Introductory Mathematical Methods for Biologists
Prof. Ranjith Padinhateeri
Department of Biosciences & Bioengineering
Indian Institute of Technology, Bombay

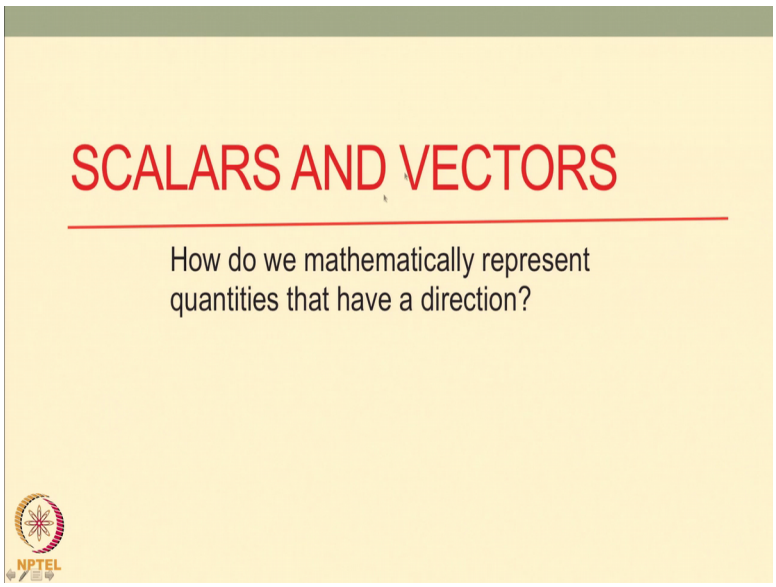
Lecture - 22
Scalars and Vectors

Hi. Welcome to this lecture on Mathematical Methods for Biology. We have been talking about various functions and how to play around with these functions that is finding slope, given slope for defined functions, and we learn that this has many applications and many quantities in biology are either derivatives or integrals of various functions, like various example that we saw so far.

Now, we said that mathematics is like a language and we want to use that language to convey something that we see around us in nature, and in particular things in biology. So when we want to describe biological things, things in the biological world or the living world very often you would have some quantities that has a particular direction; like there is life in many things in life has a particular direction and in the molecular level or in the there are different length scales things will have a particular direction movement will have a particular direction, many things will have a particular direction.

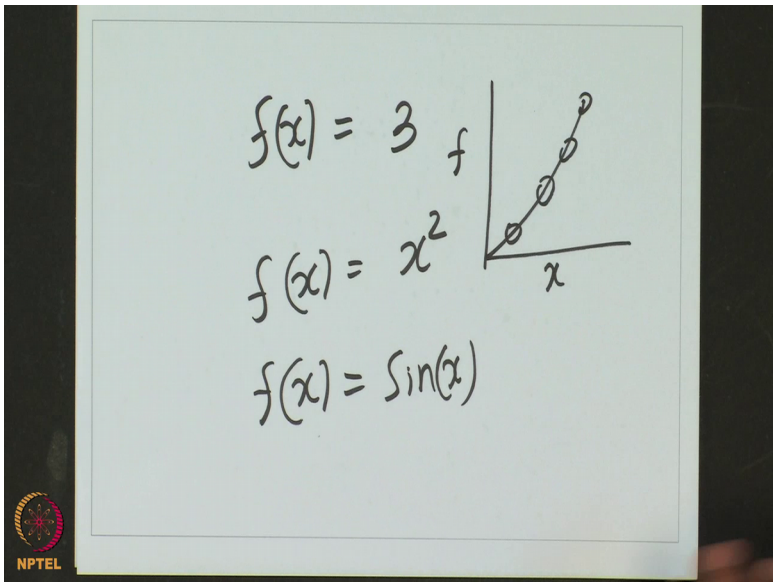
So, how do we represent those quantities which that have direction is something that we will discuss today. So, the title of today's topic is Scalars and Vectors.

(Refer Slide Time: 01:39)



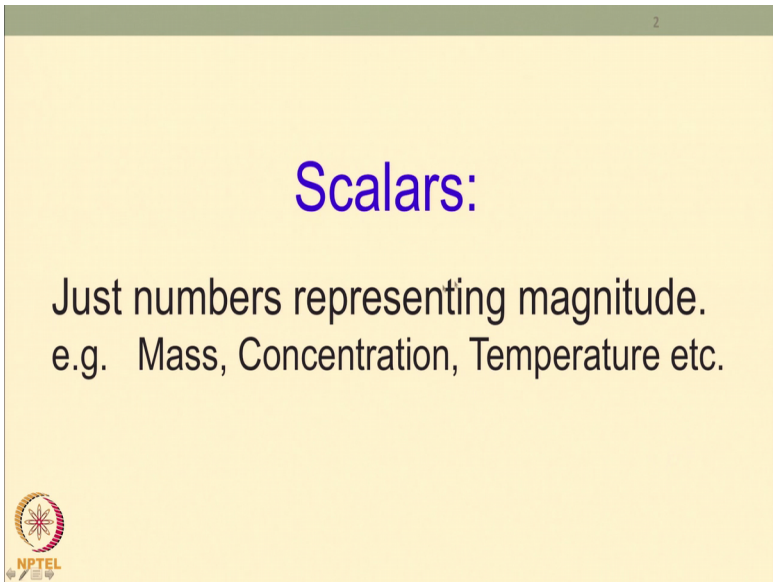
That is a topic today that we will discuss and the question we will answer is how do we mathematically represent quantities that have a direction. This is the question that we will answer today. So to do, to start let us think about things that we have been learning; like we just said f of x so far very often.

(Refer Slide Time: 02:07)



And, so when we say f of x is equal to 3 or f of x is equal to x square. These are some; I can substitute values for x when I say f of x is equal to sin x, I can substitute values over x and I will get a number. So, essentially all of these are numbers. So, some function versus x, these are all numbers. So, so far whatever we learn or some quantities that has some numbers typically that we would measure; like concentration, right where we would measure the concentration of things and many things that we measure are simply numbers. So, these things are called scalars. So, the name for them is scalars.

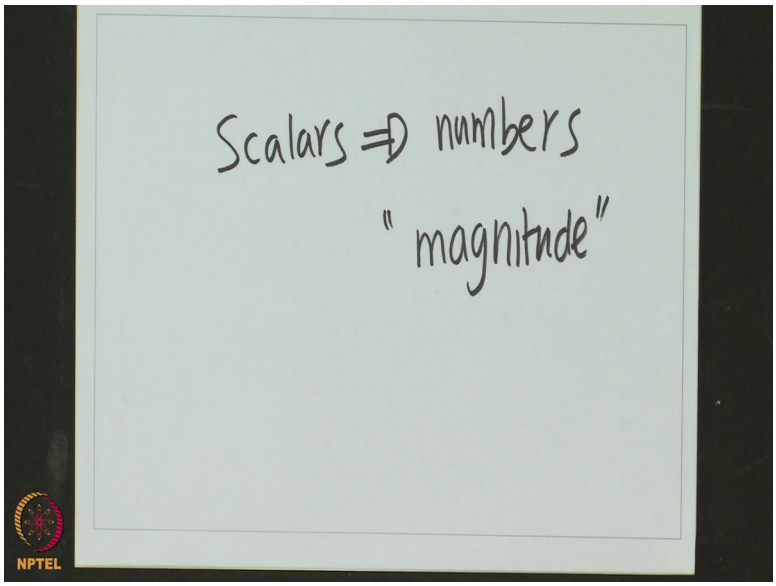
(Refer Slide Time: 02:57)



Scalars are just numbers typically representing magnitude of some quality like mass, which is the magnitude of; magnitude and the concentration which is also magnitude of number of molecules in a particular volume, temperature which is the also the magnitude of kinetic energy or hotness. And, basically all of these are some quantities representing magnitude.

Very often you would talk about numbers which is also related to concentration. So, numbers, concentration, mass, temperature these are the things that we typically would be interested or we discussed so far which are called scalars. So, the name is scalars. So, the mathematically they are called scalars, because they are simply numbers.

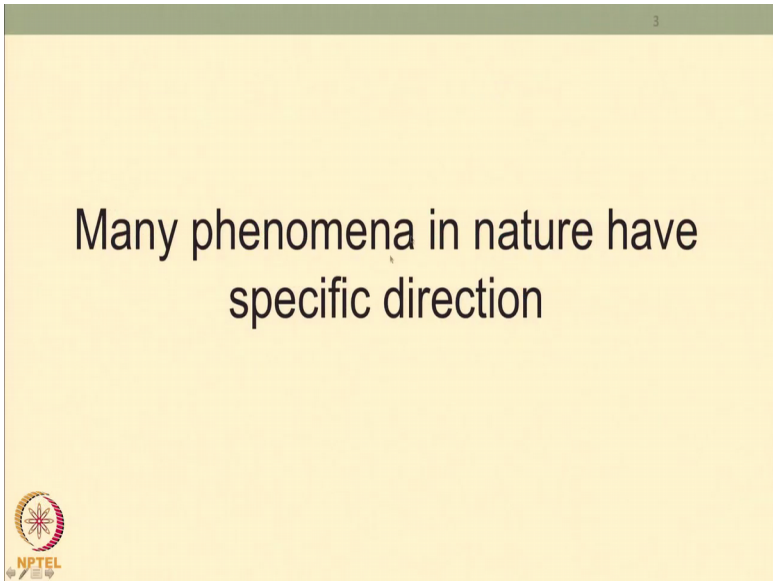
(Refer Slide Time: 03:51)



So, they are just numbers. And this would have no direction, no particular direction just magnitude only. So, this would typically represent the magnitude of something.

So, now what do I mean by all this it will be clear. Now let us think about some other quantity which are not scalars.

(Refer Slide Time: 04:25)

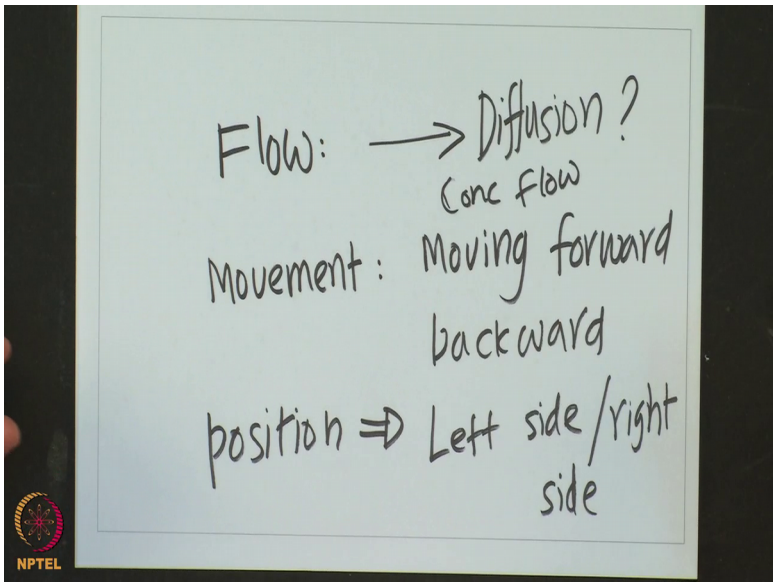


So, many phenomena in nature have specific direction. So, there are things that have direction, when we talk about direction what are the things that we would come to our mind. The first thing that we would come is something is flowing. So, flow has a

particular direction. I am moving in a particular direction, so the motion has a particular direction. So, for a position itself like I am standing left or right I am standing ahead or back, so when we talk about position relative to something or when we talk about motion which is you are moving in this particular direction or when we talk about flow, flow is in the particular direction all of these have quantities which have direction.

So, let us write down these things.

(Refer Slide Time: 05:21)



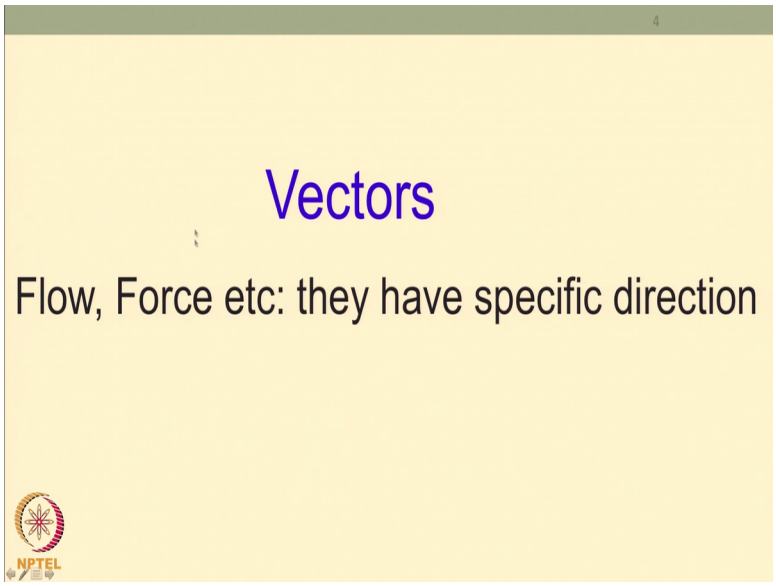
So, the things that quantities that have direction: flow. So, when we are to say it is flowing in a particular direction, right it is flowing this way or that way right; this is a particular direction.

Movement: so moving forward, backward. Position with respect to some point, right; you are positioned at the left side or right side; right left side, right side. These are things that we would typically use in English. And, when we think about flow for example, we very often molecules will flow from one direction to the other direction something that we would come to everybody's mind is related to diffusion like diffusion is some related there is a flow related diffusion. So, this was something which would come in our mind typically is the diffusion. What is the things that is related to diffusion which has there is a current flow associated with diffusion which we will discuss or concentration flows right; concentration flow there is a flow of concentration. Flow of molecules; flow of

molecules from one place to the other direction particular direction higher concentration to lower concentration is what is diffusion.

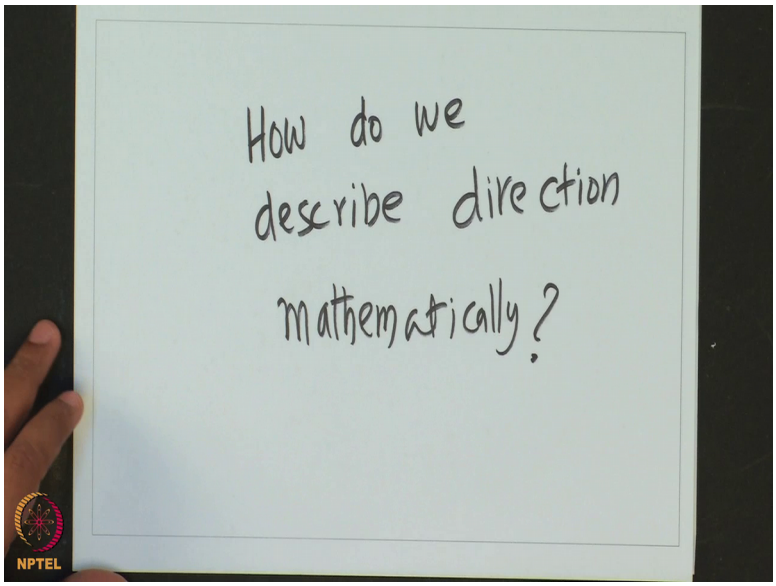
So, these are the things that are quantities which that have a direction. So, examples here vectors.

(Refer Slide Time: 07:11)



Are flow a force; force is a particular direction you apply a force you push somebody in a particular direction. So, they all have a specific direction. Now the question that we want to discuss is how do we represent this mathematically, as a language when we want to say something that is mathematically how do we represent this. So, this is the first thing that we; so the question they are going to answer is how.

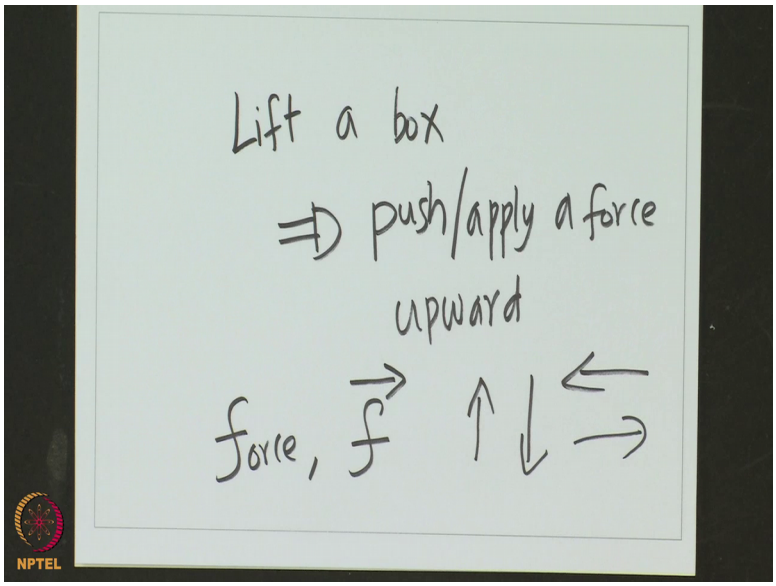
(Refer Slide Time: 07:42)



How do we describe direction mathematically? So, this is the question that we would answer. Now let us think about something that we would want to typically convey in an experiment or in a particular context where which has a direction. Simple thing to say is that: push in a particular direction or let us say I have this pen I would want to lift this up. So, lift this up in this direction, right. So, how do I say that lift up or pull down, right?

So, let us say lift this up or pull down. So, these are the things that we would want to tell somebody. So, apply a force in a particular direction.

(Refer Slide Time: 08:47)

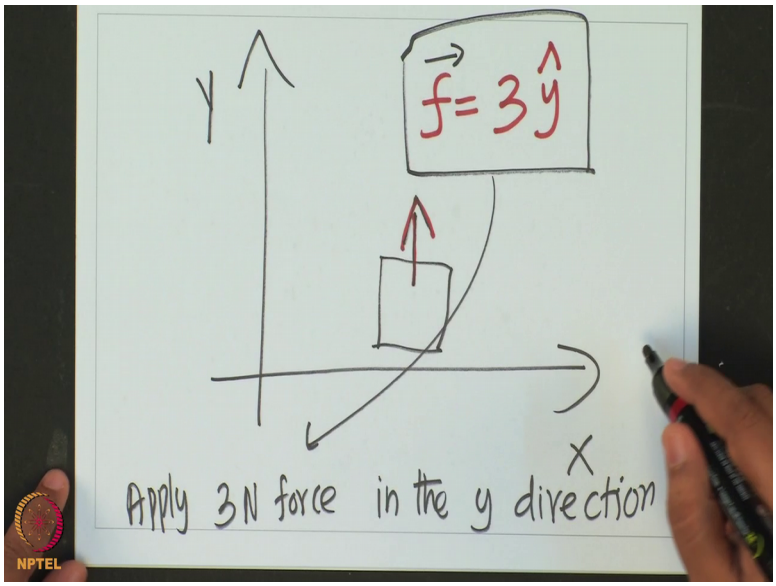


So, how do we tell somebody that lift a box; lift would mean that push upward right push or apply a force upward. This is what we want to tell somebody. So, we could say that go and push or pull; in the door we would have push pull. So, this is something that we want to say in a particular; you are instructing somebody to do something in a particular direction, if it is the opposite direction it will not work.

So, we wanted to have force f and this should be in a particular direction. So, to show that this has a direction one would typically use this notation, this arrow which is a symbol which is representing that the force is a quantity which has a particular direction. Now we wanted to say upward this is the direction or downward or to the left or to the right, this is the simple direction in a plane in this two dimensional world of plane, we could just say essentially these are the things that we would say- upward or downward left or down. So, let us first think about how do we represent; how do we tell somebody to apply a force in the upward direction push, pull, left, right, and all that.

So, the way to do that is again going back to the graph. So, if we go back to the graph and when we draw this x and y axis we all know that we have an x y plane.

(Refer Slide Time: 10:43)

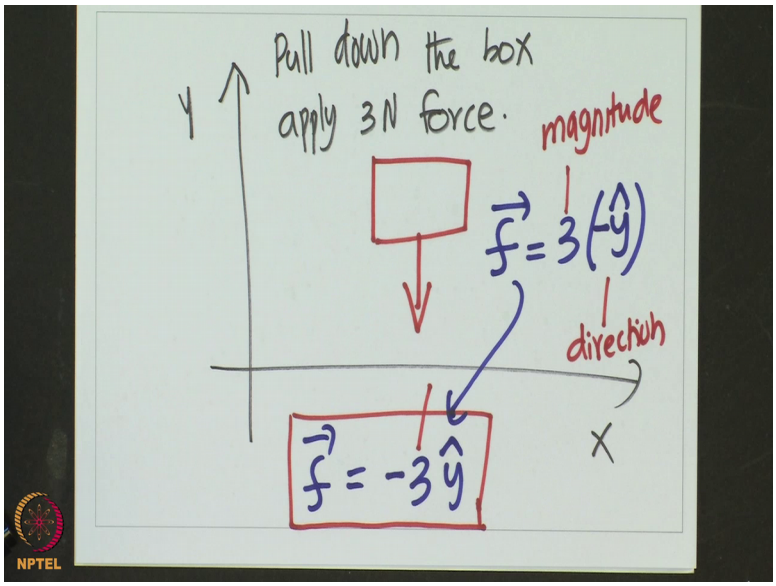


So, this is x and in the simple world this is y; in this two d this is x in this y. So, if I have a box here, if I have an object here; if I want to move this upward I have to apply a force in this direction. So, the direction of the force has to be this. So, I want to tell somebody that apply a force, I have to apply a force what is the unit of force which is in Newton.

So, let us say I want to apply a force of 3 Newton I want to apply a force of 3 Newton or 3 pico Newton or 3 whatever be the unit of force; a unit of a force of 3 Newton one would want to apply, but in this direction of upward; so to say that you should apply in the direction of y axis. So, this is the y hat. So, if I just put y and the symbol of hat, so this is called y hat. And, this would mean that when I write this mathematically, this is an instruction given to somebody which say with the meaning of this instruction is that apply 3 Newton force in the y direction.

So, this is something that we want to say in English: apply 3 Newton force in the y direction to push this up and this can be said mathematically in symbol words f is equal to 3 y, and just to show that f is a vector you will put a vector sign here; f is equal to 3 y. This would mean that apply a force upward in the direction of y. Now if we want to say- just the now let us think about what if you want to just the say the opposite. Let us say we want to say that apply a force and then push pull it down.

(Refer Slide Time: 13:00)



So, we have this and this is the increase in the direction of y and this is the increasing direction of a x. Now, we have an object here and we applied a force in this direction, but one would want to also apply a force in this direction to pull it down, right. So, we could if we want to say that pull down the box applying 3 Newton force. This is the instruction we want to convey to somebody. Apply this box pull down this box bring it down pull down by applying 3 Newton force.

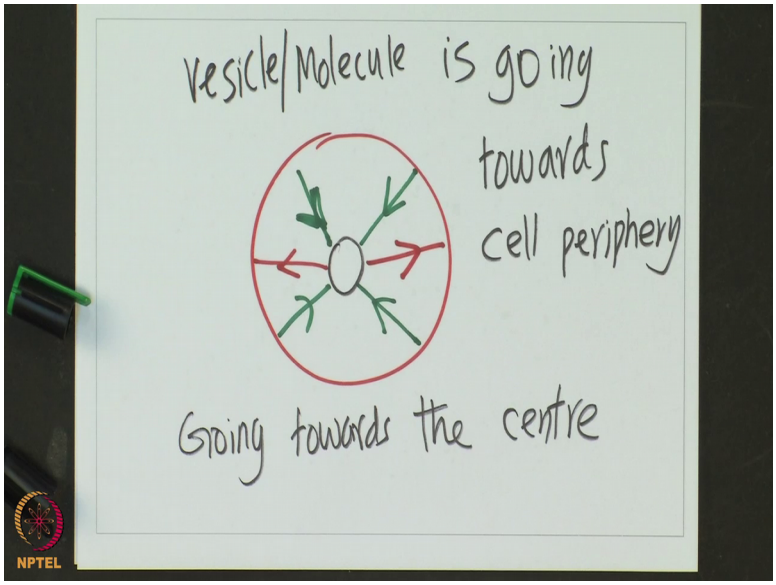
So, the way to say this is that we have to say- you have to apply a force has to be 3 has to be the magnitude 3 Newton or whatever be the unit one has to appropriately put the unit here 3 Newton in the direction downward. So how do I say that? I would say minus y hat. So, that is downward. So, this would imply this is equal to minus 3 y hat. So, when I say f vector is equal to minus 3 y hat this is essentially telling, this is an instruction given to someone that pull it down with this much force. So, this is the magnitude and this is the direction. So, this is the magnitude and this is the direction. So, this is the magnitude of the force and this is the direction of the force.

So, there is a magnitude and the direction. So, this is an example of a vector quantity that you would want to convey that how much force one should apply; is an example to of a vector where we would convey to someone that do this way.

Now let us think of some other way that same thing you want to say that run in that direction. So, we want to say that the molecular motor is moving towards the center of

the cell, this is something that you would want to tell. So, let us think about this context that you would want to say in biology. So, let us think off.

(Refer Slide Time: 15:53)



Let us think of this context we have a cell. And cell has a center which is here and this is the exterior. So, this is the cell and this is center and out. So, you have molecular motors walking along the microtubule. There are many microtubule tracts, and molecular motor would either go this way. Let us say molecular we want to say that molecular motors are going inward right, we would want to say this. How would we say this? So, this is something for example or some other molecular motor might be going outward.

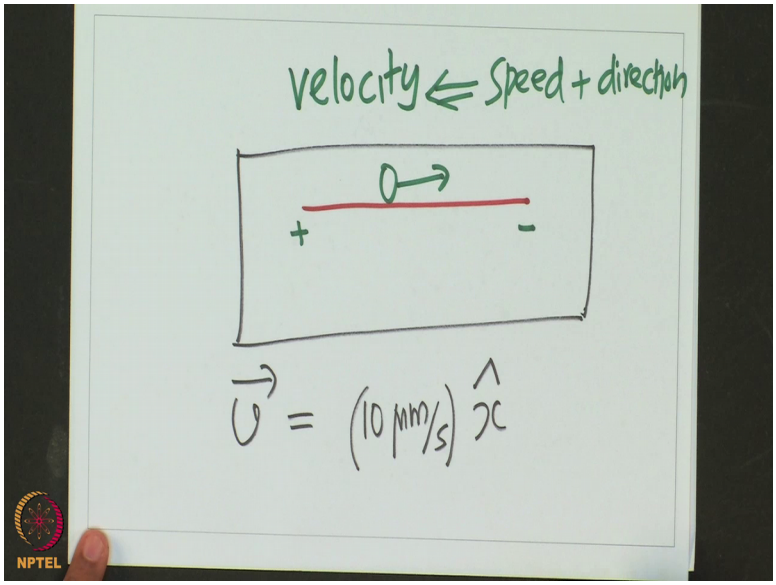
So, if you want to say there another molecular motor which is shown this red is going outward along this track. So, this motor is walking to the exterior which is this way or the vesicle is going in this way and vesicle is moving this way. So, we wanted to tell somebody look that particular vesicle of your interest or a particular molecule of that interest is going inward; look that particular molecule of your interest is going towards the cell membrane, cell periphery.

So, this is something you would want to convey like, vesicle or molecule of your interest is going towards cell periphery. This is be very often you would want to say. We would also want to say that it is going towards the center: molecule is going towards the center of the cell. Now, if you want to say the how do we mathematically say this, this is the

training that you we are getting through understanding vectors we are training ourselves to say this thing mathematically.

So, now we will reach here, but before reaching here we will think of even simpler thing which is moving left and right. So, moving towards central periphery is easy to say, but we need to get a training to do that. Before that, let us think of a simpler case which is moving to the left or to the right. So, where we would think of an in vitro experiment; let us think of an in vitro experiment where you have just a setup.

(Refer Slide Time: 18:35)



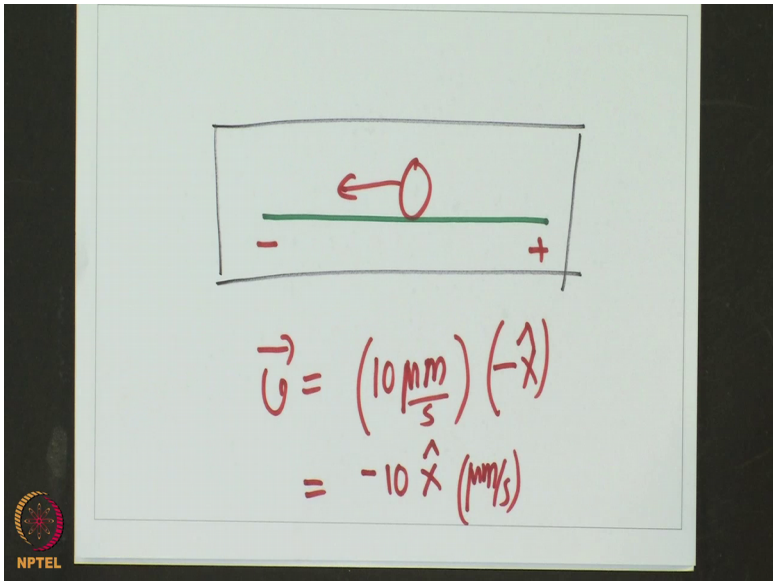
Where you have a setup, where you have microtubule track and on which a molecular motor is carrying a vesicle is moving from one direction to the other direction. So, let us say this is the plus end of the microtubule and this is the minus end of the microtubule and a motor is moving in particular direction. So, if you want to now to say that this is moving to the right word or to the left word how do we say that- we would say that it is moving. So, moving is motion is often represented by this thing called velocity. Velocity is something that represent motion. What is velocity? Velocity is nothing but speed with a with a direction; speed plus direction together is essentially velocity.

So, we want to say that a molecular motor is moving this way we would want to use the following that we want to say that velocity which is a vector it is moving with a velocity of 10 micrometer per second. So, the velocity magnitude is 10 micrometer per second; this is the magnitude. And it is going along the x axis; so you put x cap. So if I write this,

this means that the motor is moving along the x axis towards positive x. So, this would mean that the motor is going towards positive x with a speed of 10 micrometer per second.

So, this is what would mean. The same thing, if we want to say in the negative x direction; so the opposite context would be an in vitro experiment again.

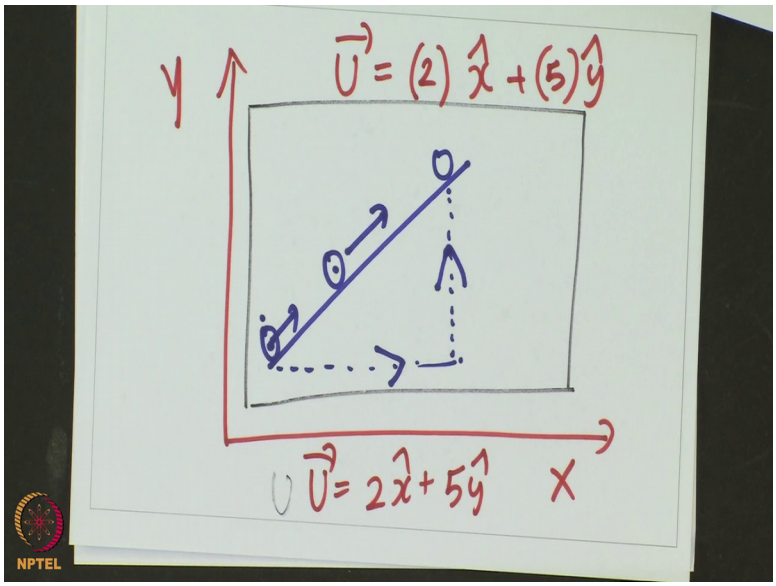
(Refer Slide Time: 20:34)



Where you have a track in which a vesicle cargo would move this way. So, this was minus and plus ends of the microtubule. So, you have a cargo which is moving towards this way with the same speed. So, we would say that as the following v is equal to 10 micro meter per second is the velocity or the speed, but it is going in the minus x direction, so minus x cap.

So in other words, I would write 10 x cap with a minus sign and the unit is micrometer per second. So, 10 x cap would mean the 10 whatever be the unit it is going in the minus x direction. So, this is the first thing you would want to convey; something this is the simplest thing we would want. But very often we would want to use little bit more complex things which is think about this.

(Refer Slide Time: 21:50)



You have a plate which is like this and we would define x axis and y axis like this. So, this is my y and this is my x. And you have a microtubule which is lying like this, and you have a cargo which is moving this way. So this cargo, if we consider the edges of their plate as x axis and y axis and you have a cargo moving in this direction; its neither moving along the x nor moving along the y, but something which is a combination of x and y.

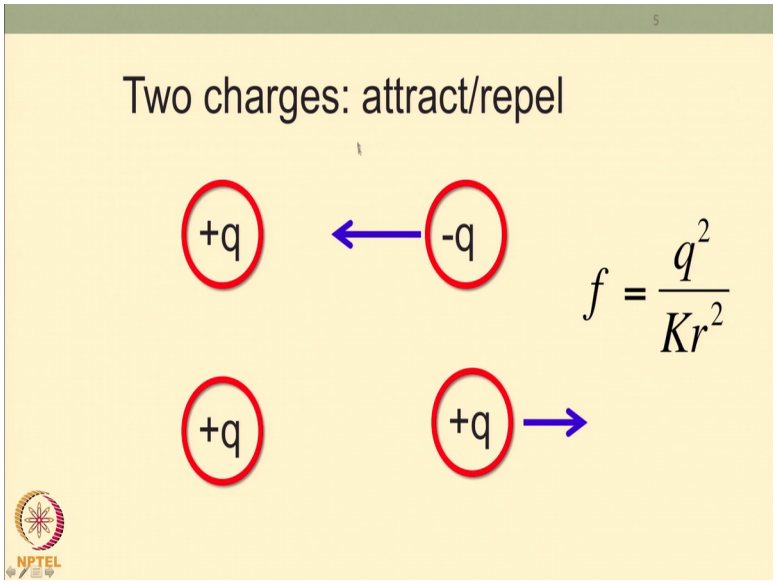
So, what one would say is that this is going; so if I start from here and it reached here. So, this is my initial position and it moved here later it reached here. So, it is moving along this direction. So, it is moving along the x and y. So, it is moving this way a little bit and this way. The way to say is that, if we want to reach from here to here we have to first go along this and then go along this; if I go first this way and this way I would reach from here to here.

So, if I just want to say that it is going in this direction and this direction is a combination of x and y. So, it is going a little bit along the x direction little bit along the y direction that then it would be a movement which is a diagonal like this which is a mix of both x and y. So, then we would say that this has some velocity v which is some amount of motion along the x direction and some amount of motion along the y direction. It is moves a little bit if you go a little bit along x direction and go little bit along the y direction I would get this y.

So, this is the magnitude which I have to put let us say 2 and 5. This has some two numbers, and what are this mean; we will think about it a little bit. But if I just say v as 2 x plus 5 y, this represent movements like this which is both a combination of x and y. So, this is a v vector, generally any vector would be like this in a two dimensional plane it will have some x and y direction. Now, how to get this x and y direction; is something that we want to understand. To understand that let us think about position and let us think about other things.

Now, talking about force which we said as a vector, we talked about a force.

(Refer Slide Time: 24:44)



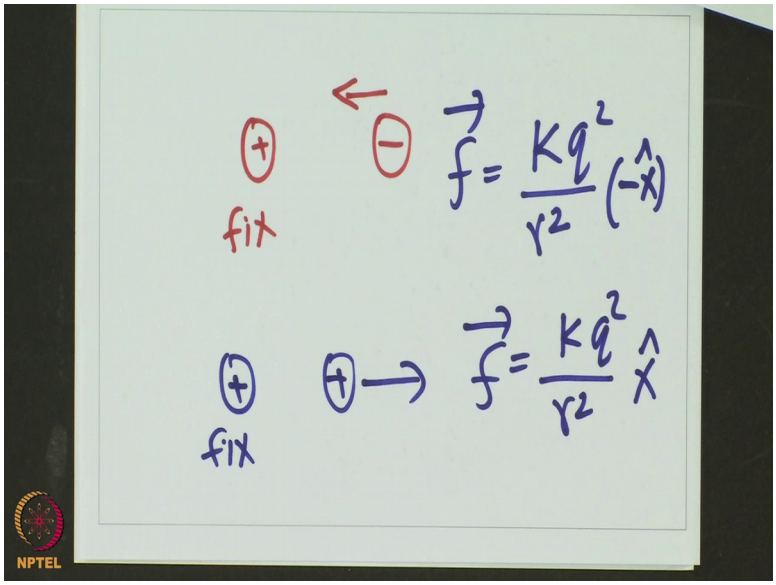
And very often what one thing that is force is two charges. So, two charges typically attract or repel; for example, you have plus charge here and a minus charge here, and this minus charge will be attracted towards this plus charge. So, if I keep this charge fixed here this charge will move along this direction to come towards this plus charge. So, if you want to say that the direction of movement of this charge, again we would want to use idea of vectors. So, this is the magnitude of the force and this is the direction of the force.

Similarly if we have two plus charges, one of the charge if assume that if this charge is fixed at some location and the other charge is free to move then this free charge this charge will move away from this charge and the force of repulsion on this charge would

be q square by some constant times r square; where r is the distance. And therefore, the magnitude of the force is this and the direction of this would be this.

And if you want to write this thing mathematically what we would write typically is that.

(Refer Slide Time: 26:05)

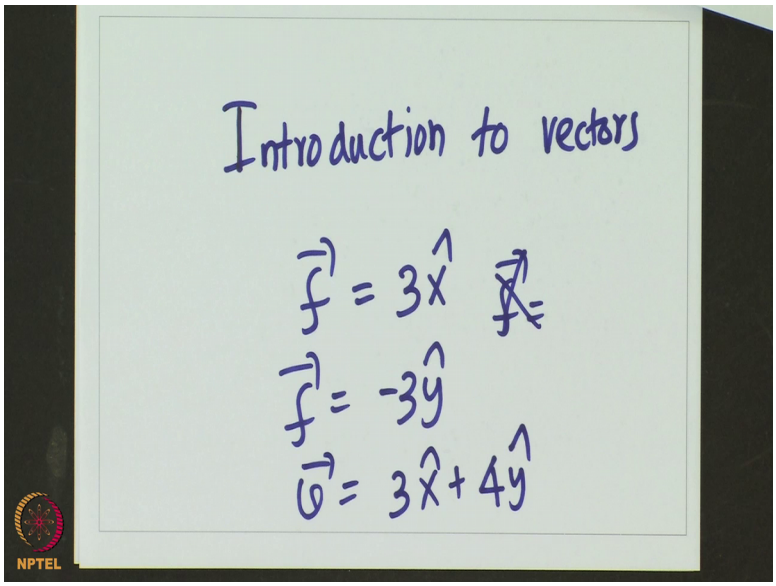


So, you have this plus charge and a minus charge, and we know that if we fix this charge this charge will move in this direction. And the force feeling felt on this charge is some constant q square by r square, it is this constant typically could be written as below also one over k does not matter it is some constant. But it has a direction, in this direction the direction is minus x; it will move in the minus x direction where q is the magnitude of the charge; q is the magnitude of the charge in this will move in this minus x direction without a sign here. So, this a whole thing will have a sign.

Now if you take a two plus charges and if you fix this charge, this charge will move in this direction and the force will be repelling and that force again can be written as kq square by r square along the x direction; it will be along the x axis, so this will be written as x cap. So, these are the ways to represent vectors in an x y plane.

So, to summarize what we said so far is; what is the simplest way of representing direction. So, we just had an introduction to vectors.

(Refer Slide Time: 27:40)



So, we introduced: we had introduction to vectors that is what we did in this module; introduction to vectors. And we said that any vector force can be 3 x cap or force could be minus 3 y cap; that means, 3 along the negative y direction or it could be some combination in principle. In the in the case of velocity we saw that velocity could be some combination of 3 x cap plus 4 y cap.

So, these are the simple ways of representing a quantity that has magnitude and direction. We would discuss this in more detail in the coming lectures and understand this, and try to answer the big question, but how do we say that something is going inward or outward of the cell and so on and so forth.

So, with this we will stop this lecture and continue in the next lecture. Bye.