Introductory Mathematical Methods for Biologists
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Lecture – 10
Understanding Derivatives

Hi, welcome to this lecture on mathematical methods for biologists. In this lecture we will discuss the topic understanding derivatives.

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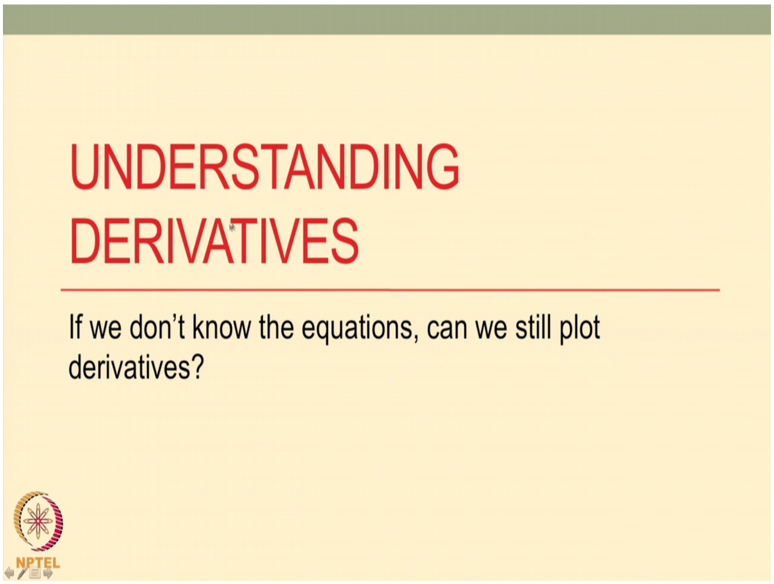

Even if we do not know the equations to compute derivatives, can we still plot the derivatives as a graph. So, this is the thing, this is the question that we will try and answer in this segment in this part of the lecture, in the last lecture we learn formulas to compute derivatives.

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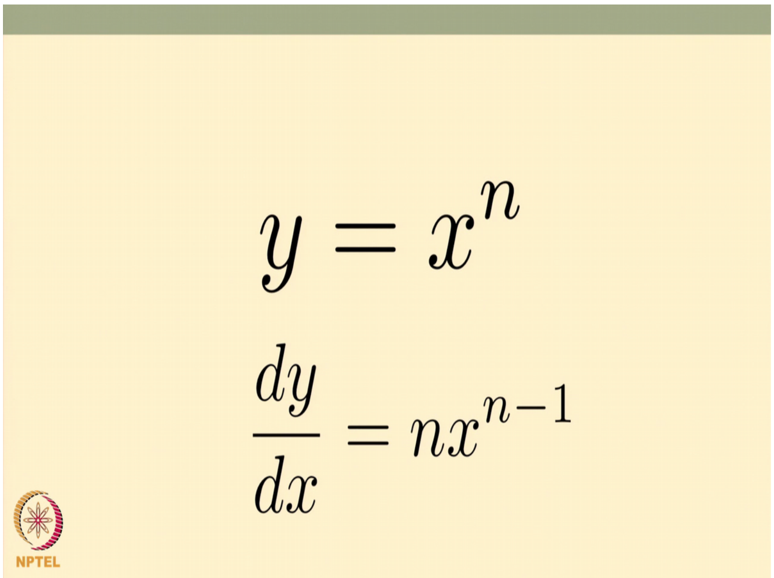

For example we learned if we have a function y which is x power n, its derivative is nx power n minus 1 and we learned derivatives of many other functions e power x and so on so forth.

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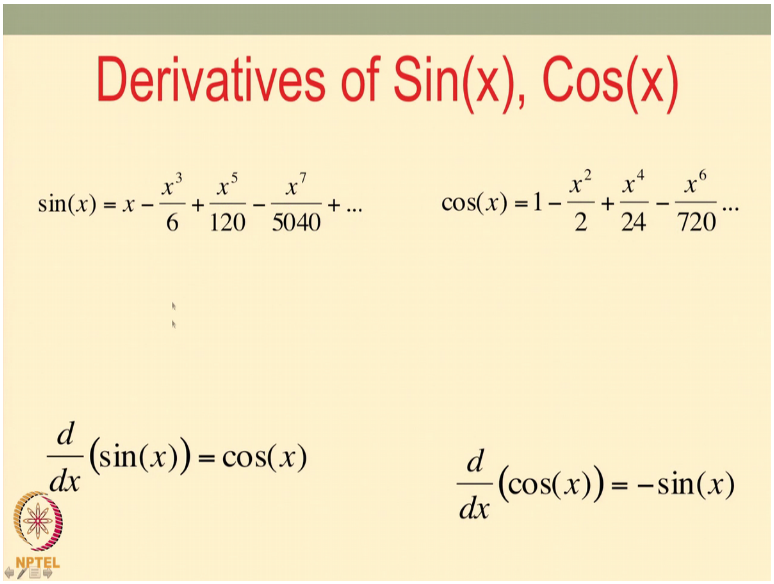

We also briefly mentioned that if we have functions like sin x and cos x, its derivatives can be computed, let me little bit elaborate on this for a few minutes and then we will come to the play main topic.

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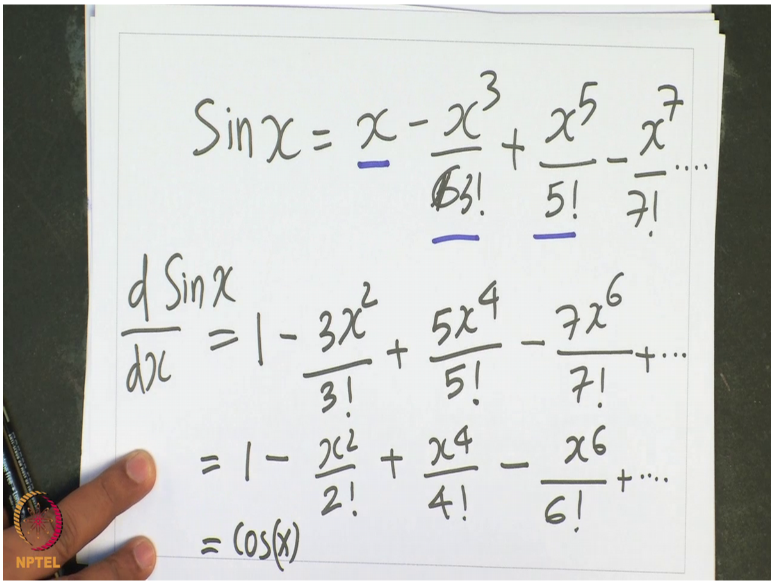

So, like we mention in the last lecture, if we have the function sin x, which is sin x which can be written as x minus x cube divided by 6 minus x power plus x power 5 divided by actually 5 factorial. So, this is 3 factorial. So, this will be 5 factorial. So, 6 I can write as 3 factorial let me write this as 3 factorial and minus x power 7 by 7 factorial and so on and so forth.

So, this is the this for sin x, and the derivative of sin x if you want to calculate, then we said in the like we said in the earlier class, we can find the derivative of each of this terms. So, x has a derivative 1, and x cube has a derivative 3 x square. So, minus 3 x square divided by 3 factorial, plus 5 x power 4 is the derivative of x power 5 divided by 5 factorial where 1 5 factorial is a constant. So, I keep the 1 over 5 fac[torial] 5 factorial as it is and find the derivative of x power 5 only then I will get this minus 7 x power 6 divided by 7 factorial plus dot dot dot.

Now, this is 1 minus 3 divided by 3 factorial is 2 factorial. So, this is x square by 2 factorial plus this is x power 4 by 4 factorial 5 divided by 5 factorial is 4 factorial minus x power 6 by 6 factorial plus dot dot dot dot. And this will turn out to be just the cos x expansion. As you can see cos x is 1 minus x square by 2 factorial plus x power 4 by 4 factorial, minus x power 6 by 6 factorial which is 7 20 and so on so forth. So, it turns out that derivative of cos x sin x is cos x, which you can directly do this and see.

Similarly, we can also take this function cos x, and find the derivative of each term if you take the derivative of 1 you will get 0. So, if you find the derivative of x square it will be 2 x and 2 x and 2 x will cancel with the minus sin, but here you have a plus x. So, derivative of cos x is minus sin x, if you do for derivative of each term here, you will find that it is same as this series with a minus sin therefore, derivative of cos x is minus sin x and derivative of sin x is cos x.

So, this is something that we can do from the equation that we have already learnt. Now the question is if we know how to plot this function, which we have learned plotting this function because I have said we discussed that equations are like graphs. So, if we plot this function and by looking at this plot by inspecting the plot, can we guess the derivatives; without doing all these complicated calculations can we guess how the derivatives look like.

So, let us learn that that is the training that all of us should get. So, the training is to look at a function and guess in plot its derivative at least how the derivatives would look like, how would they vary, this is something that we should able to guess and plot. So, this is something that we will see. So, we let us take these examples of sin and cos we know that derivative of sin is cos.

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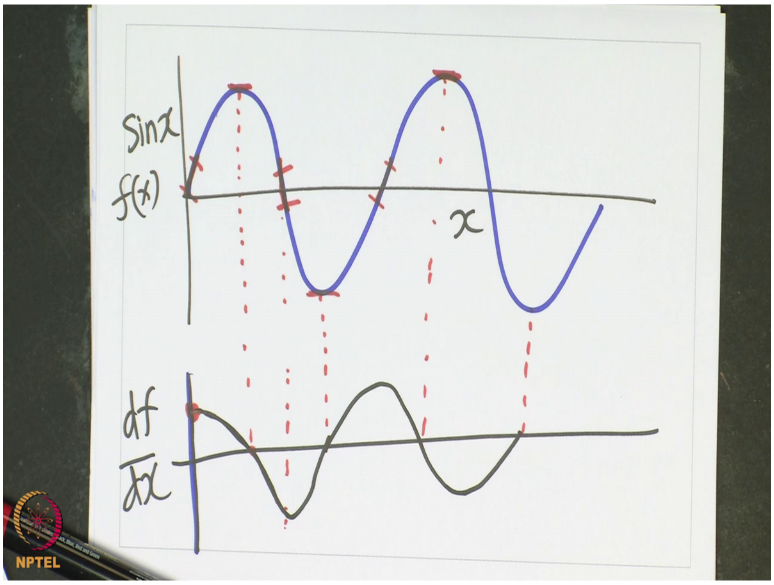

So, let us now remind us as how does the curve look like. So, we all know that if we plot sin x. So, let us plot sin x. So, if we plot sin x versus x, it would look like an oscillatory curve like this there is starting from 0 then there is a peak and so on and so forth.

Now, by looking at it, can we plot the its derivative below here. So, the derivative of sin x if I plot here what will I get. So, let us get a train ourselves to look at this curve and plot its derivative below here. So, first let us look segment by segment, we said that derivative we should take a small value of delta x and look at its derivative. So, if we take this small segment here, this small segment if we consider the small segment this segment has a derivative which is a slope which is like a positive slope. So, we know that this segment has a positive slope. So, the derivative of this will be some positive number here, and if we look at here this local segment here, here it is almost like a flat line. So, it is almost like a flat line.

So, we know that the derivative here, derivative of flat line like this will be 0. So, the derivative of this will be 0. So, the derivative of this segment is positive, derivative of this is the derivative of this part where there is a maxima is almost like a it will be 0 there. So, the derivative of this function is going from a positive number to 0. If you look at this segment corresponding to here, we know that this will have a negative slope, this is a negative slope the slope locally here, if you look at the slope this is a negative slope its a tilted function to this side therefore, this will be negative value somewhere here this segment if you look at this part this again has a 0 slope, here there is a positive slope. If you look at this small segment here it will have a 0 slope again.

Similarly, here also there will be a 0 slope. So, wherever there is a maxima or a minima the derivative of it will be 0, this is something that we will learn in little bit more in detail, but if a function has a maxima it is just changing from a positive slope to a negative slope, it has to go through a 0. So, therefore, this will be have a this this point will have a slope 0. So, if I look at it first I have a positive slope, then I have a 0 here then I have a negative, then again 0 here, then they have a positive and then 0 and then again 0. So, the curve roughly without knowing much about how the derivatives looks like. We by looking a zeros and positive and negative pass, we can guess that the curve would roughly look like this.

So, this would this is going to be if this is your f of x, this is going to be the df by dx. If I plot it little bit more carefully it would look like this like. So, if I plot it more carefully, it would look like here. So, this is the precise plot.

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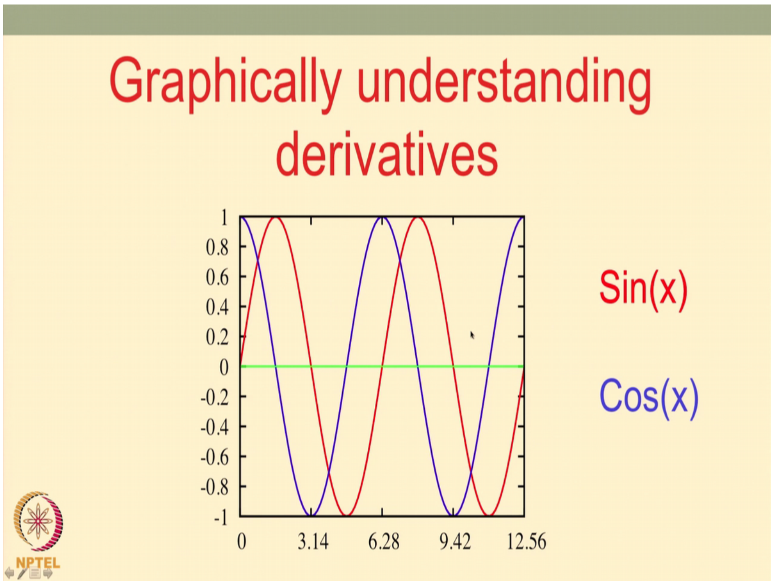

So, the red curve is sin x and the blue curve is cos x. So, as you see here, here the derivative the slope here is positive therefore, cos x is positive there. So, at 0 the slope is positive near 0 therefore, the cos x value near 0 is positive, which is actually close to one indeed these are it is 1 at 0 and this as a slope decreases and here the slope is 0 therefore, the cos function will be 0 when sin has a maximum there.

And here after pi by 2 this will have a negative slope therefore, the cos value will be negative and as further here the function will have another minima. So, another minimum. So, here in this minimum, again the derivative will be 0 therefore, the cos x will be 0 and so on and so forth. So, just by looking at small small segments whether the positive slope is positive or negative, one can guess and plot the derivatives of many functions.

So, this is some exercise that before learning, how to precisely compute the derivatives we should try ourselves to guess the answer, what the answer would be and this is an exercise that all of us should train ourselves, because when we do experiments or when we do simulations, we should have a educated guess on what the answer will be. Of course, the answers could have surprise us, but if only we train ourselves to look and think about what the answer will be what how what can we expect from our scientific knowledge so far, only then we will that will that will direct us guide us in the right direction always

So, now we will think about some biological examples some applications in biology. So, that is was we will do next.

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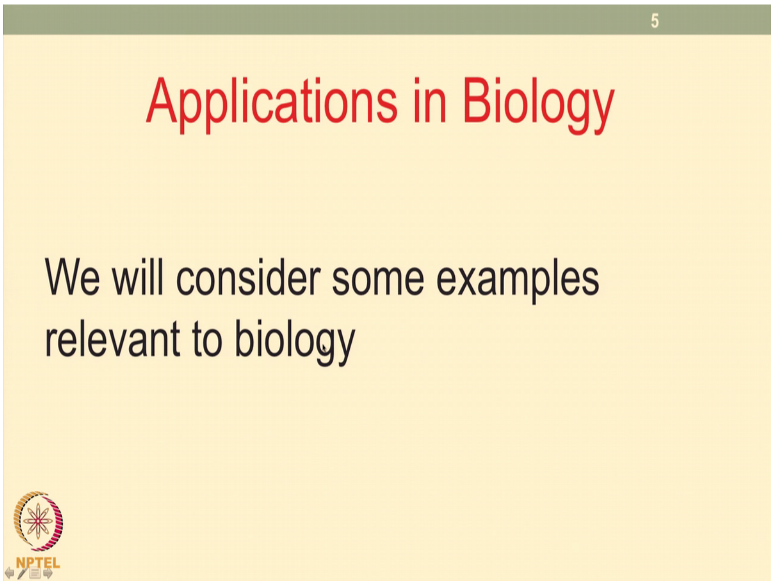

We will consider some examples relevant to biology, and to understand and get some picture about derivatives.

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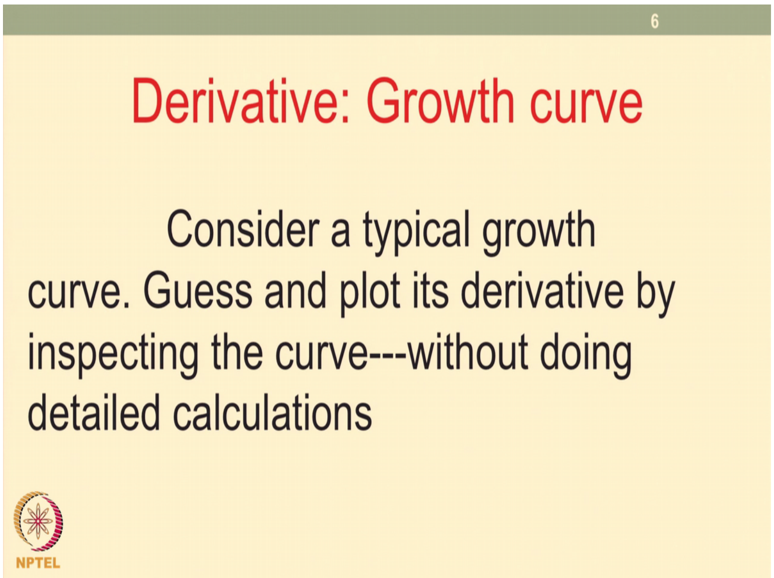

So, the first thing we will do is, something which is everybody knows in biology is growth curve. So, if you consider if we consider a typical growth curve, and can we guess and plot its derivative by inspecting the curve without doing detailed calculations. We have not yet learned how to calculate the derivatives of a curve given to us, if we given experimental data if we given experimental data, how to compute the derivative is something that we have not learned yet, we will learn this soon in the coming class as. But before doing it can we guess and plot the derivative by inspecting the curve by looking at it. So, this is something that we will do.

So, first we will do an idealized growth curve. So, let us we all know how the growth curve looks like let me first plot a function.

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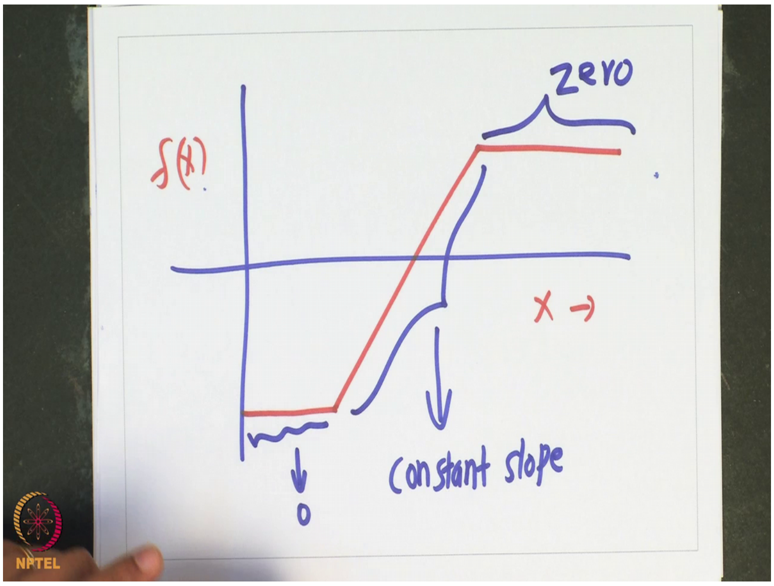

If we have an x axis and y axis, I will plot a function which not a growth curve, but it is somewhat related to growth curve or it might it is a if I draw a function like this something which is small, then this increases then it is a constant. So, this is not a growth curve, but it has roughly the shape of a growth curve, except that I have a sudden transition at some points.

So, if I have a function like this, what would be the derivative of this function? So, this is my f of x versus x, what would be its derivative, can we inspect this and plot the derivative of? This function this is the question that we want to learn first. So, let us try doing this. So, let us first think about it, if we have this the derivative of this part. So, here this the function is flat, the function is not changing for a for many values of x. So, the derivative here will be 0, we know that the derivative here be 0 because the function is not changing its a flat the function is a constant. So, its derivative of this part has to be 0; this is like a straight line. So, this part will have a derivative, which is which will have a constant slope. So, this is like y is equal to mx plus c. So, this will have a derivative m which is same everywhere like here or here or here, if you take a straight line the slope here is same as the slope here this is the same as slope here. So, slope of this curve is same everywhere and here for this part, again the function is a constant. So, the derivative of this will be 0, because the function is a constant is not changing with respective to x.

So, we expect 0 a constant again 0. So, this is something that we would expect. So, let us redraw this say for our purpose.

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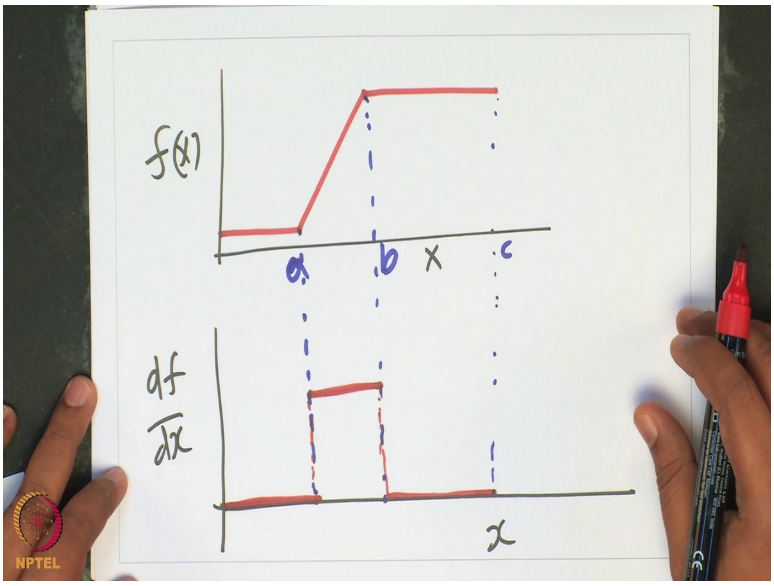

So, we have the function. So, let me draw this like that I am just slightly redrawing it. You have this function which is small value increases and saturates. Now if you this is your f of x versus x, now what we want? We want its derivative df by dx versus x, this what we want to plot and what we know is that up to this point, the function is constant between though between these point let me call this a and this point b between a and b, it is like a straight line and then beyond that the function is 0o. So, here up to functions up to c and up to this the function as 0. So, this is the function how it would look like.

So, now we plot the derivative for this here the derivative is 0. So, I would draw the derivative here as 0. So, I am drawing a 0 here there is a constant value. So, let us that constant some positive constant. So, let me draw some positive constant. So, it is this this is the positive constant. So, from here to here the derivative is same is a constant, let me if it is 3 3 everywhere, if it is 4 is 4 everywhere whatever be the value some positive value and here onwards again 0.

So, again 0. So, the it is almost like a stuff function if you wish suddenly changes. So, the derivative would look like this, suddenly the derivative changes at this point. So, almost like discontinuous, sudden change which is typically does not happen in nature, but even though there is a sudden change and then again suddenly falling, and then we have this. So, this if we are given this f of x, the derivative is df by dx which can be plotted like this.

Now, that we did it is easier for us to think about a growth curve, because growth curve is a smooth version of this. So, if we smoothen this curve, what would be the derivative how will it look like? Try this yourself, but we will quickly look at it.

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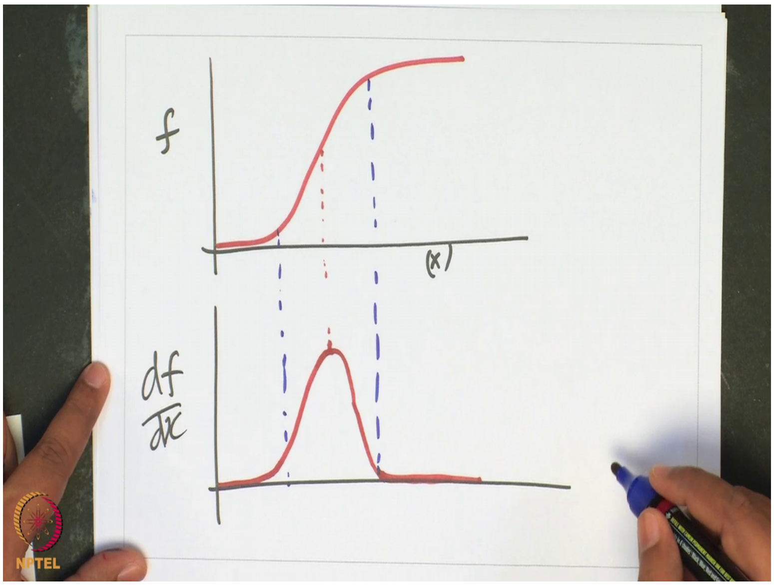

We quickly see this growth curve that a typical growth curve would look like this. So, is a smooth transition now and if we want to plot its derivative. So, this is our f and f of x and now we want df by dx to be plotted.

So, we know that roughly up to here there is almost somewhere here, even though I cannot precisely till the point somewhere here there is a transition somewhere here roughly. So, when the transition is very smooth therefore, if I plot the derivative early will be 0, but it will slowly pick up and here it will be maximum. So, somewhere here you will have maximum derivative because this is a smooth function, its not like a straight line and again it will hits a 0 here. So, it has to reduce and come down nicely to 0 and then it will stay closer 0 slowly it raise 0. So, something like this. I did not plot very nicely, but it is a smoother version of a function that we drew, which is as have we more symmetric may be there is a slight asymmetry what I plotted, but nonetheless the idea is what I described.

So, please draw yourself, look at it in and train yourself to plot the rough derivative if you have this derivative will roughly look like this the precisely how it look like? That we will learn how to precisely understand plot derivative of a function given to us, but before doing it we know while inspecting different parts for this part it has to be 0 for this part it has to be positive increasing. So, note here it is slowly increasing from 0 to a positive number therefore; it is slowly increasing from here. Similarly slowly decreasing, here the slope is slowly decreasing. So, here also slowly decreasing somewhat like here also this curve will come down. So, this is the idea.

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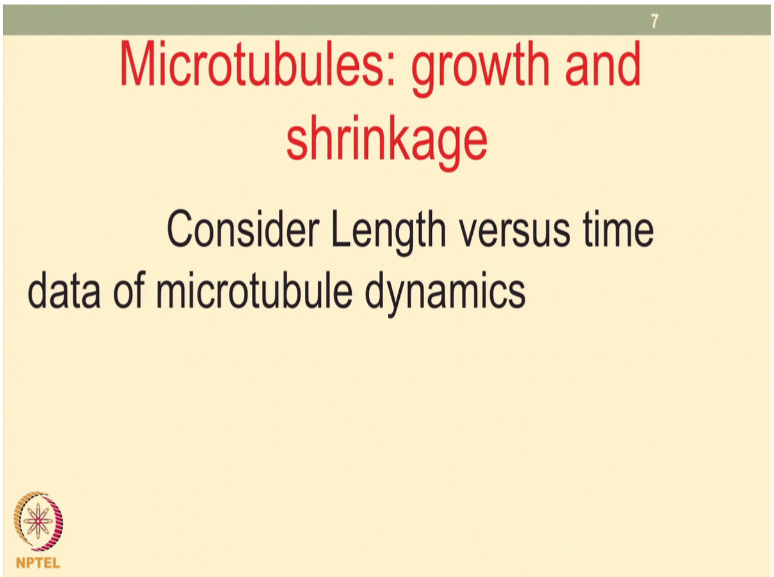

Now, let us go to the next example. So, next example is microtubules growth and shrinkage. So, many of you who has studied biology has heard of this polymer call microtubule, which is a polymer in a cytoskeletal filament in our cells, this has a property at least in (Refer Time: 21:29) experiments, when you look at microtubules what one would see is at microtubule would grow and shrink rapidly. So, they polymerize and depolymerize.

So, experiments have shown as how the length versus time data for microtubule would look like. So, if I plot the length versus time data for microtubule, typically it would look like the following.

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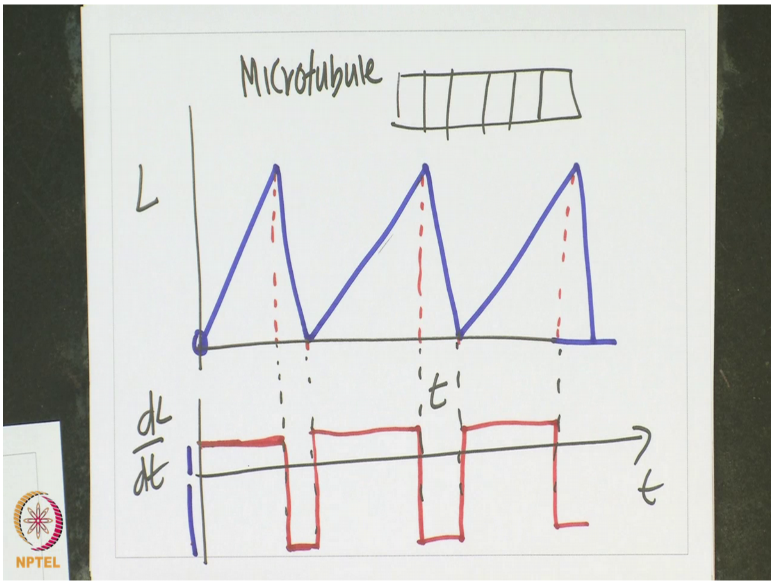

So, let me plot here length versus time data for microtubule. So, microtubule is a polymer of course. So, it has start with a small length and it grow slowly up to a point, then suddenly it shrinks almost rapidly it shrinks, then it grows slowly then pretty rapidly it shrinks, then grow slowly and shrinks rapidly.

So, this is how typical length versus time curve for a microtubule would look like. So, this are this points are called the this are called catastrophical shrinkage. So, this point is called the catastrophy, where sudden shrinkage happens and after some point it grows again and this is some time called rescue. So, this is the typical does not matter the details, even if you do not know the phenomenology of microtubule, just understand that microtubule is a polymer; microtubule its length increases with time it polymerizes, suddenly it starts depolymerizing and shrink, then it again polymerizes sudden depolymerization polymerizes depolymerizes.

So, when I say sudden depolymerization at the slow shrink, slow growth and sudden depolymerization what do I mean? When is a slow growth I mean what I am trying to say is that the length increases with time slowly; that means, takes a long time for a to polymerize, compare to this the shrinkage is very fast. So, if you look at the time here this is would be if you think take in minutes this could be in some like some many hundreds seconds for example, it could be some minutes and this is time, and here there is increase shrinkage. So, if you look at this length versus time, what we would see is that there is a slow growth. So, this takes a long time to reach and there is a sudden shrinkage this is shorter and this is long this is short long short.

Now, if you look at this in plot its derivatives. So, here we could plot is derivative. So, what I am going to plot? I am going to plot dL by dt versus t this what I am going to plot. So, plot dL by dt versus t. So, here there is a positive here there is a positive slope, which is a small positive number because the slope is not very big some positive number. So, I will draw a positive number, this is like a straight line.

So, I have a same slope up to this point. So, I will just extend this. So, let us extend this to this points down. So, what do we have? Up to this point we have a straight line. So, we have a constant slope, which is a slope of a straight line which is this. And there is a sudden shrinkage sudden would mean large negative number, this what sudden shrinkage would mean the slope is large compare to this it is like larger and shrinkage would means a negative slope.

So, then its a small negative number. So, big negative number compare to this, this is much bigger then there is it is a constant. So, almost like a straight line, then suddenly is a positive again. So, is again is a large negative small negative number its like this, and then again this this, this this this this. So, this is how the dL by dt would look like. So, slow would mean that this part this number is smallm which would mean slow this is much larger this this is much larger therefore, it is a fast shrinkage.

So, the training that we want to give you is basically to study to tell you to think about a given data or a given phenomena, let would be biological or any phenomena that we see around us we saw 2 examples a growth curve and a microtubule. Given this phenomena how will we draw the derivative. So, what is the meaning of the derivative? That is 2 things that we learn. So, here think about what is the meaning of dL by dt is like the rate of shrinkage and we will come to that in detail and try to understand.

So, we want to summarize what we learned now, in this.

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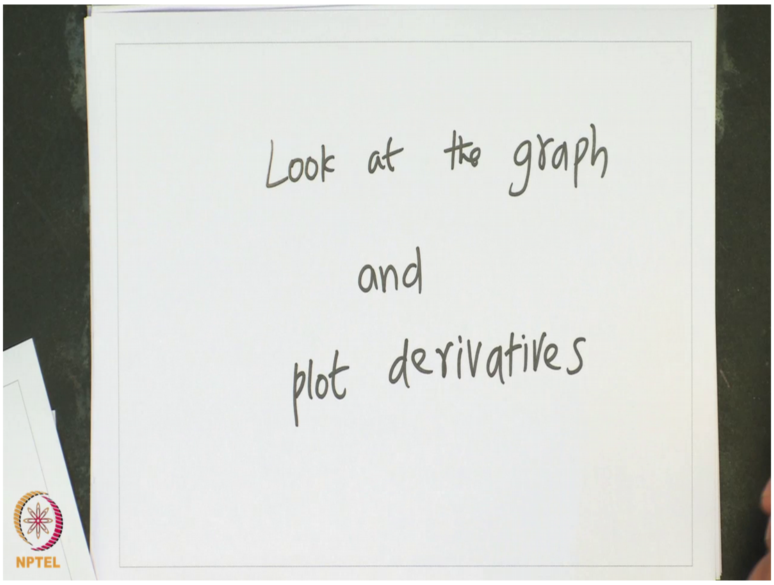

So, we learned essentially one thing, look at graph, look at the plot, look at the graph and plot derivatives. This is the thing that we learned how to plot derivative just by inspecting any graph given to you and in the coming lectures we will learn, how to precisely calculate this how to quantitatively calculate this given a data set. So, with this ill stop todays lecture.

Thank you.