Introductory Mathematical Methods for Biologists

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Lecture - 02
Graphs and Functions

Hi, welcome to the lecture on mathematical methods for biological sciences or for biologists. Today, the first thing we will discuss is graphs and functions.

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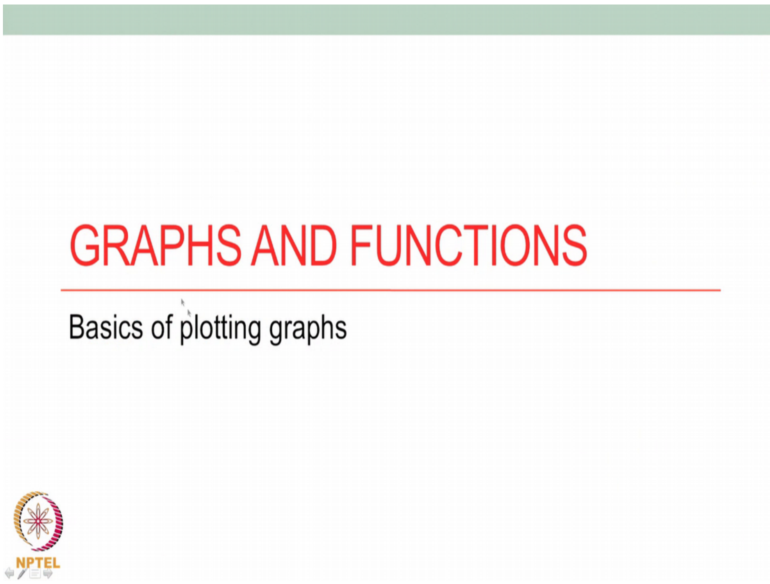

So, topic for today's lecture is graphs and functions. We will discuss the basics of plotting graphs, that is, we will learn how to plot graph. So, we will learn a couple of things in this.

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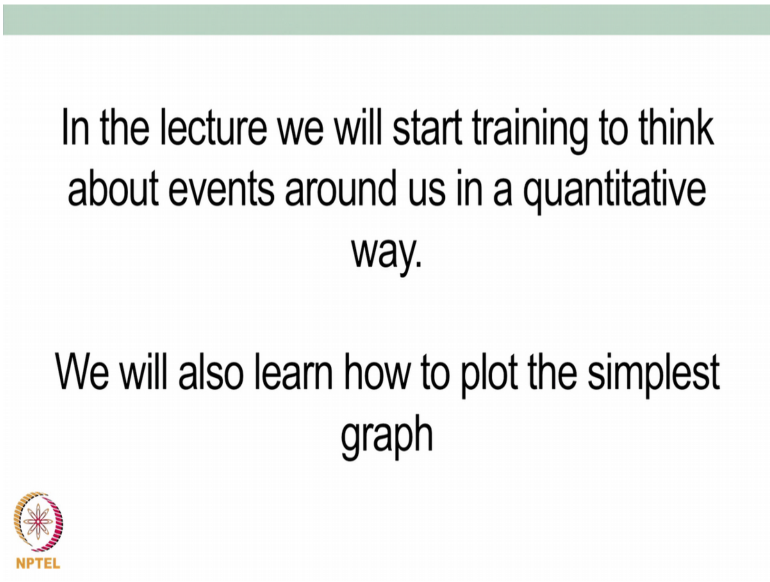

So, the first thing that we will learn in this lecture is we start training ourselves to think about events around us in a quantitative way. We said that mathematics is a language to start thinking about that language which will help us to express things in a quantitative way. So, we will train ourselves on how to think and express things in a quantitative way that is the first thing. We will start the training today and with some through some examples and we will also learn how to plot the simplest graph possible.

So, this is the first 2 things that in todays lecture that we will do. So, as I said mathematics is like a language.

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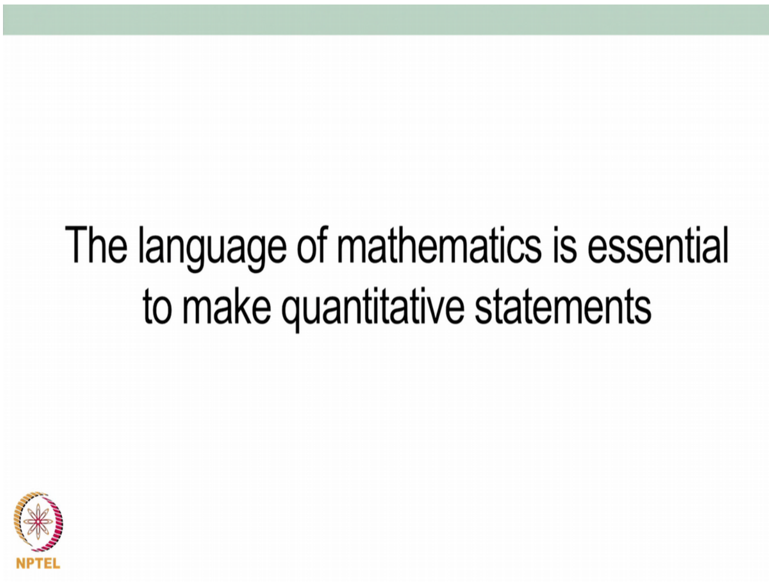

And, the language of mathematics is essential to make quantitative statements. These things we know.

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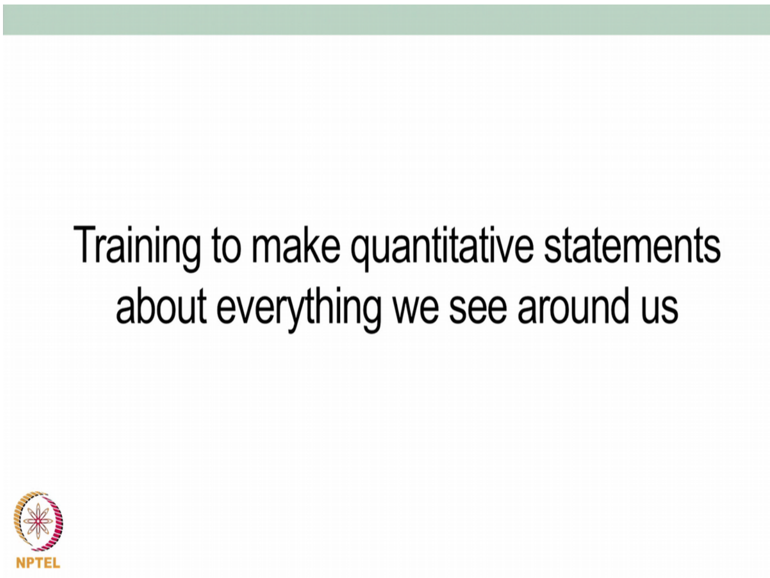

And, the training to make quantitative statements are all about everything around us. Repeat, we need to get training or we need to train our brain to make quantitative statements about things that we see around us. We need to think about the things that we see around us in a quantitative manner. Often we see of course, things around us and we understand them, but how to express that in a quantitative way is something that we have to train and the training is a part of this course.

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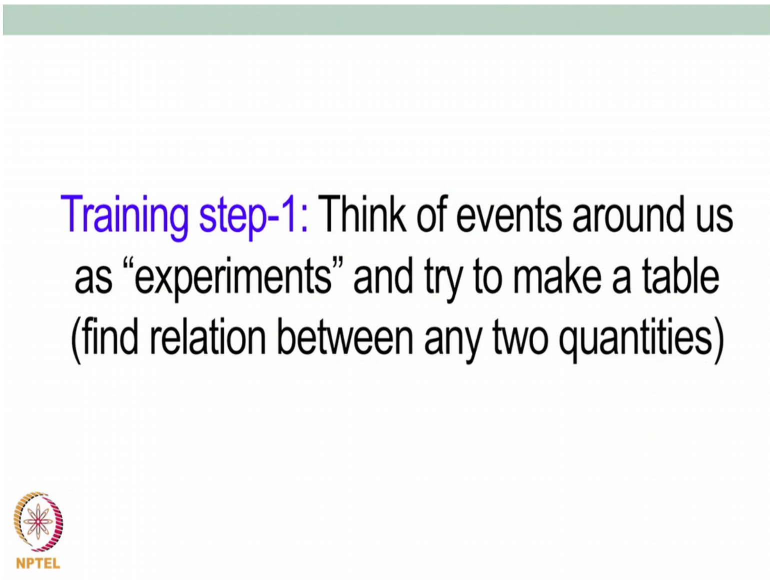

So, the first step, what is the first step? So, in the training, the first step is, think of events around us as experiments and try to make a table first and find the relation between 2 quantities. So, I will explain what does this mean and it will be clear what does this mean. So, let us take some example of things around us.

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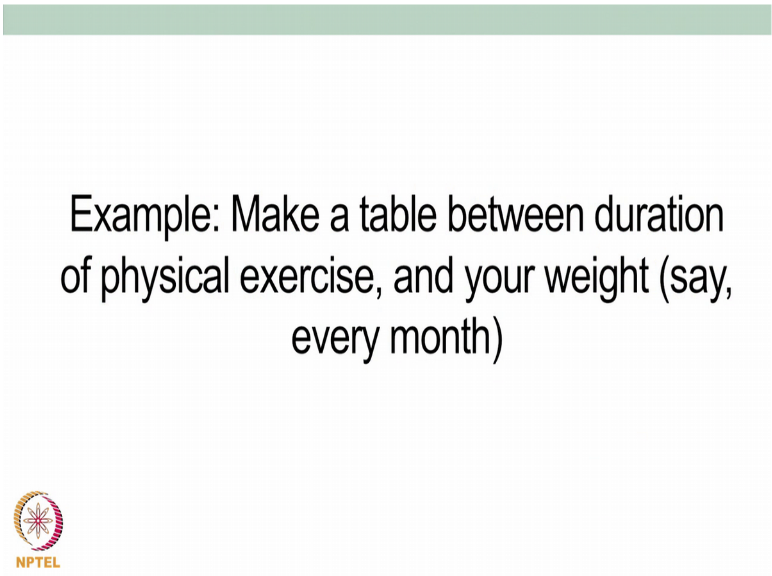

So, for example: we do physical exercise every day or most of us many of us do exercise, we run or we walk or do jogging. We all do, for example, let us say we run for 10 minutes, 20 minutes, 30 minutes. We do this every day and we can quantify this. We can express this in a quantitative way. So, how do we do that? That is the first thing that we learn.

So, we can make a table based on the things we see, for example, make a table between the duration of physical exercise and your weight say every month. So, every month we can look at the how long we do our physical exercise and we can also measure kind of average weight every month and we can make a table of these things. So, this is quantifying some things that we do in our day to day life. We run or we jog or we go for walking that is that we do very often, but we have never quantified it. We know roughly, but we have never written down a table based on the things that we do. If we can write down a table and we can express this in a quantitative way, we will learn many new things from this.

So, what we want us to make a table between duration of physical exercise and your weight for example, this is an example.

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Duration of your daily run versus weight

Duration of run per day (min)	Weight (kg)
0	80
10	78
30	75
60	70

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So, let us say, I made this table and I am going to start running from today. So, till now I have not been running. So, I am going to make a table being duration of run per day, that is, how many minutes will I run every day typically and the weight what is my weight for that month. So, this is an average weight in that month and how long I have been running. So, let us say I have 0 now. So far I have not been running, that is 0, so that means, I have not been running at all. 0 minutes I have been running and my weight is 80 kg.

Now, from tomorrow onwards or today onwards I start running 10 minutes every day for a month or 2 then I measure my weight for a period where I run 10 minutes and I find that my weight is about 78 kg which is reduced by 2 kg. Then, I run for 30 minutes every day, and then I look at my weight. It has further reduced, and then I can increase my run time, I run 60 minutes every day for a period of time and then I see that my weight is further reduced and so on and so forth.

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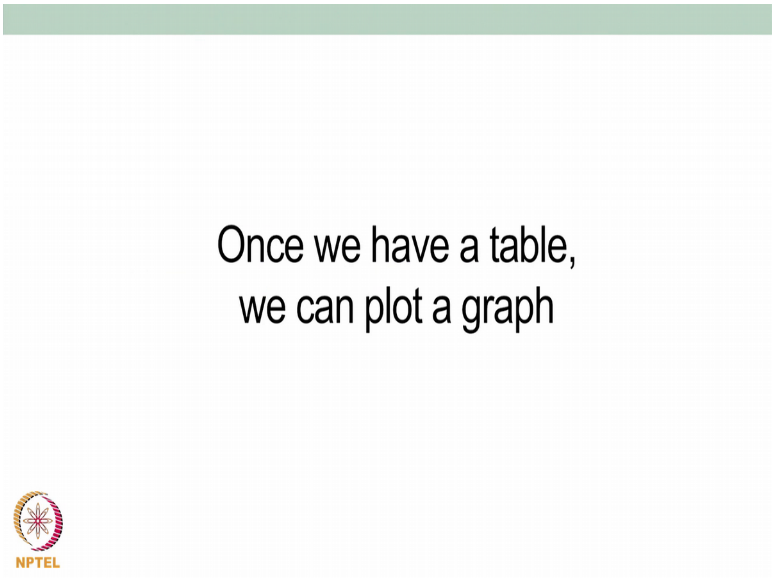

So, I can note this is day to day event that we see I can note I can make a table and then once we have a table we can plot a graph. So, if we plot a graph.

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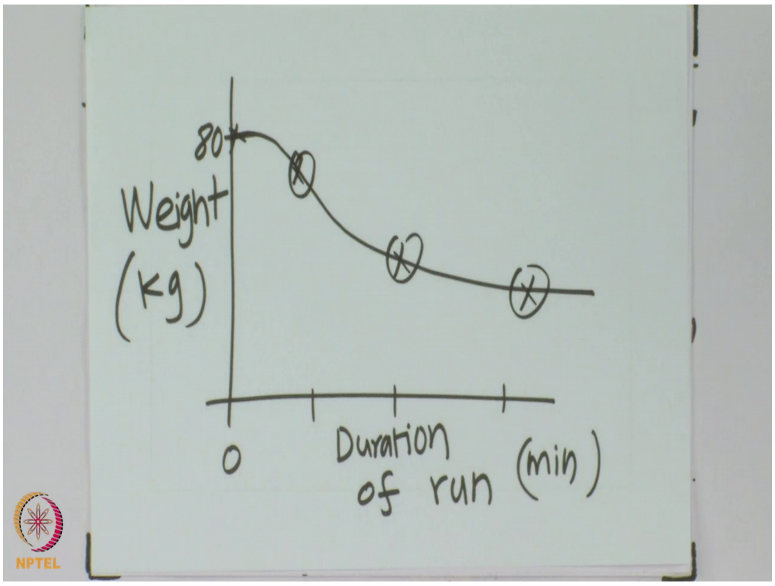

So, we can plot the in the x-axis, let us say duration of run and weight. So, this will be in kg and this will be in minutes.

So, in different people this graph will look like different. So, depending on the person this graph will have slightly different starting point and all that. So, 0, I said we have been about 80. So, when I do not run I have like 80 kg weight and it will slowly decrease as I run 10 minutes, 20 minutes and so on and so forth. So, this has to decrease and it can't, for the it has to saturate somewhere something like the, I do not know how the graph will look like, but it might look something like this, that is my guess and once you do proper experiments you will get some data points and you can plot a nice graph based on this.

So, this is the one example of quantifying some things that we see in our day to day life and plotted as a graph make a table and put it as a graph. Why are we doing this, because as we go ahead in the course we will learn that making such quantifying things will help us to understand many things better; so, called experiments we define, we design experiments so that we can understand the nature around us in a better way. So, that is why we design specific experiments and we can quantify things and then we can make predictions.

So, this is one example and let us see the next example. So, we see for example, let us say we buy things for vegetables every day we saw buy or a week we buy vegetables or rice or wheat from market. So, we can for example, look at the price of vegetables or price of rice. So, that is the next example this is another example note down vegetable price every week or every month and I can make a table. So, the first week the price of onion was this many rupees, for the second week it either increased or decreased or it didn't change, third week what is the price of onion, fourth week what is the price of onion.

So, if we can make a table and then we can plot a graph from this table and we will learn many things from this and in fact, economists use this graph to find out something all inflation and they make many of our economic policies for example, are made from such graphs. So, such graphs will be very useful and we will learn in this course. The quantitative thinking will help us to decide some action plan in the case of economics or example and we learn how to do this in biology.

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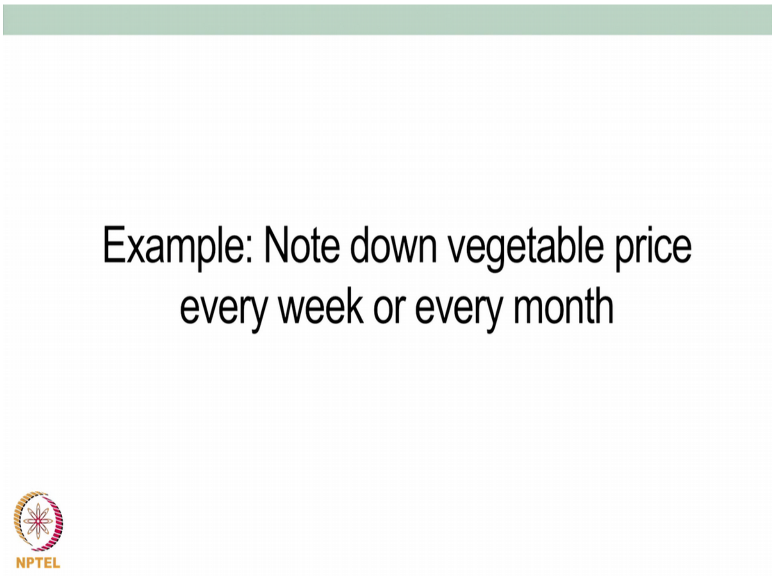

So, one example is note down the price of vegetables.

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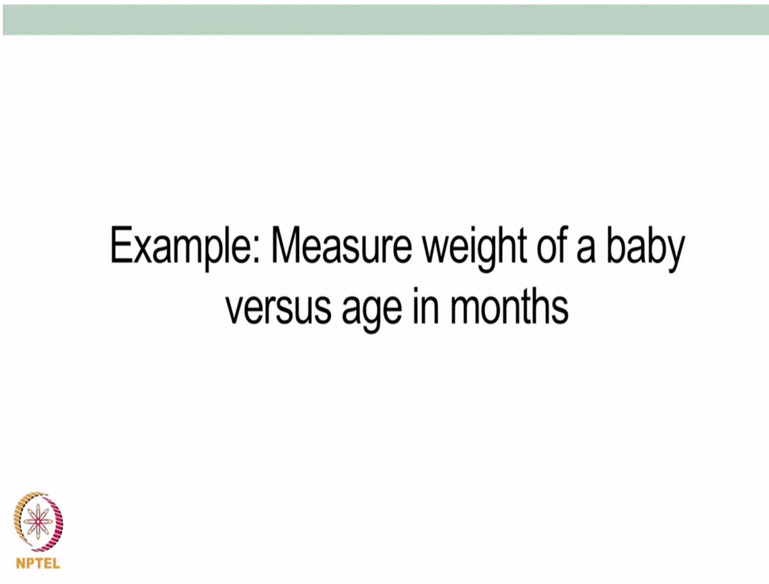

Another example is measure the weight of a baby versus in months. So, let us say you have newborn baby and what is the weight in month. So, as every month versus the baby will grow and it will have weight will increase. So, we can plot we can have a graph we can have a table in this case. So, this is all the doctors will have such a chart.

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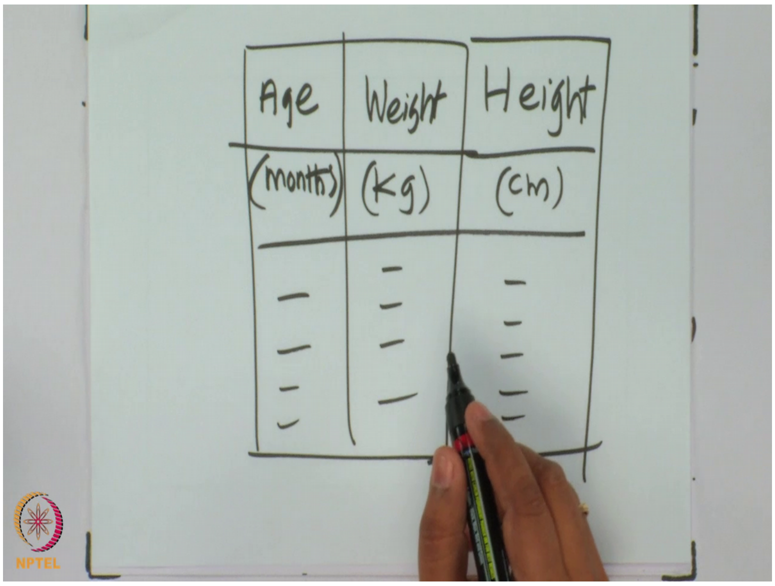

So, you can have a chart. So, age of the baby in months and weight or it could be height also. So, it could be either weight or height. So, either you can have weight or you can have height also as another column height or length sometimes this is called length for small babies. So, weight will be, this will be in months. That will be the unit, the time period, unit of time will be months here, weight will be of course in kg and the height will be in centimeters for babies let us say and it could be so, this is in months and then we can have entries here. And corresponding to first month what is the weight and height second month, what is the weight and height and one can make such chart and all doctors pediatricians will have such a chart and we can see how it look like and from this there will be a standard chart and if the weight or height is too small; that means, there is a sub growth problem.

So, one can infer some things. So, I do not know exactly how this graph will look like as of now.

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But, if I have to guess, for example, let us say the length or height of a baby versus age, if I have to plot it will start with some length or height and then it will slowly increase in little lot something like this is what I would guess, even though I do not know how it what has to get the exact data to plotted. But this is one guess that I would make that it would increase and then finally, at some point it has to saturate and all that. So, this is my guess and one has been one can get data and you should get data and plot this and see how it look like and let us say this is the standard curve that one has and if this happens much below or much above this there are some growth problems. That’s what this would mean.

So, one can have a standard curve. In fact, WHO will have a standard curve like this one can go to the WHO website or go to any doctor and ask a pediatrician ask what is the standard curve, typical average curve for a baby, age versus height or age versus weight and from doctors use that and then they mark baby's weight or height periodically and then compare with this chart and then they infer whether the growth has a problem or not.

So, this is very useful for doctors. So, this is another importance of quantitatively displaying or quantitatively understanding things and that can have very that can be highly useful and as in this course we will learn how to make use of these things and what all things that we can learn by quantifying the phenomena that things that we see around us and how to learn new things from by quantifying things that is the aim of this course and that is what we will learn in this course.

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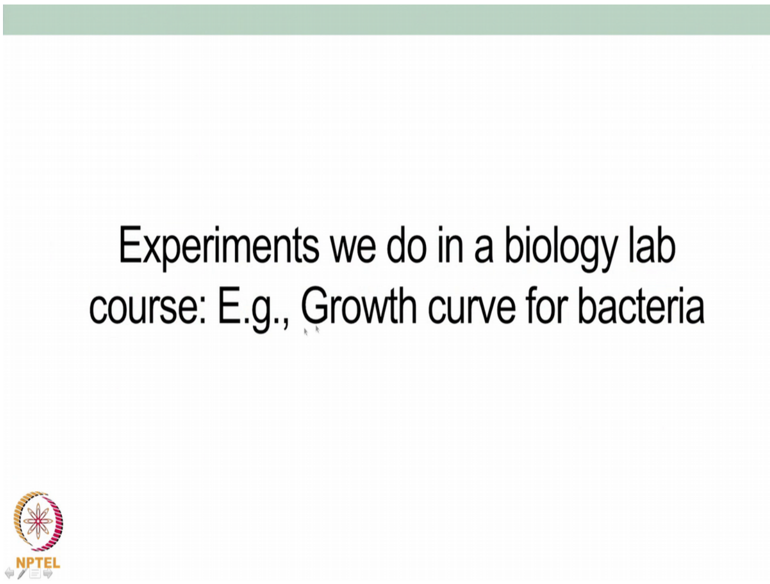

So, these are things that we see around us if we come to a biology lab, we do many experiments in the lab. In the biology lab one typical example is growth curve. So, the growth curve is some standard experiment that everybody would do in a biology lab. For example, growth curves for bacteria.

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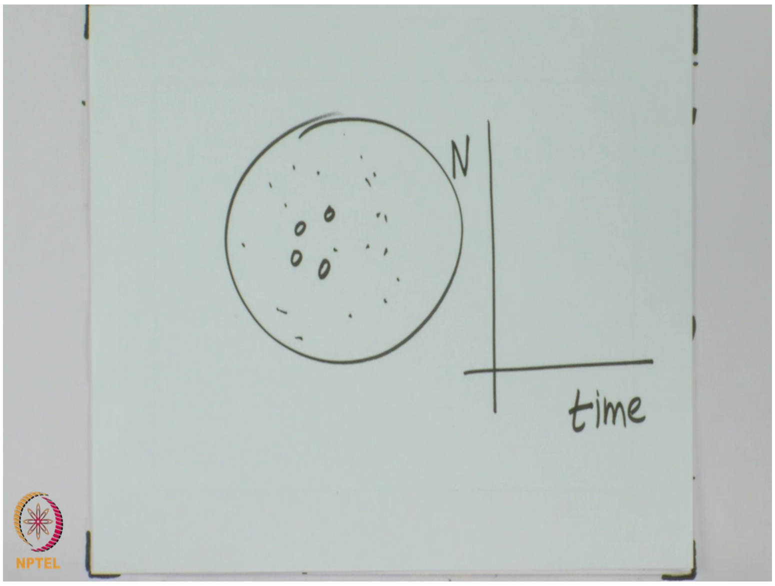

So, as we you already know we all will grow, we will take a plate and you will you start from a few bacteria where few bacteria and then give provide nutrition and this bacteria will grow and we can essentially plot something like number of bacteria as a function of time. Different ways this number is measured through optical density or various ways one can measure the number, but this is something that we will plot and we will come back to such plots throughout this course.

But what I wanted to convey now, is that these are some examples of experiments that we do in the lab and the result of an experiment is essentially a graph.

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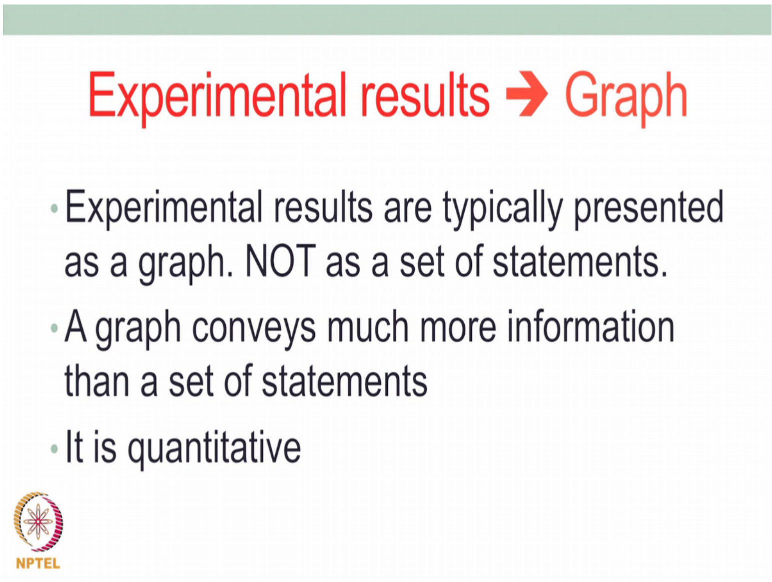

So, the first point is that from experimental results we get a graph. So, the experimental result is directly converted to a graph. Experimental results are typically presented as a graph, not as a set of statements. We do not do an experiment and make a set of statements we can we of course, also make a statement, but more importantly we do plot the data that we get from experiments as a graph. Why do we do that? The one reason is that a graph conveys much more information than a set of statements. We say typically that an image or a picture is equivalent to thousand words or many words. Instead of saying many sentences you present a picture, it conveys much more message than many words or many sentences would be in a typical language.

Similarly, a graph if you wish graph is equivalent of a picture and the equations are equivalent of word, sentences. So, one can write equations and one can plot graphs and a graph can convey a lot of information and that is why we plot graph and it is quantitative. The graphs present the outcome of this a particular experiment in a quantitative manner and we will see that. So, that is the reason we typically plot graph.

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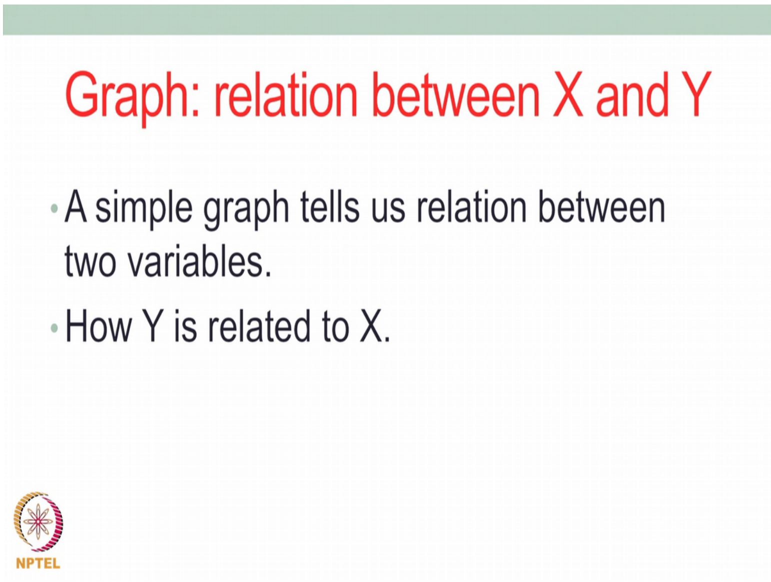

Now, what is a graph? So, graph is as you all know, graph is a relation between X and Y. A simple graph tells us the relation between 2 variables typically X and Y, how Y is related to X. So, that is what we have it in a graph. So, if we look at any graph there are various graphs of various shapes.

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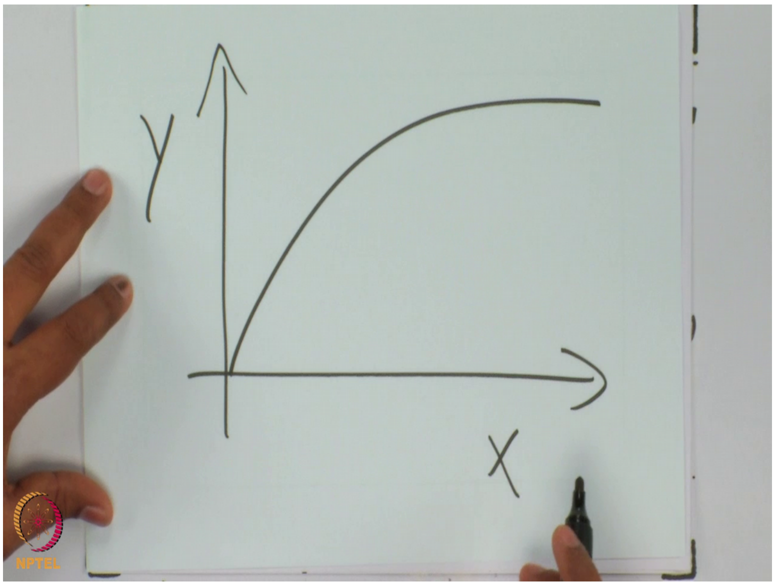

So, we have an X axis and some variable in the X axis and Y axis and we will see and you can have various relations like some graph like this or we have seen many kinds of graphs and this tells how Y changes as we vary X and there are many examples that we know and we will discuss many examples as we go along this.

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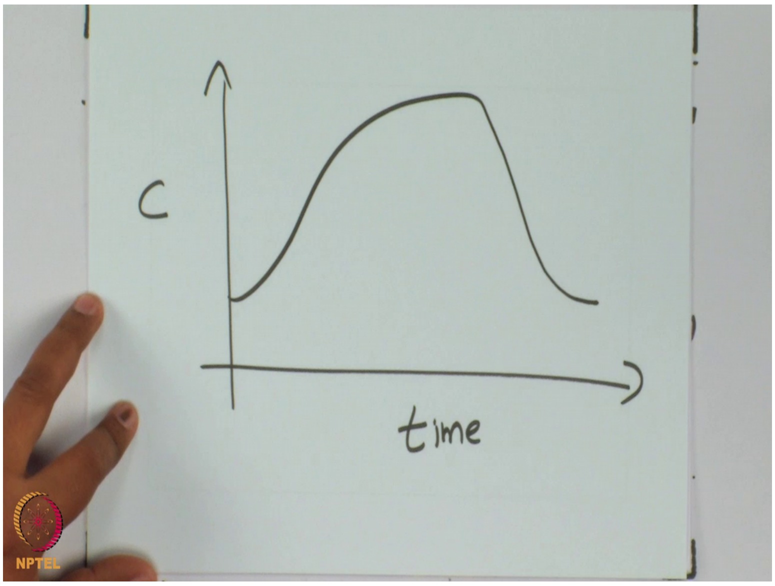

So, simplest things we would do in biology is concentration. For example, how does the concentration of a particular protein changes with time. How the concentration changes with time the concentration could increase or decrease, so it could first increase and then saturate and then decrease and all kinds of things can happen depending on the context. So, one can have a relation between concentration and time or any 2 variables. If we are moving, we can have position and time which we have for example, learn twelfth class the position time graphs and all that. So, any 2 variables can be plotted as the graph if they are related with each other.

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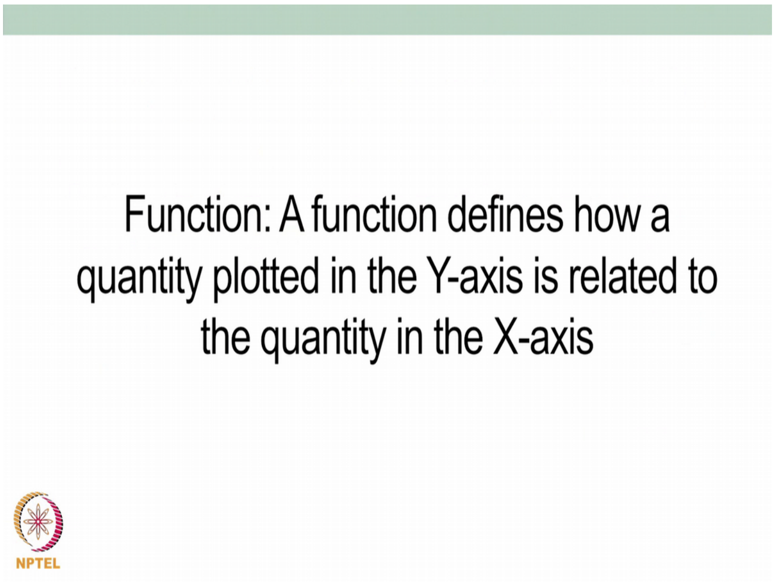

And the mathematical term for this is function. So, the word function is something that all of you should learn.

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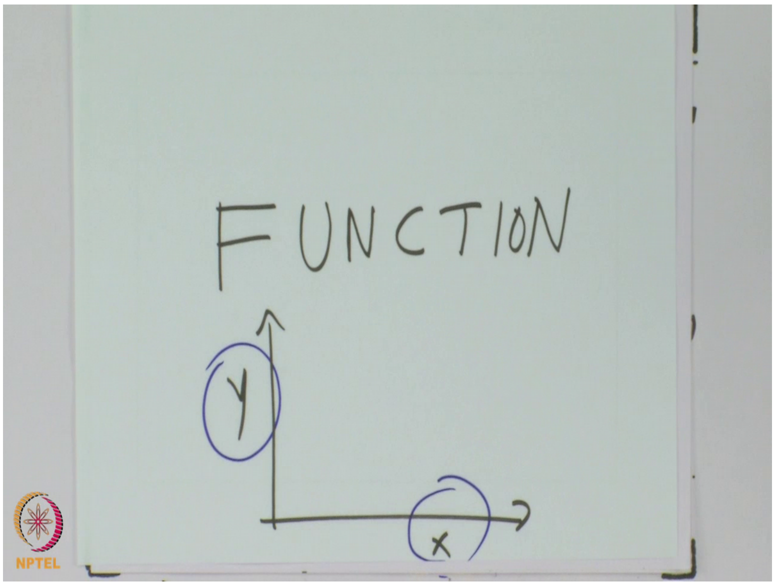

So, the key word that all of us, all of you should learn in this lecture is function. So, what is function? A function defines, how a quantity plotted on the Y axis is related with the quantity in the X axis. What is the relation between the quantity in the X axis and Y axis. How these 2 quantities are related. So, this relationship is function and this relationship can be written as an equation and we will see, what is the simplest function that we can think of soon, but that is the function is nothing, but a relationship between X and Y and the simplest, what is the simplest function that you can imagine? Think about it.

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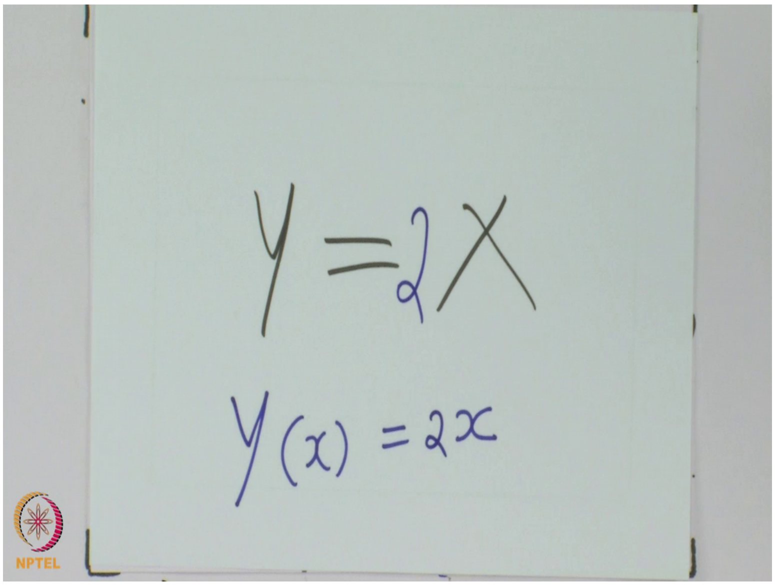

So, you have Y, you have X. You have the Y variable and you have this X variable, what is the simplest relation that one can imagine between X and Y. So, I am going to write down the simplest relation I can imagine or it is nothing, but Y equal to X. This is simplest, one can think of.

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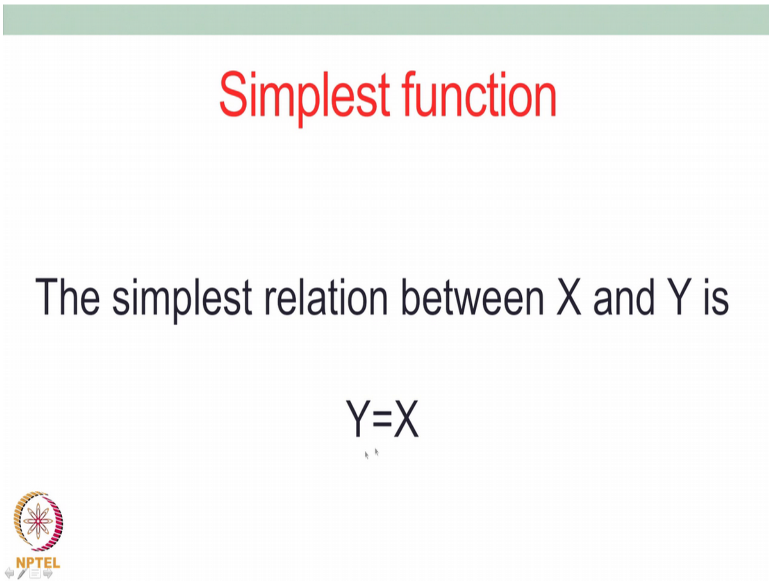

So, the simplest relation is Y is equal to X. So, what is that now mean. So, this is the simplest equation this is also an equation. So, what is that mean? When we say Y is equal to X it means if you have some 2 variables and we will see those examples they are equal, that is what it means.

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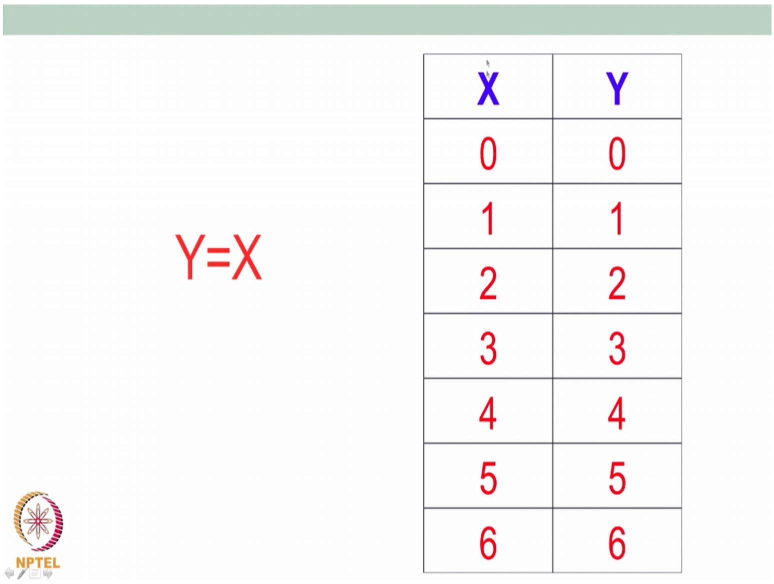

So, let us take an example from this. So, we have a X variable and Y and when they are equal when X is 0, Y is 0, when X is 1, Y is 1, when X is 2, Y is 2, when X is 3, Y is 3, 4; 4, 5; 5, 6;

6. So, this is what it means. When you have 2 variables they are equal, you will get a table like this and one can plot this as a graph and that is what one would see in the next slide.

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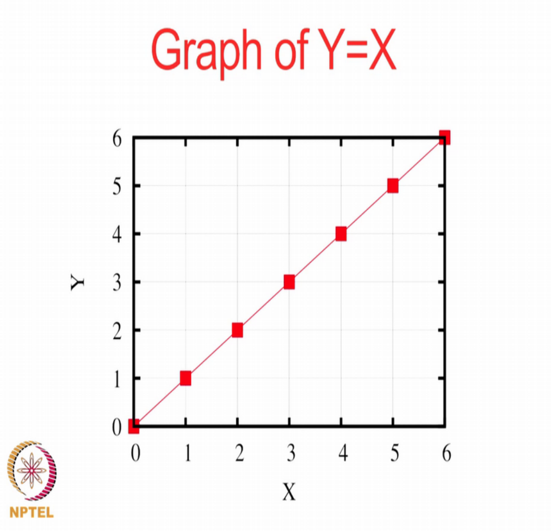

X	Y
0	0
1	1
2	2
3	3
4	4
5	5
6	6

If you plot Y is equal to X is a graph this would look for example, the next value is 1 the corresponding Y value is 1. When X value is 2, the corresponding Y value is 2; when the X value is 3, the corresponding Y value is 3. So, you put a dot there and you all know how to plot a graph, but I am just explaining this is the simplest graph that you can think of which is Y is equal to X.

So, this is a straight line, which is passing through the origin. So, and we have essentially plotted this table as a graph. So, there is a table and a graph and they are related with each other and this thing is this relation is called a function. This is all the simplest equation that one would typically think of.

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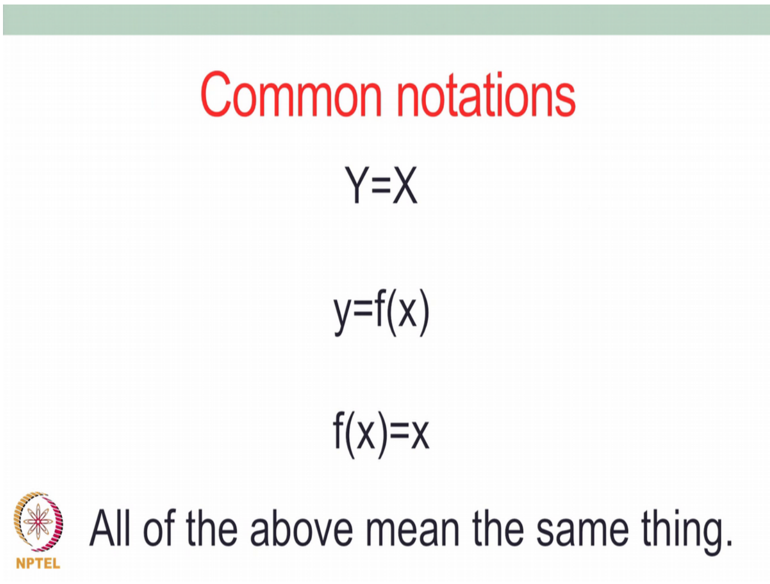

In textbooks, commonly this can either be notated Y it is common notations to represent this is typically Y is equal to X. Sometimes you would write Y is equal to f of x that means Y is a function of x and that function is nothing, but x, f of x. So, one would see sometimes any of this and this all means the same thing. So, f of x equal to x is exactly the same as Y is equal to X and sometimes you might even some places you would also see Y of x is equal to x, you would also see something like all of this would mean same thing; this means that Y is a function of x and that function is x, that is what it typically means.

So, all of this implies the same thing. So, this is the simplest thing you can think of. Now, given this simplest thing, what is the next complicated thing that is you add likely more complexity to this, how would that, what would be the function, that is, makes one step ahead of this. So, you look at here you have Y is equal to X, now, we can have Y is equal to 2X. So, I can put Y is equal to 2X.

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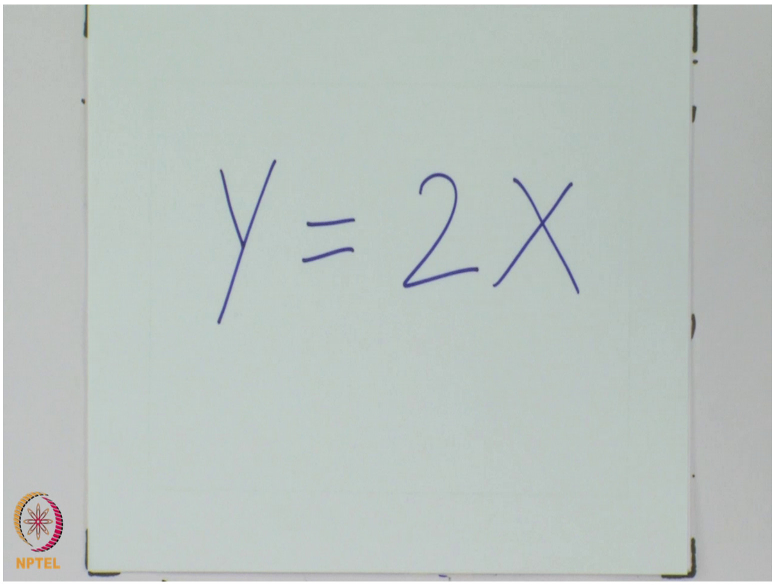

So, this is the next level of complexity we can add. So, this is Y is equal to 2X. So, that is what we would see next in the slide Y is equal to 2X.

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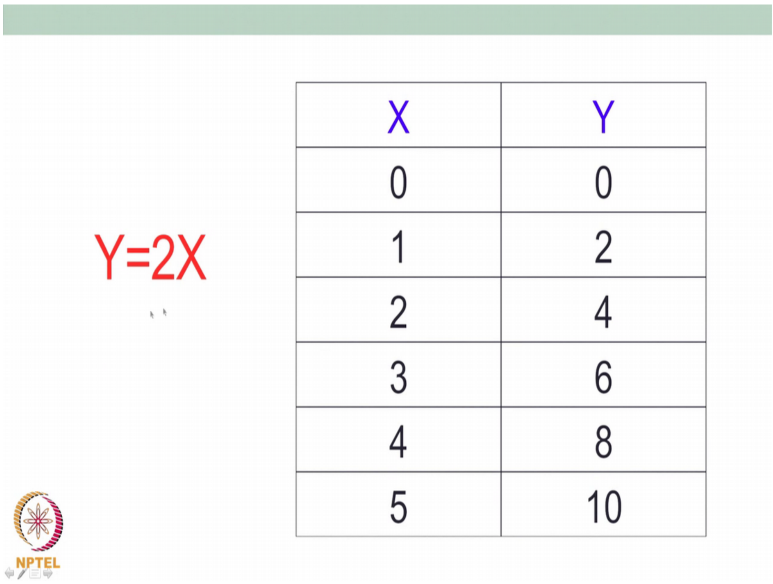
Y=2X

X	Y
0	0
1	2
2	4
3	6
4	8
5	10

So, then what does it mean for every X value there is the Y value is 2 times that of the X value. So, this is what here when X is 0, 2 times 0 is 0; when X is 1, 2 times 1 is 2; X is 2, 2 times 2 is 4, then X is 3, 6; 2 times 4 is 8; 2 times 5 is 10. So, this is stable essentially represents this equation that Y value is 2 times that of the corresponding X value, that is what this means.

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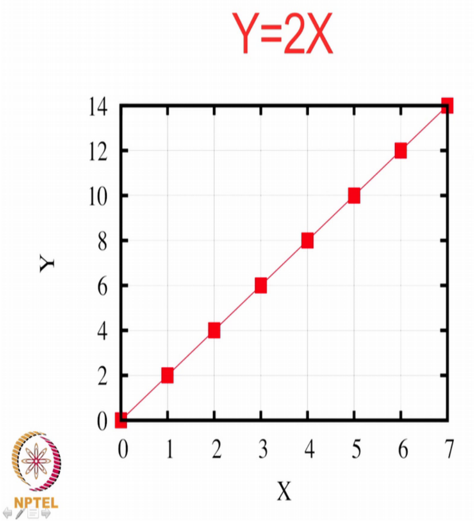

X	Y
0	0
1	2
2	4
3	6
4	8
5	10

Now, if you plot this, you will get a graph like this. So, you have 1 and 2 times 1 is 2. So, 1 and 2 correspondingly, you put a dot and 2 times 2 is 4. So, wherever 2 and 4 comes together we put a dot there, 3 and 6, and 4 and 8 and so on and so forth. 6 and 12, so, one can plot a graph given this table and the graph would look like. This is also a straight line passing through the origin. So, both X and Y of 2X are straight lines passing through origin; origin means 0, 0. It passes through X is equal to 0, Y is equal to 0. This point is called origin. So, this passes both are straight line, but what is the difference between Y is equal to X and Y is equal to 2X.

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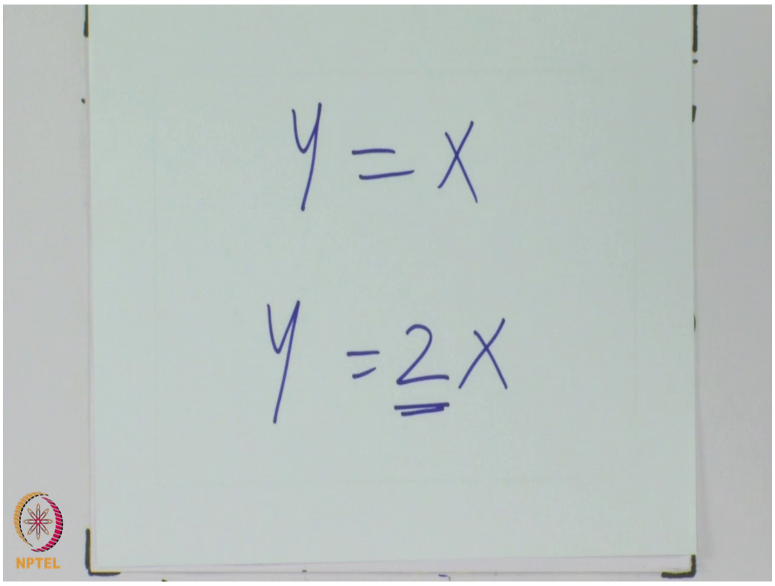

So, if we look at it the difference between Y is equal X and Y is equal to 2X. So, you have Y is equal to X and Y is equal to 2 X. Of course, this is the differences we are multiplying this with 2. So, if you plot Y is equal to X and Y is equal to 2X together in the graph it will look like this.

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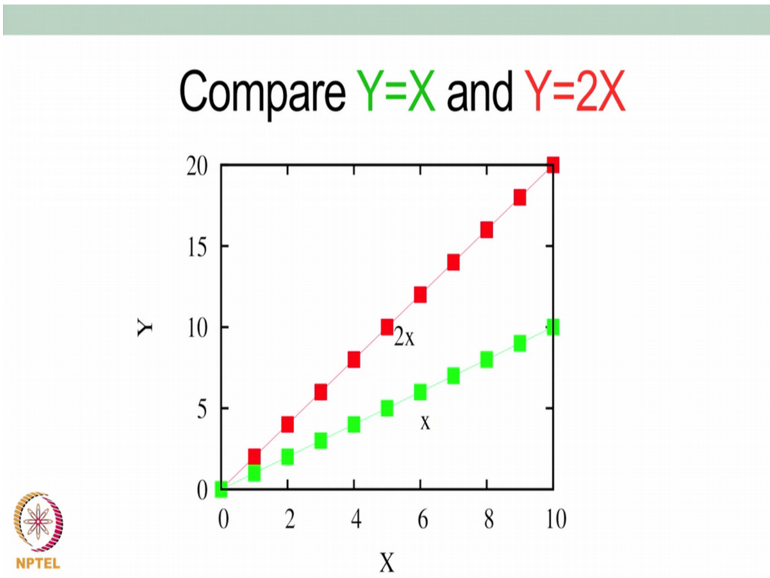

So, what we have here is, Y of x plotted in the. So, in the green line here is Y is equal to X, the first graph; the red line and the dots are Y is equal to 2x.

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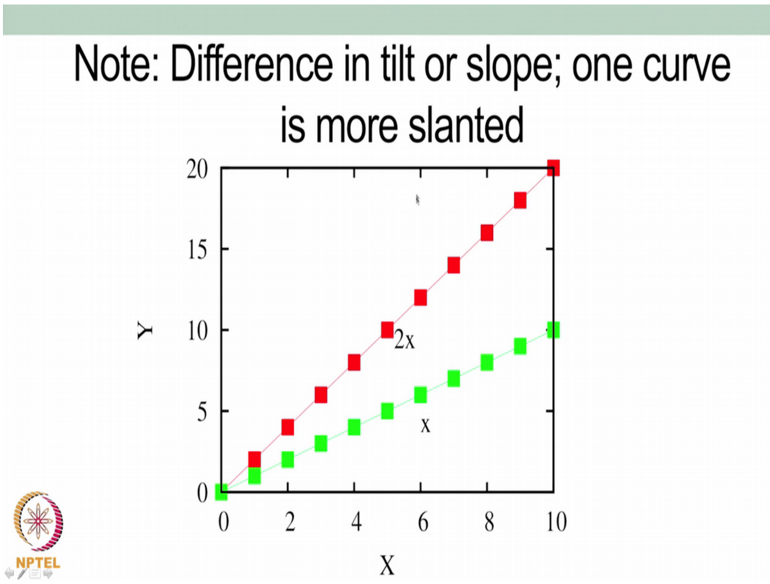

So, what one can see is if you compare these two, Y is equal to x and Y is equal to 2x, what do we see? We see that we can note that the differences basically in the tilt. So, this is more tilted than this the slope or the tilt or the slant, one curve is more slanted than the other.

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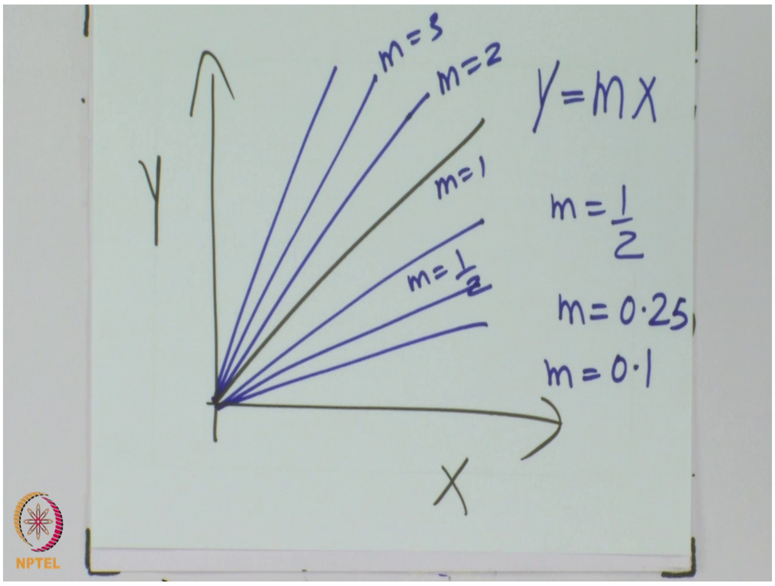

So, if you take and plot Y is equal to. So, we have X and Y and first we have this is let us say Y is equal to x and we said Y is equal to 2x is slightly more slanted than this and Y is equal to 3x, try to plotting this will be more slanted 4x. So, this is as I increase, in general I can write

Y is equal to mx. This is generally, this is m is equal to 1, m is equal to 2, m is equal to 3, so, that is Y is equal to 3x and so on and so forth. As I increase m goes, as m goes larger and larger this will become slanted in this way. This will tilt and become closer and closer to Y axis. So, think about what happens at Y axis.

Now, I can also have m is equal to half. So, then this would come below this it would be can look something like this and m is equal to 0.25, m is equal to 0.1, I can have all this various curves. I can have curves like this. So, think, do this and write down an equation over all of this curve. So, this is an exercise for you.

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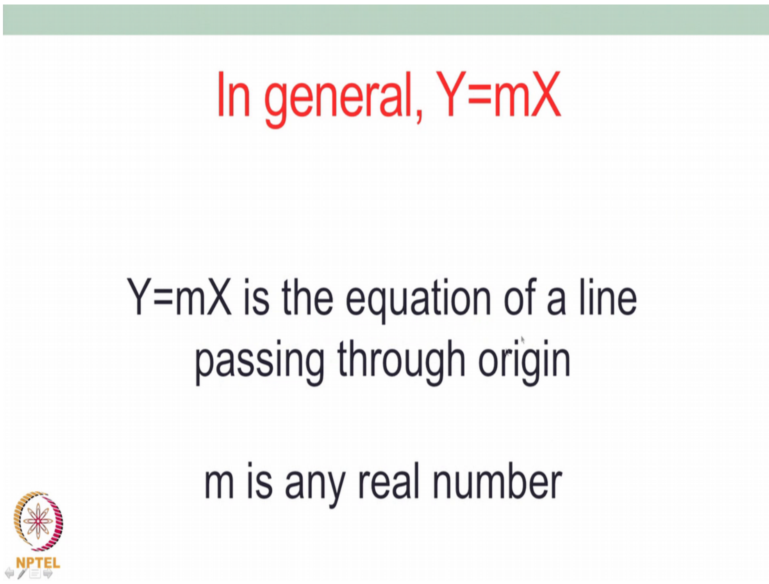

So, in general, we can write Y is equal to mX and you will all get straight line. So, this is Y equal to mX is the general equation of a line passing through origin. m is any real number; it could be half, it could be 1, 2, 3 and all that.

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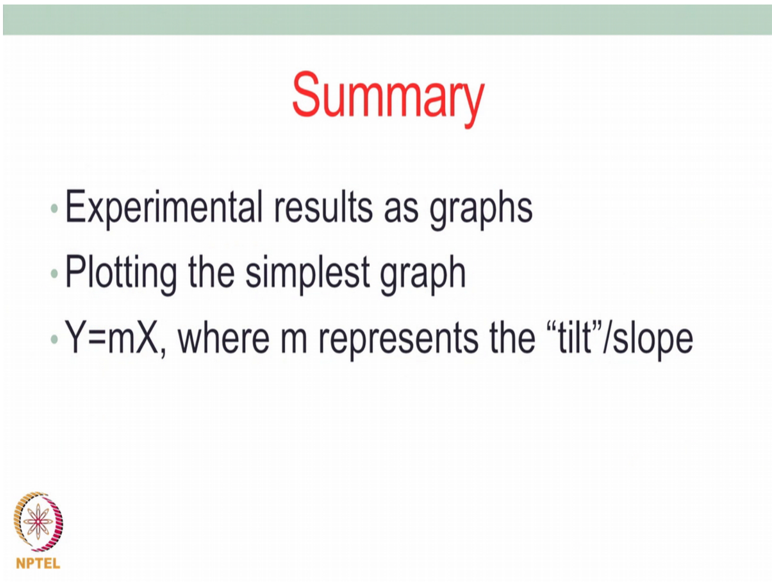

So, this is what we learned in today's lecture. So, we learned how to think about things around us in a quantitative way and how to plot the simplest graph possible. So, let us summarize what we learned today.

So, we learned that experimental results are typically drawn as graphs the reason is graphs are quantitative. It helps us to infer more things. By plotting this we learn we express them in a quantitative way and probably you can infer a lot more than a bunch of numbers or a bunch of sentences that we have. And we also learned how to plot the simplest graph; in general, Y is equal mX and we also saw that m represents the tilt and I want you to think about this and in the next lecture we will further they already go deep into this, how various equations are connected with graphs and how more complex complicated functions, how different functions can be plotted as graphs and so on and things such things we will learn in the coming lectures. So, with this I will stop today's lecture.

Thank you.