Introductory Mathematical Methods for Biologists
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Lecture - 12
Plotting Curves

Hi. Welcome to this lecture on mathematical methods, today we will discuss plotting a function, and the question that we will answer in this lecture is; what is the recipe to sketch a function. We have learned many things about the about mathematical functions and recently we will learnt about derivatives first derivatives the second derivatives we learned that the second derivatives give us the curvature.

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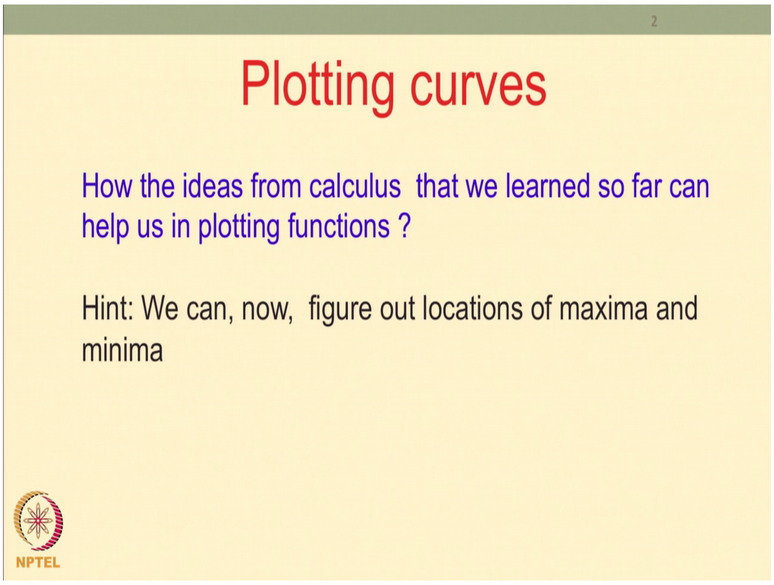

So, using the idea from what we learn so far; we will learn about plotting curves. So, what we will learn is how the ideas from calculus that we learned so far can help us in plotting functions. So, this is we would learn, and the hint is we can now figure out locations of maxima and minima, something that we learned after learning second derivatives.

So, first question is what is the minimal; what are the minimal details that we need to sketch the function.

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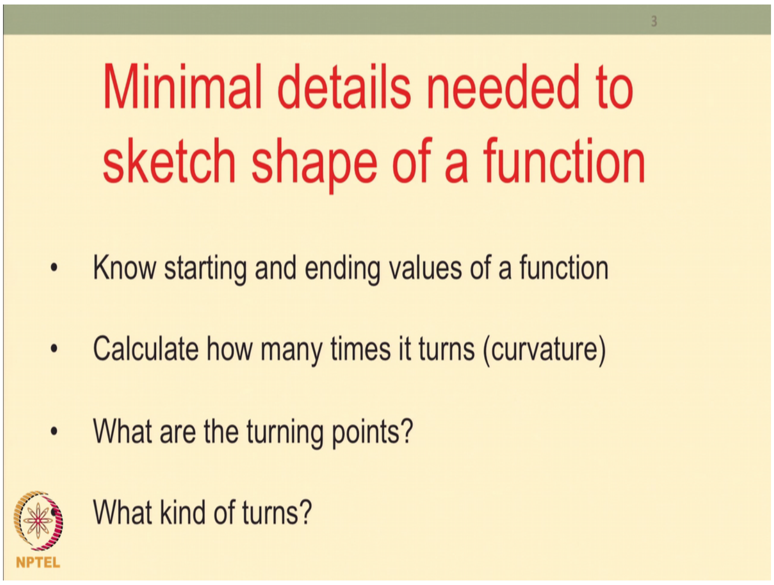

So, now the question is what are the minimal details needed to sketch the shape of a function. So, function we do not know how that the shape would be, but what are the minimal things that we need to know. So, the details would be that, if we know the starting and ending point of a function; if we know the beginning value and the ending value of a function that would be the first thing needed. So, if we think about it, we want to plot.

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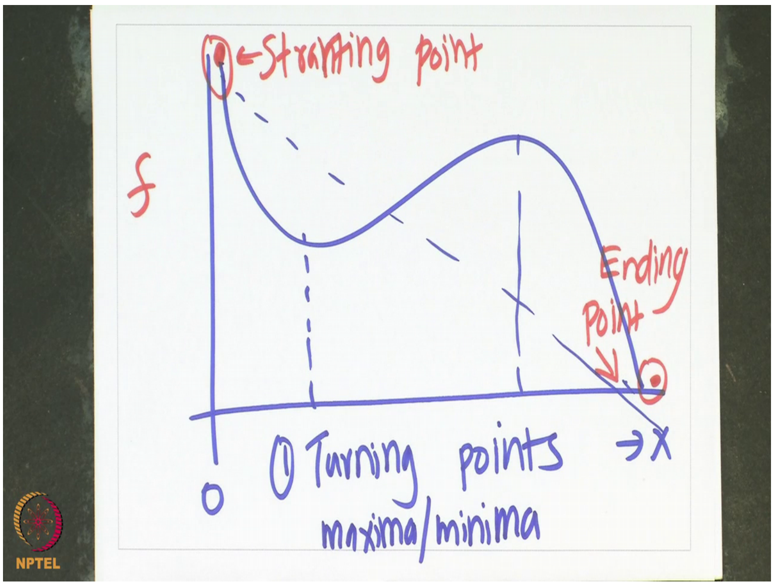

Let us say we want to draw from 0 to some large value of x and one thing we know is that at 0 at very small value of x is somewhere here is a very large value of the function as a very large value and when x is very huge.

The function would reach almost 0. So, if this is the; if you know the starting point and ending point. So, this is the starting point and ending point. There could be many curves which would satisfy this I could draw a straight line or I could draw many kind of curves then what we want to know after that is that this is the curve turn at all is it a straight line or is there some turns. So, now, second thing we would want to calculate first after you understand turning point and ending point, if you would calculate whether there are any turns. So, if the function is one possibility is a just a straight line, if it was not a straight line it could have some curves like this right. It could have a starting point and it could have some turns. So, if does this curve have turns or not.

So, turning points. So, these turning points would be typically maxima or minima right. So, these are the points where the function is coming down and is going to turn here. So, to turn this if this turning points are maxima or minima and there are some way of finding out maxima or minima. So, turning points or maxima or minima we will understand how each of this can be found. And one we have to know what kind of turning points does it have, I could draw I could have different kind of turning points, I could have a similar curve exactly is similar very similar curve, but with a different behaviour let us say I could have some function.

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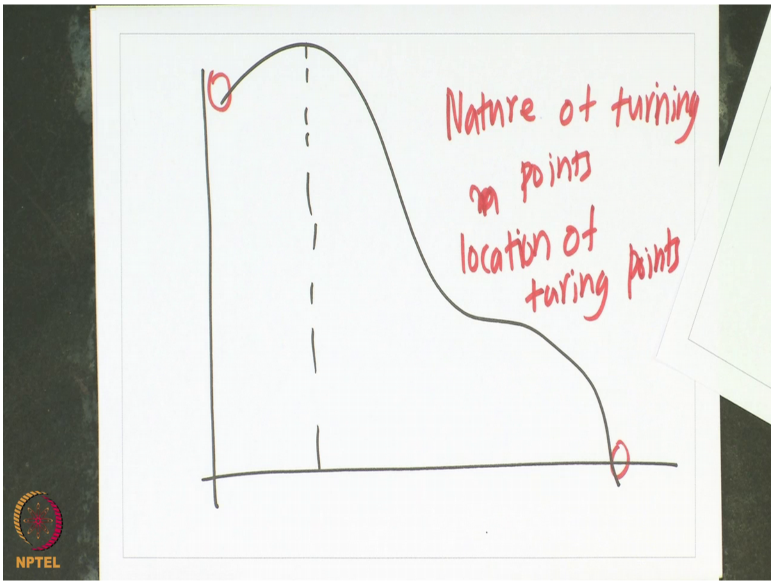

Which is like this which is a starting point here, and ending point here, instead of like this I might have the function like that or something we do not know. So, in other words the turning point here is different, from the turning point that we had drawn previously here. So, the nature of the turning point it is a maxima or minima and where are the turning points. So, a nature of maxima or minima turning points right which would say you which would tell whether is a maxima or minima that could be turning points of different kinds and the location of the turning points. So, if you knows this kind of stuff, we would be able to draw different curves.

So, now let us the take some example and think about it. So, the point here is that these are the factors that you would typically would minimal details that you would like to know, now the starting and ending value of a of values of a function, if you know this is important to know this calculate how many times does it turn right. So, that is essentially the curvature what are the turning points like where are the turning points, what is the location of the training points, what kind of turns are there they maxima or a minima maxima or a minima or maxima or minima. So, these are the details that you would want to know. So, let us take a simple example right. So, let us take a some simple function f of x is equal to x cube minus x.

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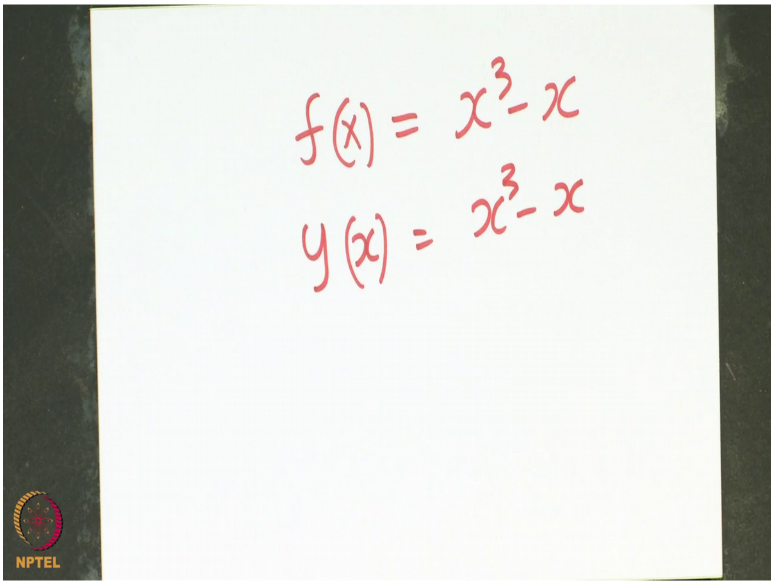

So, I would write that is our function or you could write y of x is equal to x cube minus x. So, whether you like x of y, f of x or y of x both are same representation. So, now, if we want to if we want to think about this function, let us consider this.

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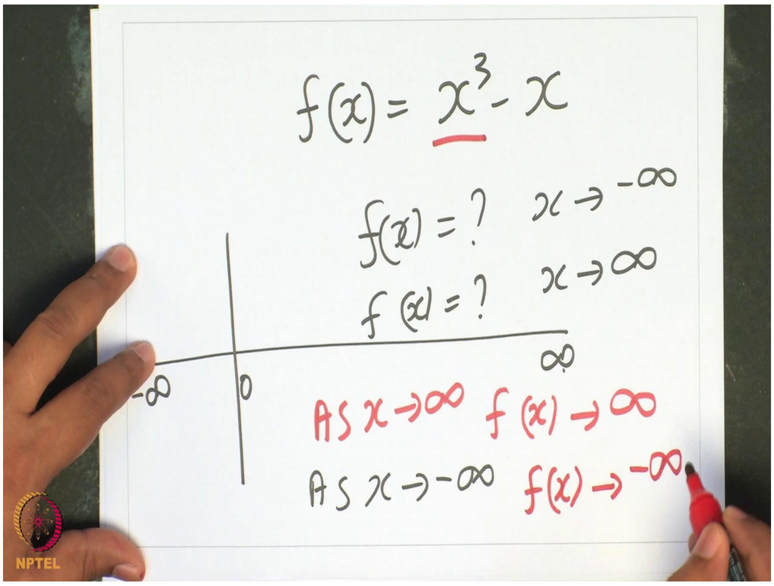

Function which is our f of x is x cube minus x this is our function. Now first thing we want to know is that, where is this function if you plot if we think of this as a plot, what are the values of this function at different? So, whether if we could plot this from minus infinity to infinity. So, let us say minus infinity to infinity minus infinity far away to infinity we could plot this or we could plot from 0 to infinity, depending on what we want to plot we would have a different starting point.

So, let us say if we want to plot from minus infinity. First thing we want to know is; that what is the value of this function, at minus infinity. So, when f of x in the when x is what is the value of f of x, when x goes to minus infinity and what is the value of x f of x when x goes to plus infinity. These are the first things you would want to know as the starting point and the ending point. So, if you look at it surely as x goes to infinity, a cube will be larger and larger and larger if substitute first one. So, it will be 1 cube minus 1 is 0 if it is 10, n cube minus 10.

Is a huge number 1000 minus 10 is like 990 if it is 100; 100 cube minus 100 is a very large number. So, as x tends to infinity this f of x value will also go to infinity, this is clear from this. Now if x is minus infinity right as x goes to minus infinity, what will happen? So, when x is very small minus infinity minus infinity cube. So, if it is minus x is minus 10 minus 10 cubed is a huge negative number minus, minus 10 will be plus 10. So, this will be the dominating term always. So, if since this is if x is highly negative, x cubed will be also a large negative number; and will be much larger than x.

So it will depending as x is x cube would go to minus of radius a function the function also will go to minus infinity, that is as x goes to minus infinity, x cube will go to an infinity therefore, f of x will also go to minus infinity. So, we understood the 2 points the starting point and the ending point at minus infinity the function is minus infinity, when x is plus infinity the function is plus infinity. So, let us mark let us take this paper and mark this properly.

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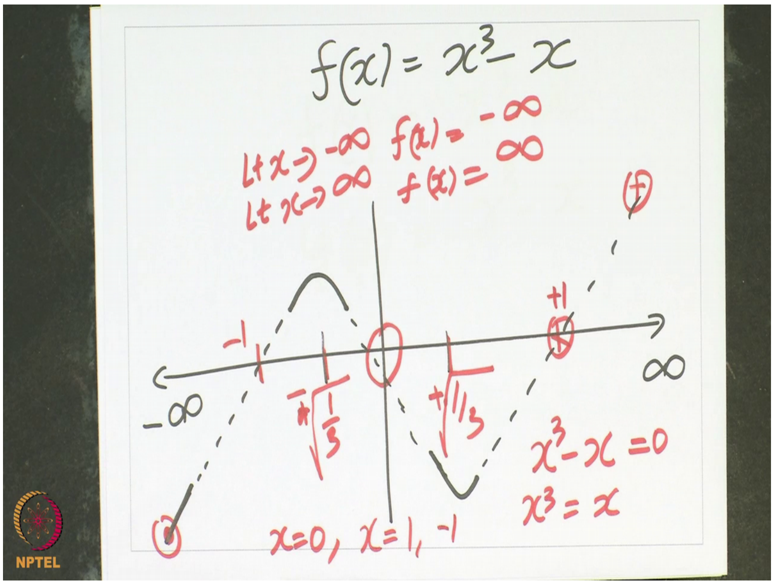

So, our function is f of x is x cube minus x. Now we are going to draw this from minus infinity to plus infinity. So, minus infinity far away here and plus infinity far away here.

And what we found is that at minus infinity in the limit as x going to minus infinity, the function of function is minus infinity. In the limit x going to plus infinity in the limit x going to plus infinity the function is value infinity. So, here the value is minus infinity some large negative number. So, this is one of my starting point and as x goes to infinity here I will have a large positive value. So, I have to grow from here to here and I can think it I could draw this a different many different ways from here to here; these 2 points I could draw this many different ways , but that is not enough now. Now we want to know whether this has any maxima or minima.

Any turning points right whether will it go like this and turn and come down or will it just directly go. So, the first thing is whether will it turn right that is what the first thing we have to know, if it turns where will it turn; right. So, let us think about this question. So, how do we find out the turning points?

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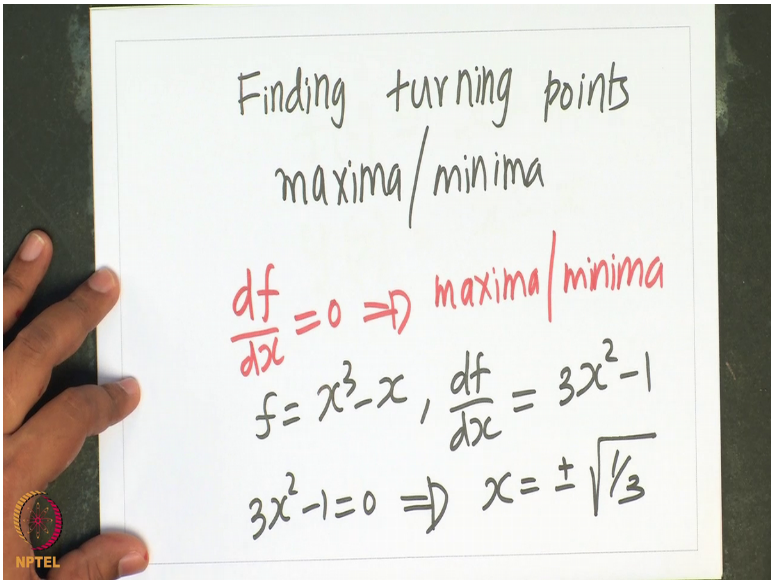

So, this is the thinking finding turning points, in other words maxima or minima. When the function f of x. So, it I we know is that if at maxima way at maxima or minima wherever if there is a maximum or a minimum the df by dx will be 0 this would imply maxima or minima we do not know what it will be, but both at a maximum or a minimum their first derivative will be 0 .

So, if you take this function x cube minus x and calculate first derivative, it is going to be 3 x square minus 1. In other words and were if where will this be 0. So, 3 x square minus 1 equal to 0 this means df by dx is 0 this would imply that x is equal to plus or minus root 1 by 3, 3 x square is 1 x square is 1 by 3 and x is root of 1 by 3. So, there is the 2 roots plus 1 by 3 and minus 1 by 3 root plus 1 by 3 root and minus 1 by. So, these are the 2 points where there are 2 turning points we will have.

So, we have 2 turning points which is basically 1 by 3. So, let me draw here this as 1 by 3 and this is also. So, this is plus 1 by. So, this is minus minus 1 by 3 root and this is plus 1 by 3 root. So, these are the 2 points the function will turn now if it has to turn here, how many ways what are the way you think you draw different ways this would happen, but the function has to be come and it does turn here, and it has to turn here and go. So, no it could this could happen in different ways, but this would roughly give us the shape of the function. So, the rough shape of the function would be it has to come somewhere turn here and then go and turn again back here.

So, if now itself we can draw the rough type of the function, even though we may not be able to draw this precisely. So, now, next question is in this turning point and in this turning point whether it is going to turn upside down or down up right is it a positive curvature or a negative curvature, that is something which we have to still find out second derivative will tell us whether it is a positive curvature or negative curvature, but let me just guess. If it is the function is going to start like this and it does to turn at this point and there is no turning point before this, right there is only the 2 turning points only it has to go then it has 2 only it will get turn down.

So, if it comes like this it has to turn like this here that is clear; and if it turns here this has to turn back here. So, these things are clear that you can guess and I am guessing there it is it is going to be like this is going to turn here like this and the turn here like this and go like this. This is some gas that I am making, but we have to confirm this guesses that if this is starting here, I am going to make a turn here and only 2 turns it can be it has to turn like this, and then it is going to come and turn again here and then reach the value this is one possibility. Of course, we have no way of knowing whether it will turn at the moment we do not know whether it will turn here or will it turn below 0.

Will it cross 0 here will it will it cross 0 here, this could be the places where it is crossing 0 could be horribly wrong, but the shape could be rough like this is something that I would guess at this moment, without doing much of the calculation right. So, let us now understand this in detail where the turning points are, and what is the values and all that. So, for this we have a very clear recipe instruction and instruction set it is like a set of instructions. So, what is the recipe for sketching the function f of x Evaluate the f of x at 3 points?

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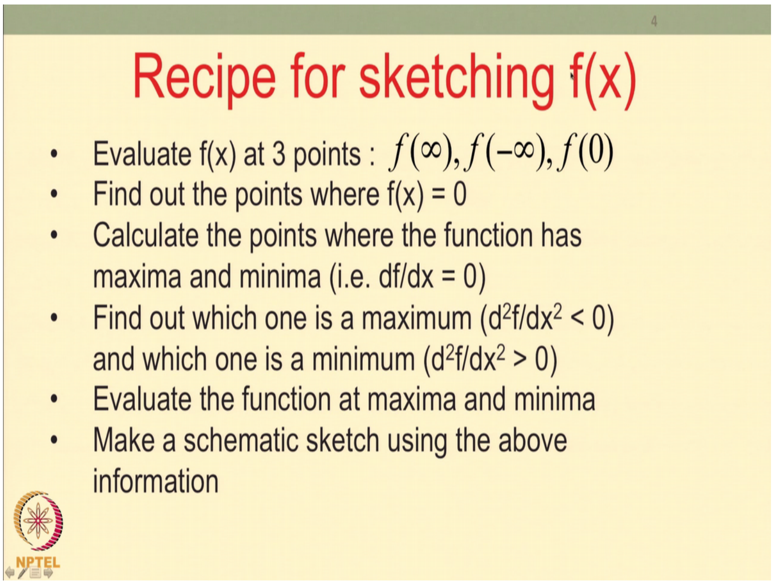

So, at infinity minus infinity and 0, this would help us even though we have not done it so far the function as 0 will help us plotting it more precisely; find out the points where the function is crossing 0. So, when f of x equals 0 if you know the point where the function is 0. So, let us come back here, we want to know where the function x cube minus x will be 0; that means, x cube is equal to x which would mean that this is possible one it is 0 both are 0, x is 0, when x is 0 x cube will be equal to x. So, one possibility is x equal to 0, when x is equal to one of course, this is one cube is one. So, this is another possibility is x is 1. So, as 0. So, what I have drawn here is not fully correct, because this has to cross it 0 the function will hit 0 at 0 that is what this would imply then it will hit at 1.

So, this point is this point is likely to be 1 (Refer Time: 17:18) it will be a one right and it has to hit one more place 0 because it will have 3 roots. So, one root is this we with the one root this is 0, the other roots are x square is equal to 1. So, x is equal to plus 1 and minus 1; x is equal to plus 1 and minus 1 this will be minus 1. So, if I cancel 1 x it will be x square is equal to one and then we will have minus 1 and plus 1 if you see this. So, to do this to see this let us go ahead and understand this, but this is the important of find out the points where the function is 0, calculate the points where the function has maxima and minima that is you calculate the df by dx 0. Find out which one is maximum that is where is d square f by dx square less than 0, and which one is minimum d square f by dx square is greater than 0, evaluate the function at maxima and minima make a schematic sketch using the above information. This much information will help us to make a nice schematic sketch. So, let us do this step by step.

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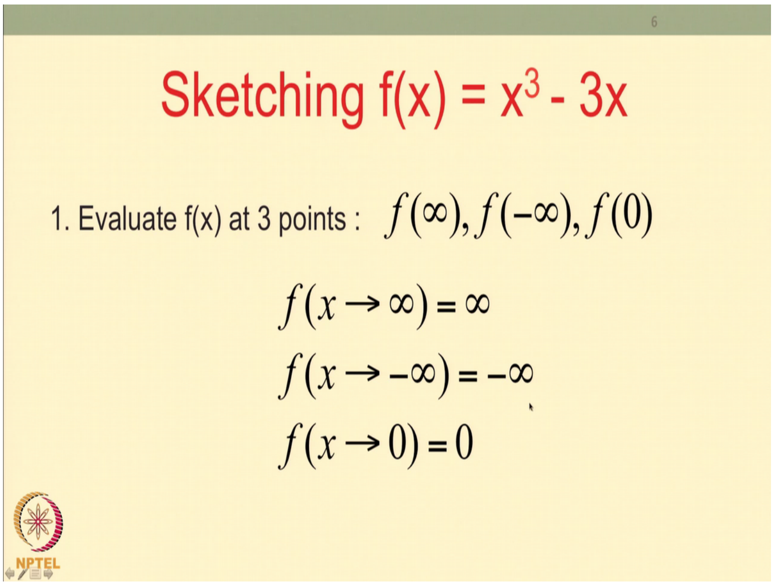

First we have to evaluate the function at 3 points infinity, minus infinity, and 0 and as we saw this is x cube by 3. So, we will do a different function here x cube minus 3 x right. So, let us take this function x cube minus 3 x. So, if you take this function f x cube minus 3 x similarly at infinity when x is going to infinity.

This function also has infinity and x is going to minus infinity this function will have minus infinity, when the function is 0 this function will when x is 0, the function will be 0. So, the x is infinity the function is infinity; when x is minus infinity the function will tend to minus infinity, when x tends to 0 the function will be 0 right. So, these are the limits that we have. So, we know the starting points an ending point.

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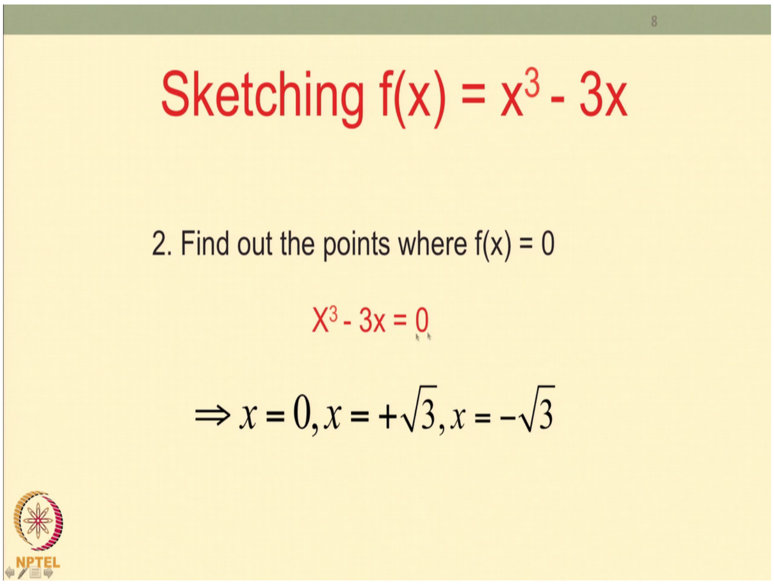

Now, we want to find out the points where the function will be 0. So, note the function is slightly different here is x cubed minus 3 x. So, if we want to find out the values of x at which x cube minus 3 x is 0. So, when x is 0, x cube minus 3 x is 0; when x is plus root 3 this function will be 0 when x is minus root 3 the function will be 0.

As we know like if you have this function x cube minus 2 x, 3 x.

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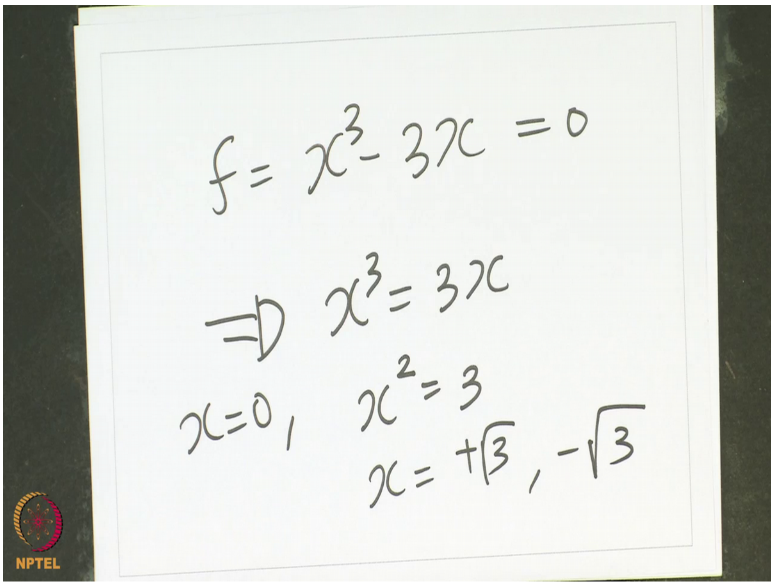

So, let us take this function that we have which is x cube minus 3 x this is our function and this would if this is equal to 0 implies, x cube is equal to 3 x this is what it implies right this would imply x equal to 0 when x equal to 0 this was possible, otherwise if this is also possible when x square is equal to 3, that is x is equal to plus root 3 or minus root three. So, these are the 2 places where this would be 0.

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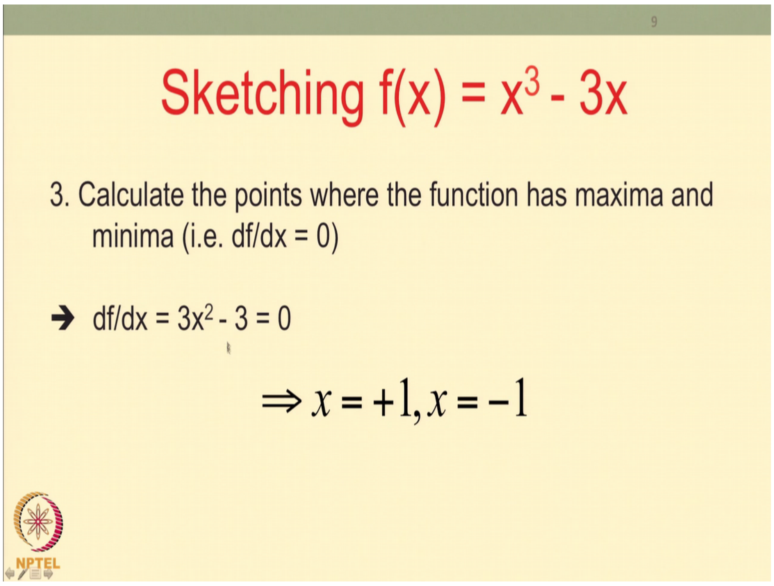

So, now then we calculate the first derivative.

The first derivative is df by dx is equal to 0, you can calculate the first derivative at 3 x square minus 3 equal to 0 is the first derivative, and you equate this to 0 and solve for x and you will find that x is equal to plus 1 and minus 1 are the 2 points this have 0; that means, that these 2 points the function will turn. So, these are the 2 turning points.

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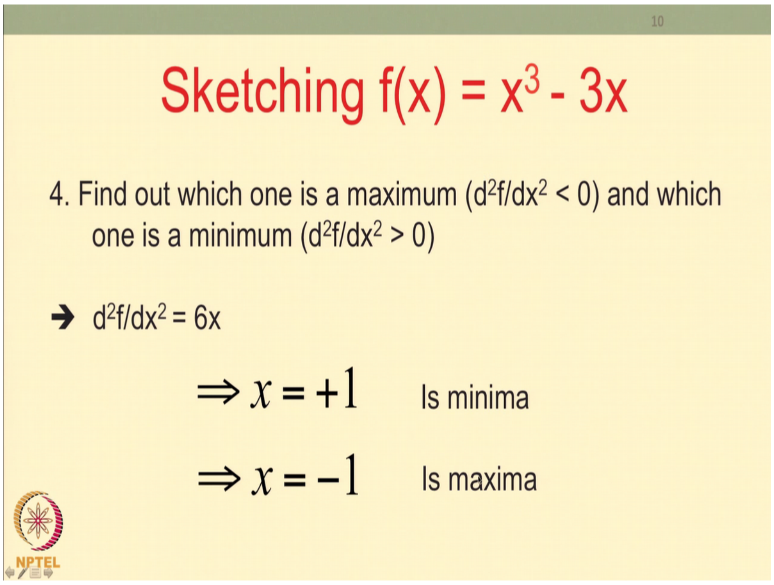

And you calculate the d square f by dx square, and at this turning points what is the d square f by dx square. So, let us quickly look at this d square f by dx square.

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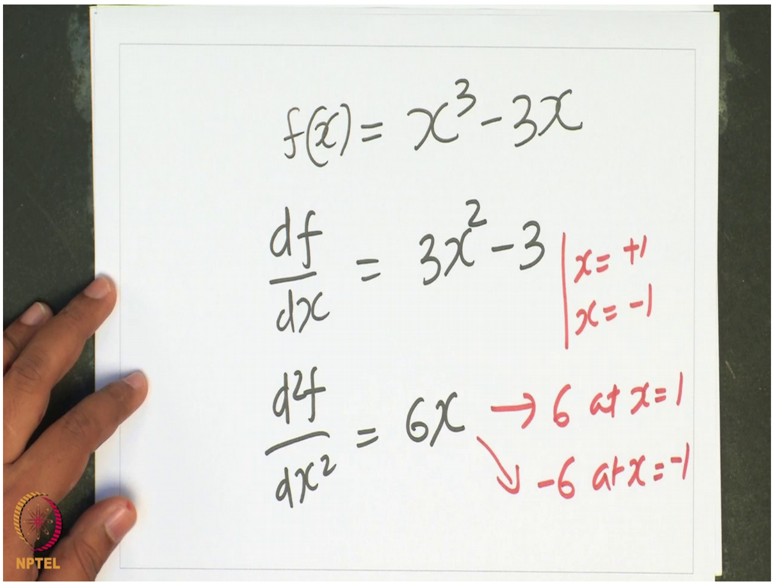

So, you have f of x is x cube minus 3 x, and df by dx the first derivative is 3 x square minus 3 the second derivative d square f by dx square.

This is 0. So, the derivative of this is 6 x. So, the second derivative is 6 x. So, now, we know that the turning points are x equal to plus 1 and x equal to minus 1, these are the maxima minima points and when x equal to plus 1 this would give 6 at x equal to plus 1, and this would give minus 6 at x equal to minus 1. So, when x is minus 1 the value second derivative is negative, when x equal plus 1 the second derivative is positive. So, this is what an x equal to plus 1 the second derivative is since it is a positive it is a minima, and x is equal to minus 1 the second derivative is. So, we will give us that the function is has a maxima.

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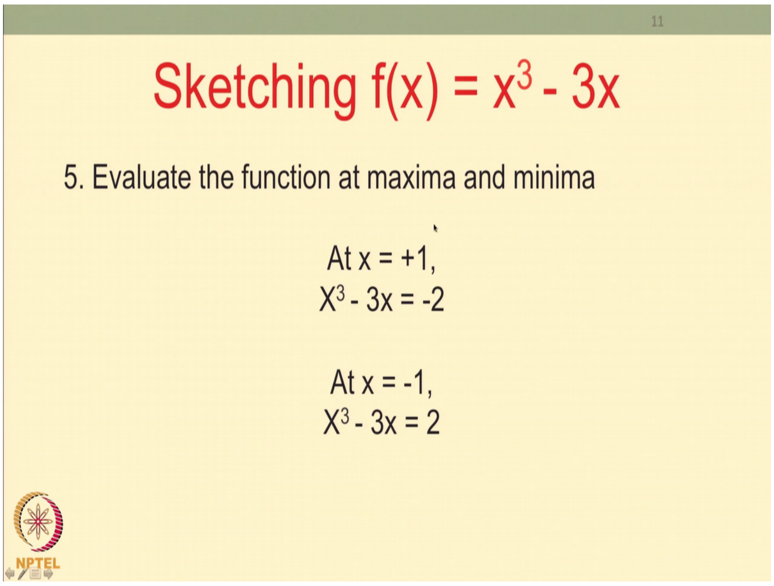

So, now if we want to sketch evaluate the function at x equal to plus 1. x cube minus 3 x equal to minus 2 and you can calculate these values at x equal to minus 1 x cube minus 3 x is 2 and then.

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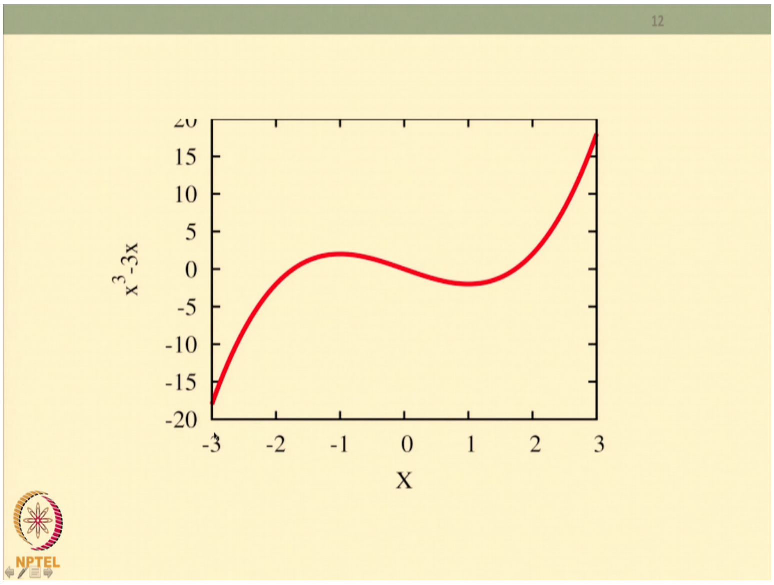

We plot this and what we would find is that is start from minus infinity and it will come up and it will turn here at minus 1 here, and it will come and again turn here back and then it will go to 0 and here there is at minus 1, there is a maximum and at plus 1 there is a minimum. So, this is the recipe to plot any function. So, to summarize what we have is that.

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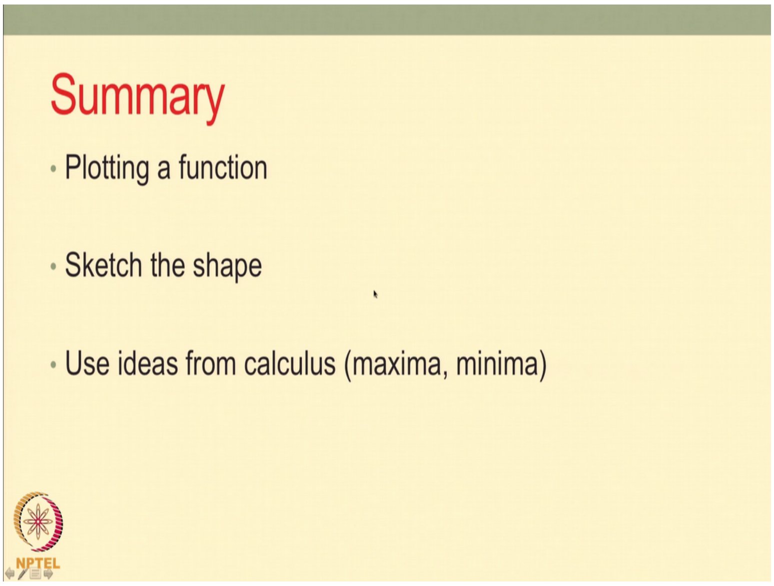

To plot function what this would what we are going to learn we learned this about how to sketch a function.

So, to sketch the shape of a function, we may not know all the points precisely, but we would know the rough shape of a function by following this recipe. So, for this we have to use ideas from calculus like the idea of maxima minima derivatives, that the idea that the first derivative of 0 would mean there are maxima there are turning points there is maximum or minimum or maxima or minima. And whenever the second derivative; if the second derivative is positive it will be a minima and when the second derivative is negative it will be a maxima. So, these are the ideas from calculus that we learned this we could directly apply and learn about how to plot how to sketch a function and this is going to be extremely useful when we have to use those mathematical ideas mathematical equations to describe some biological phenomenon.

How would things vary in a cell this function would be concentration. So, where is the concentration high where is the concentration law how is the concentration profile would look like in a cell, if we want to know all this details what is the shape where is it curved whether it is curved or not. If you want to understand these things for a biological system, this would be extremely useful for us we would descries discuss some of these examples soon and therefore, with this. So, it is important to learn this with this I will stop today's lecture see you later.