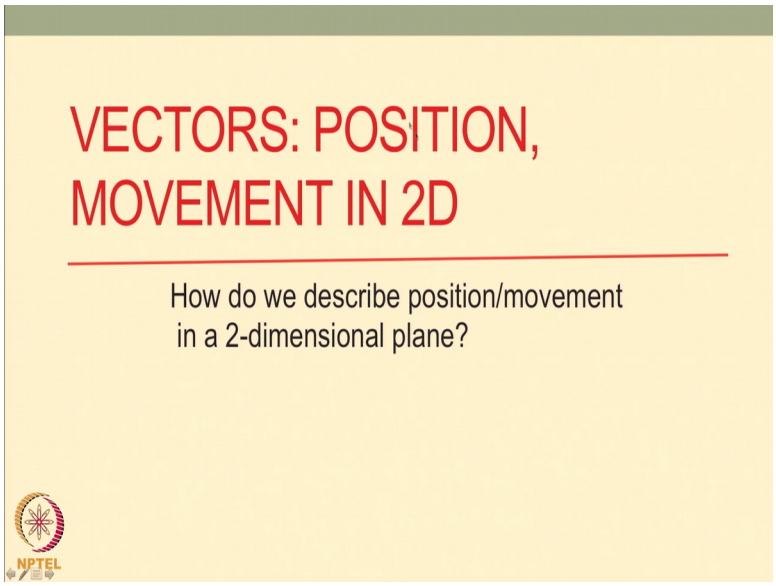

2D means 2-dimension; how do we describe position and movement in a 2-dimensional plane. This is something that we would describe. And if we understand this lot of things we understand because, many things in nature then we want to describe we are essentially dealing with position of things. For example, if you caught about talked about structure, structure of a molecule; what is structure? Structure is basically the position of atoms or molecules. If you have structure of a protein how each amino acid is organized in 3D; how each atom is organized in 3D.

So, the position of each of these atoms with respect to the other atoms, the relative position of each of this that is what is describing the structure. If we want to know whether the concentration of molecules is higher than somewhere lower somewhere;

what is concentration would mean? Concentration in what location they are positioned, like they are concentrated, which position they have they have a higher density. That is what essentially concentration. At that particular position where is the number right or how much is a number. So, how do we represent position mathematically properly? If we understand that that is important to understand many things because, many things that we study are function of position.

So, let us this is something that important to understand.

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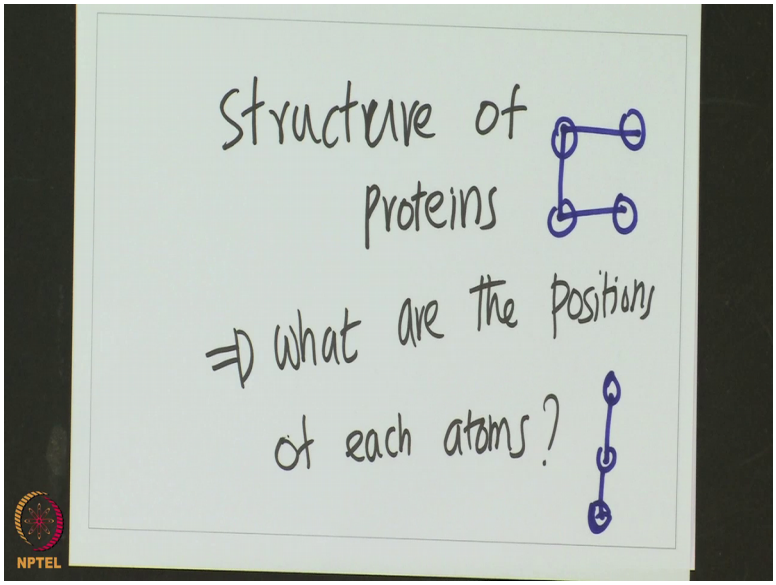

Most of the things that we learn for example structure; structure of molecule this is something that is in biologists structure of protein or any, right. This would mean that what is the position of each of; so the question is: what are the positions of each atoms? So, if the structure is this way; for example if you structure is this, where the atoms are positioned in this particular fashion as opposed to the structure is like this, so this I have two different structures and we want to describe the mathematically properly. This is one example that we would want to understand.

Another example is the concentration itself is the position and a function of position.

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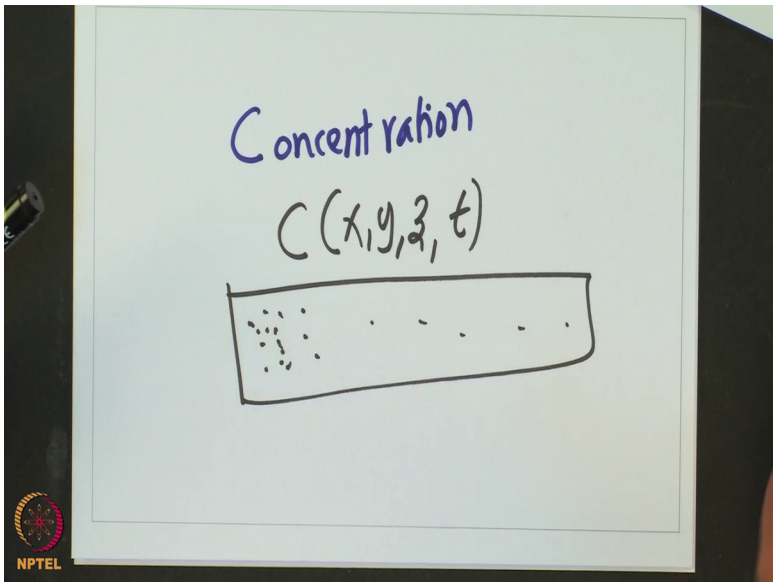

If you think of the tube there is higher concentration here and lower concentration here the concentration is a function of position. So, concentration is a function of x y in this case and in 3D, it would be z also, and it could be in also a function of time. So, we know that this important to know the position of particles even to understand the concentration and how to represent this in the vectorial manner is something that we would want to understand.

So, to represent position let us think about the simplest thing.

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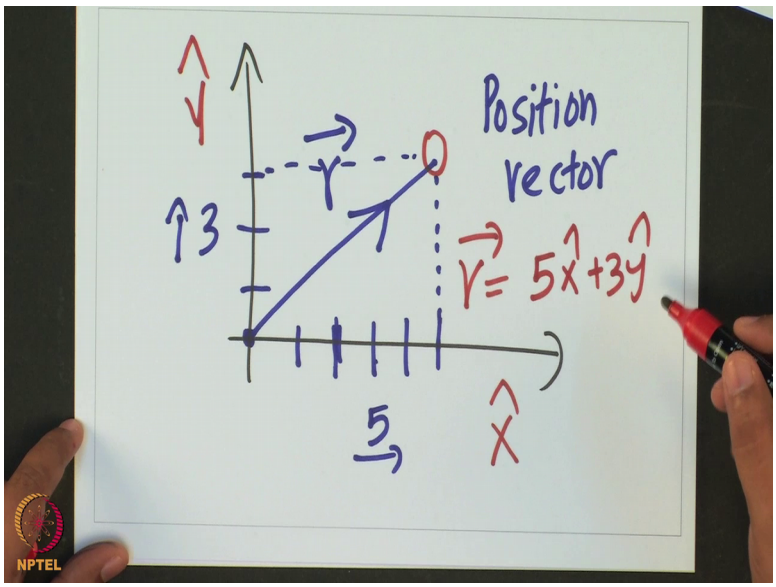

So, if you have a 2D plane and we have an x and a y. So, this is an x and y, and this is the position of a particular particle; particle of our interest and this is the position of this. So, I can draw a vector from 0 origin to here; I can draw a vector, I can draw an arrow. So, this vector let us call position vector. So, this position vector represents this position vector as r vector; r is a vector which I drew usually this arrow that I drew from here to here. Now this arrow has two things.

So, now this is one the case. Now even a how do you write an equation for this r is a question.

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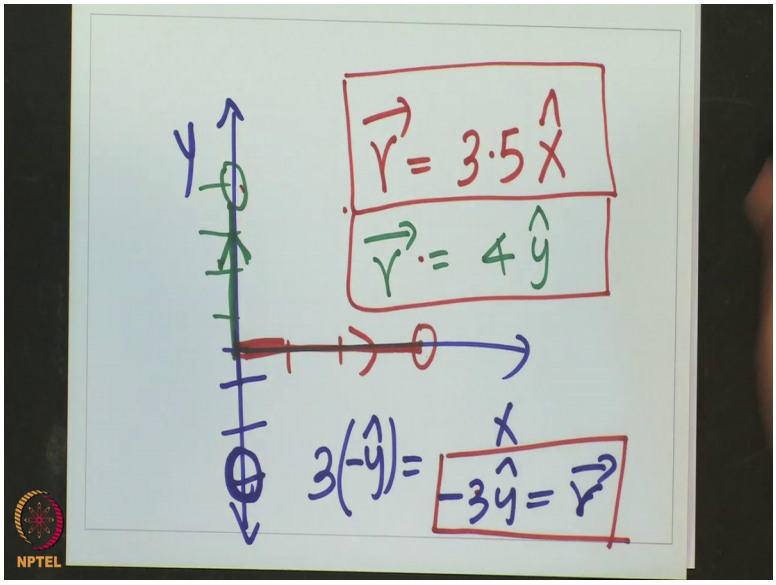

To understand this let us taken take even a simpler case where we would have just x y, and this particle which is basically going to be here in the x and my arrow is basically from here to here; this is my arrow along the x axis this is my arrow. So, if the position is here then this is my arrow. And to represent this I could say that the location of this, so this is 1, 2, 3; so let us say this location would be 3.5 distance away from the origin.

So, I could say that the position of this is 3.5 distance away from the origin; 3.5 units away from the origin along the x axis. This is the position vector for this particle. So, this is the position vector of this particle which is 3.5 units away along the x axis. This means that you have a particle along the x axis 3.5 units away from the origin that you are mentioning, that you want to start. So, you have to fix the origin and once you fix the origin we can compute this.

Similarly, if I have a particle here which is along the y; so I have a particle along the y you have 1, 2, 3, 4; 4 units away along the y then I would say my position vector is 4 y cap 4 units away from the origin along the y cap. If it is 4 units down here or 2 units; 1, 2, 3; 3 units down if the particle as here I would write particle is 3 unit down along the minus y which is minus 3 y cap. This is going to be my position vector.

So these three things: r vector is minus 3 y, r vector is 3.5 x, r vector is 4 y; so this represents this, this represents this, and this is for this. So, these three things is the simplest way of representing position which is along the x axis or y axis.

Now if it is something in between like we discussed earlier here, if you have something in between like this we want to first check how much along the y and how much along the x. So, if I just move 1, 2, so this is the 3, 4 and 4. Let us say this is 5 units 1, 2, 3, 4, 5; 5 units along the x axis. And, let us say 1, 2, 3; 3 units along the y axis. So, 3 units along the y axis then, the position vector of this r can be written as 5 units along the x; I if I go 5 units along the x axis and 3 units along the y axis I have 5 x plus 3 y.

Some of this you would have learned earlier in school. So, this is the way that we would typically represent 5 x plus 3 y, this is the vectorial representation of this. Now, if in general; what would when I say 5 x plus 3 y what does that mean. It means you have a particle if I go 5 along the x axis and three along the y axis I have a particle which is at this particular location.

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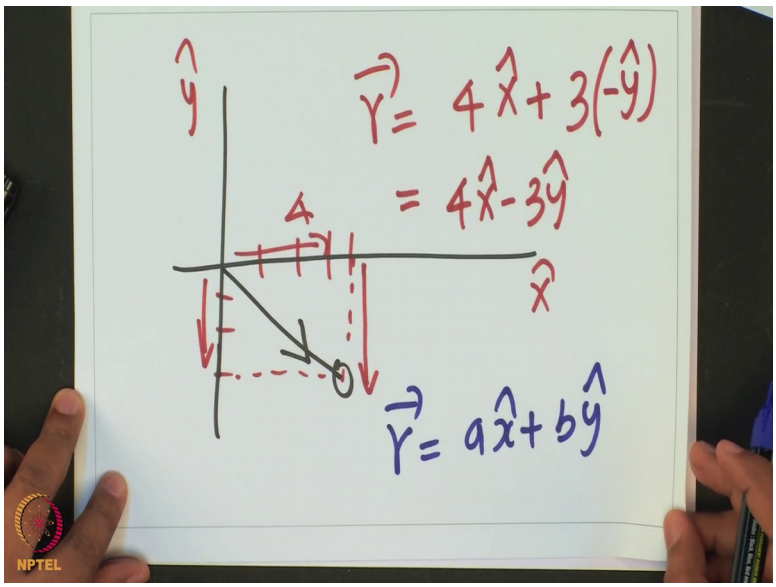

Now, in general I could write this as: anywhere let us if this is here if the particle is here I will have a vector like this and to get it you have to go first along the x direction and then along the minus y direction. So, this is my y this is my x. So, you have to first to go along this and then. So, first you have to go along this 1, 2, 3, 4 units and then you have to go 1, 2, 3 units along the minus y direction. So, I will go 4 units along the plus y direction, so r vector is 4 units along the x direction which is this and then 3 units downward plus 3 units downward 3 units along the minus y direction. This would tell me 4 x cap minus 3 y cap. This would tell me the position of this particle 4 x cap minus 3 y cap.

In general, I could write a sum position vector r is some a x cap plus b y cap. So, a and b could be some numbers, any numbers you want. So, this is the general representation of a position vector. So, a position vector in 2D will be represented by a x cap plus b y cap or this could represent; there are many ways you can represent. So, you could just another way of representing the same thing is. Now, instead of a and b I could use any symbol I want, some for simplicity we will would use r 1 x plus r 2 y and so on and so forth. But that is just a matter of representing.

This is the simplest way to represent something in 2-dimension. Now, going ahead what we want is move me; if you want to also represent movement from one place to another place you could also draw vectors like this which is obvious for us.

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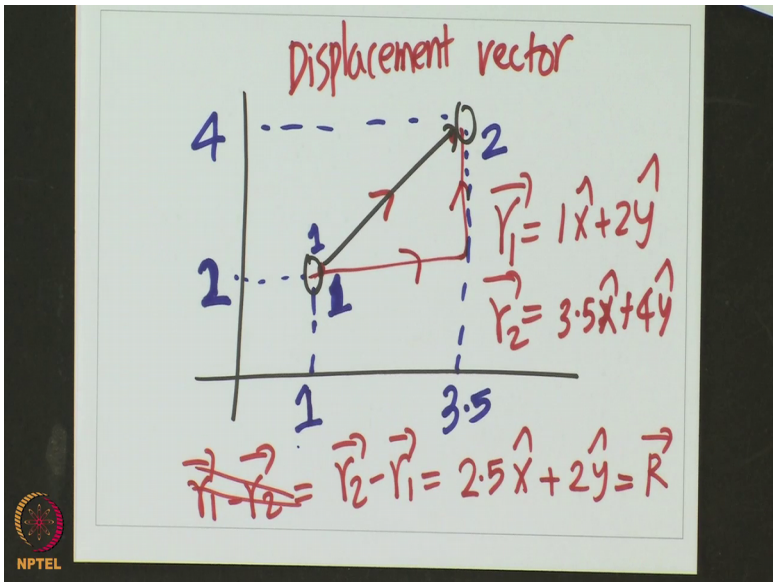

That is if you have; if I say I want to say that I move from this location to this location. So, if I want to say that this location is let us say 1 and 1; so 1 and 2; so the position of this location 1 and location 2. So, I could say the location 1 and this one location 2 is let us say; so this is 1 of particle one and this is let us say 2, 3, 3.5; and this is let us say 4.

So, the position of the first particle I would write r 1 which is a position of the first particle is 1 x 1 along the x axis in 2 along the y axis; 1 x cap plus 2 y cap, this is the position of this. The position of the second particle r 2 is 3.5 x plus 4 y.

Now if I find the difference between these two r 1 minus r 2 I could get r 1 minus r 2 which would mean this minus this or r 2 minus r 1. So, let us first do r 2 minus r 1 this is this minus this, so let us write r 2 minus r 1; so 3.5 minus 1. So, first take the x component and subtract then bake y component and subtract. So this is the rule: first you take only the x component 3.5 minus 1 which is 2.5 x cap and the y component 4 minus 2 which is 2 y cap.

So, this would be essentially this vector, this vector from here to here; this vector that we represent that vector is r 2 minus r 1 this I would call some other vector r vector which is the difference of two vectors. And this would basically tell me that if I this particular vector which is the displacement vector; I would call it displacement vector. How some particle moves from here to here? If we want to say that a particle move from r 1 to r 2 and it has a displacement r and the magnitude of the displacement vector can be written as 2.5 x plus 2 y. That means, it moved from here 2.5 along the x plus 2 along the y 1 plus 2.5 give me 3.5 and 2 plus 2 gave me 4. This essentially means that it displays from here to here 2.5 it moved along the x axis and 2 it moved along the y axis and it reach here.

So, this is things to represent as a displacement. Why is this important; because this is very important, very all the context. Like if you think of the force of two charges, right the force of two charges r 1 r 2 something which we discussed.

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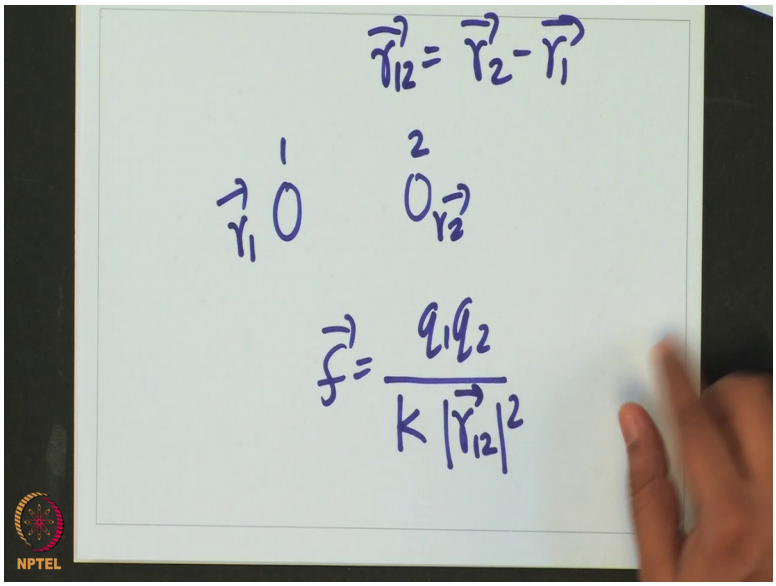

So, you have something here and something here and this is charge 1 and charge 2, and we want to know the force between this. So, what is the force between this? Force between this is some q 1 q 2 divided by some constant k times the distance between these two r 12 square.

Now, what is r 12? R 12 is basically a vector. So, r 12 is the displacement vector, this is a vector between these two quantities. So this is precisely like, if you know the r 1 vector of this and vector of this r 12 vector is r 2 minus r 1; this is r 12. So, just like we calculated we could calculate r 12 which is r 2 minus r 1; this is the way to calculate these vectors.

And now what is this? We will talk about this. Now let us come to that; if we have a vector which is displacement.

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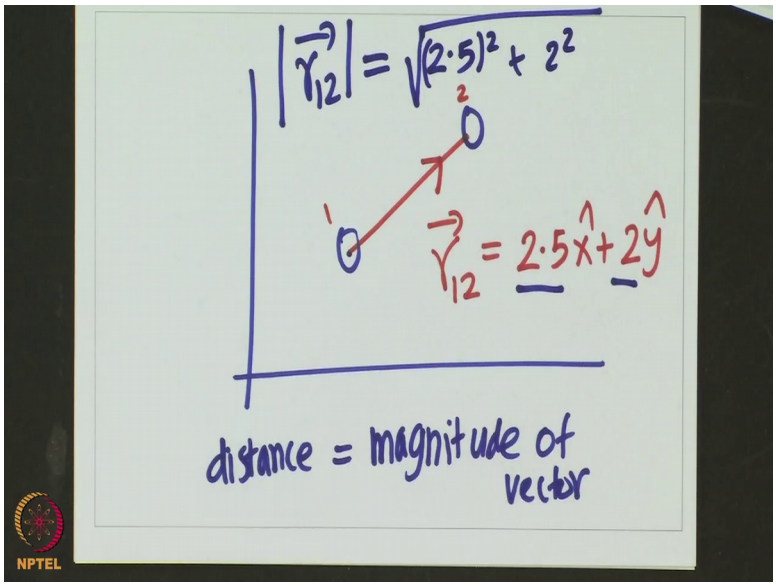

So, if you have two points here and we drew a vector from here to here, and this vector r; this vector let us call r 12 vector which we would write let us say 2.5 x plus 2 y; For example, I wrote this vector I go 2.5 along the x axis and 2 along the y axis I would get this position. So, this is the displacement vector or r 12 is 2.5 plus this.

Now, how do we compute the magnitude of this vector? The magnitude of the vector is basically the length. What is the distance this moved? How far away they are? So, this distance is magnitude of this vector. It turns out that to calculate the magnitude, if we have a vector like this the magnitude r 12 vector magnitude this would mean the magnitude of this vector r 12 is nothing but this square plus this square root; square root of 2.5 square plus 2 square. This is the magnitude of this. Why is it? Because it is obvious; what do we want?

So, think of let us think about this once more. What essentially we want is the distance.

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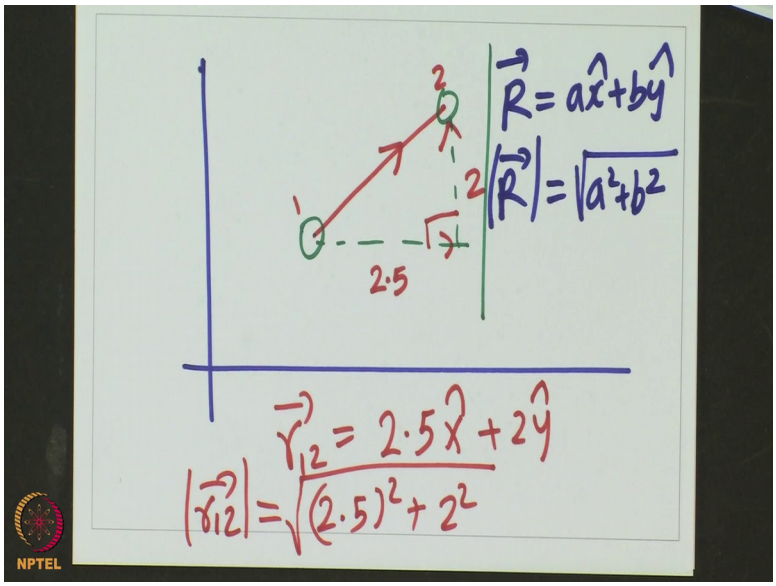

So, we want distance from this point to this point that is what our interest is from here to here. And, this distance is what we were interested in. We are interested in this red distance.

So you know that this is; if I go 2.5 in this and 2 along this I would reach here. So, that is why when I write r 12 vector which is the distance between 1 and 2 the displacement vector which says between 1 and 2, this essentially says that how do you go from 1 to 2. R 12 actually means how do I go from 1 to 2; it would say 2.5 x go 2.5 unit along the x and 2 unit along the y 2 y. Now the magnitude is this, this distance and from trigonometry we know that x square plus y square will give you this z square, because this is a rectangle triangle we have a 90 degree here. Since it is 90 degree here, this square plus this square will give you this that is why this r 12 magnitude is 2.5 square plus 2 square root. So, r 12 vector magnitude is 2.5 square plus 2 square root this is the square root of these things.

In general, if we have a vector which is R which is a x cap plus by cap, the magnitude of, so this is a vector; the magnitude of this vector which is represented by this two vertical lines like this; so this magnitude is root of a square plus b square. This is the magnitude of this vector a x plus b y is root of a square plus b square. This is the magnitude of this vector.

So, let us represent. So, you have a vector.

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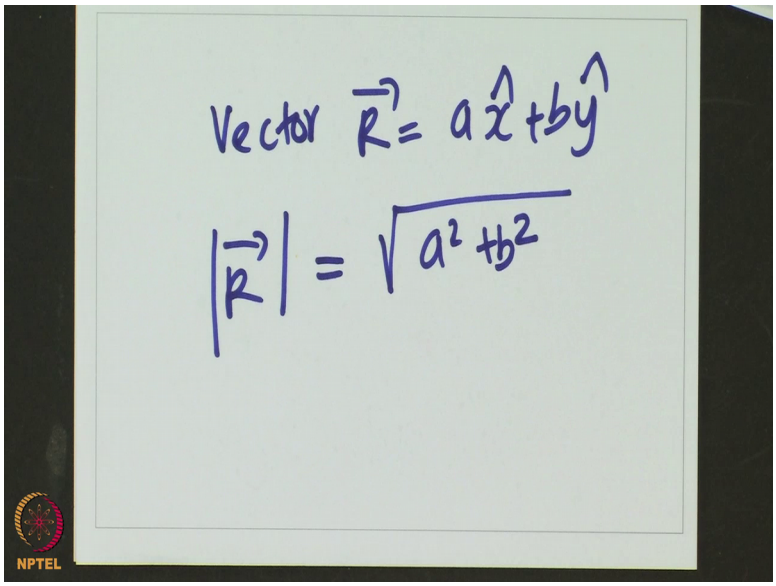

R which is a x cap plus b y cap and the magnitude of r, which is if R is the displacement this is the distance is root of a square plus b square. If R is the velocity the magnitude is speed. So, if this is the velocity in a particular direction what is the speed; that is root of a square plus b square will give us the speed in the magnitude.

Now, there are many other ways of representing this. Now, if you want to another concept that we would also all want to understand is something called a unit vector. So, again let us think of this.

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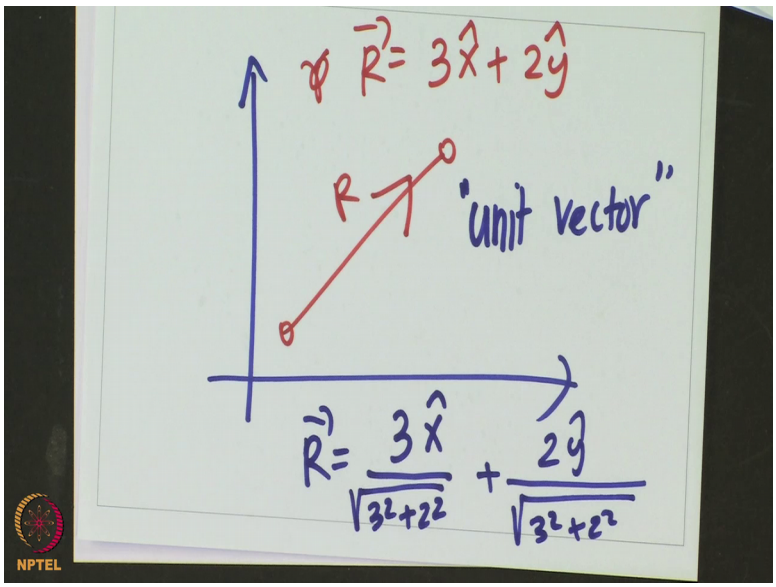

And we have some particular motion in this direction from here to here. And we said this vector along this direction R is represented by 3 x cap plus 2 y cap; 3 along this x (Refer Time: 22:07) direction and 3 along this. Now this is the magnitude which is 3 square. So, the magnitude is root of 3 square plus 2 square would be the magnitude which is 9 plus 4 root 13 would be the magnitude.

Now, what is the direction which is; this is the particular direction. Now this concept called a unit vector is a vector in a particular direction which has a magnitude unity. So, the magnitude of this vector is not 1, but if I divide this vector by root 3 square plus 2 square. So I have this R, I have 3 x cap I divided by a root of 3 square plus 2 square then I have here 2 y and divide here also by root of 3 square plus 2 square what is the root of; this is the magnitude of r. So, if I divide this by the magnitude of r I would get some vector which is going to be a unit vector.

So, let us think about it once more.

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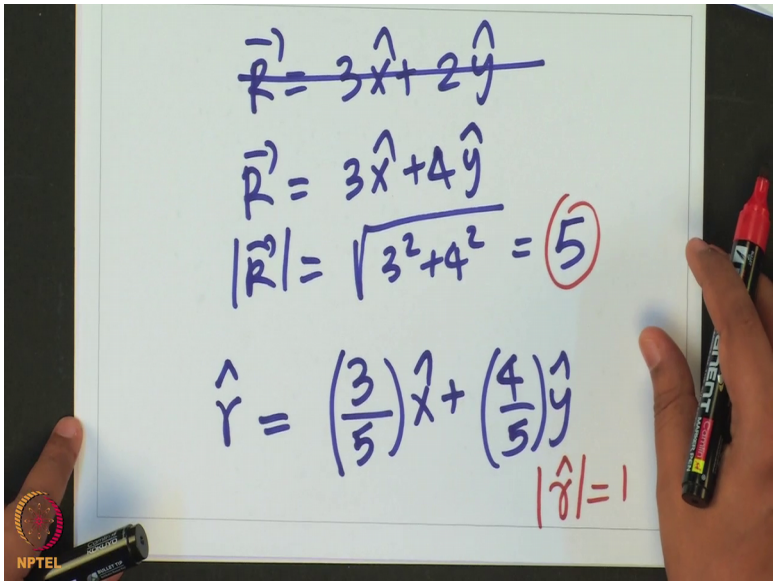

What we have is R which is 3 x cap plus 2 y cap, this is one example. So, another example I could take is instead of 2 I could write here y 4; let us me take another vector which is 3 x cap plus 4 y cap. Now what is the magnitude of this? So, I would take this one. So, magnitude of r vector is root of 3 square plus 4 square which is root of 9 plus 16 to a root 25 which is 5. So, the magnitude is 5. If I divide, if I make a new vector which I would call r hat this is my new vector this vector is 3 by 5 x cap plus 4 by 5 y cap.

So, I have a new vector which is 3 by 5 x cap plus 4 by 5, I divided I use this 5 and divided 3 and 4, and I got a new vector. This vector has a property that this magnitude of this vector is 1; magnitude of r cap is 1. So, because 3 by 5 square plus 4 by 5 square, so let us do this what do we get magnitude. So, magnitudes we have a new vector which is r cap.

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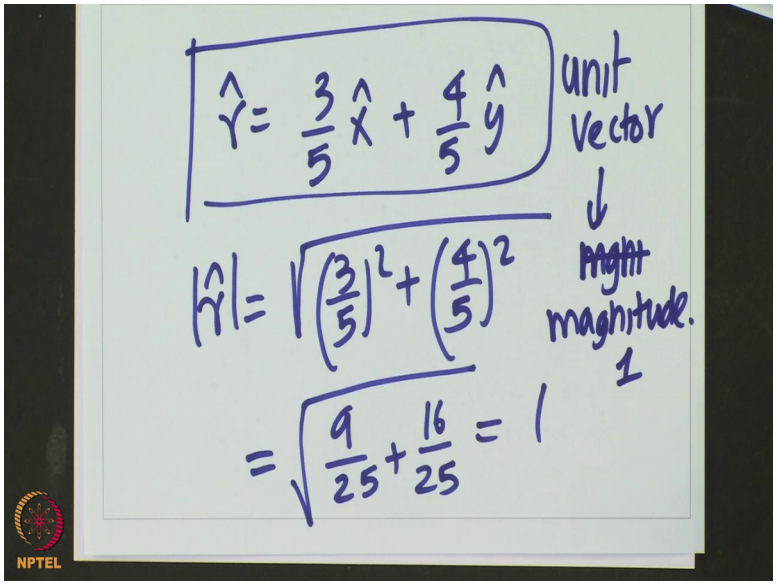

3 by 5 x cap plus 4 by 5 y cap. Now, if I just do find the magnitude of this which is a root of 3 by 5 square plus 4 by 5 square, this is the magnitude of this vector and this will turn out to be 3 square is 9 by 25 plus 16 by 25 and square root of it and this is going to be 1.

So the point is: you can design a vector, this vector, this is a unit vector which would mean that it has a magnitude 1. So, vectors with magnitude 1 and a particular direction is called a unit vector. We want to define unit vector in a particular direction, so this is the way one would define. So, if I just want to say only the direction magnitude is 1 does not matter. So, then I would say a unit vector in a particular direction. So, this is something a vector has a magnitude we know how to calculate the magnitude. If you want to say only the direction we would say the unit vector such that it represent a vector with magnitude 1, but in a particular direction. So, we just now know how to calculate magnitude of a vector and the direction of vector.

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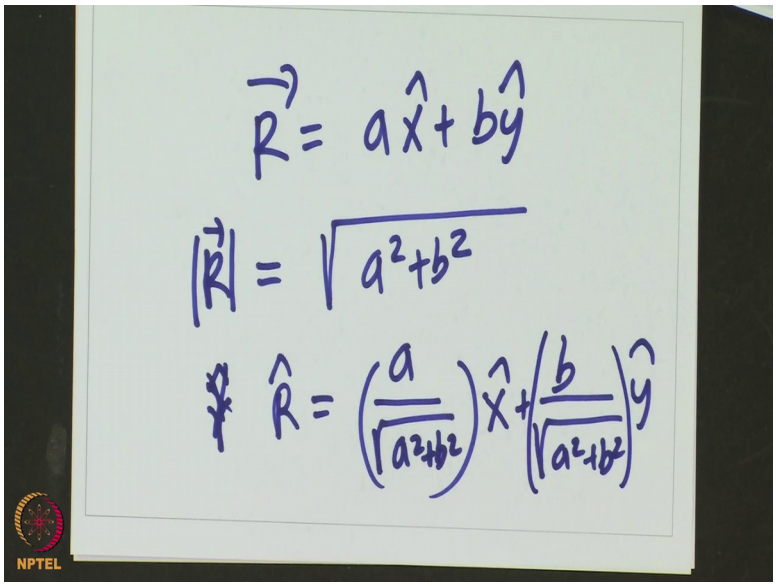

So, to summarize if you have a vector which is R which is a x cap plus b y cap, the magnitude of this is vector which is represented as R with this symbols like this is root of a square plus b square. And the unit vector r which is the sometimes it will also represented as like this unit vector R this is nothing but a by root of a square plus b square x plus b by root of a square plus b square y cap. So, this is the unit vector in the direction of this r vector and this has a magnitude which is 1.

So, this is basically of an a vectors to represent position and displacement movement in 2-dimensional plane this is the simplest way to represent. Still we are away thinking about a cell which is spherical in nature. So, cell has something spherical nature. How do we represent this cell in the spherical nature is something that is very important and interesting. And how do we do that something that we will think about in the coming lectures.

With this we will stop today's lecture. Bye.