Introduction to Biostatistics
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Lecture - 32
Test of Hypothesis-2
(1 tailed and 2 tailed Test of Hypothesis, p-value)

Hello and welcome to today's lecture. Today we will start discussing about testing of hypothesis. So, in testing of hypothesis as we had concluded in last class, there are 5 components of a test of hypothesis.

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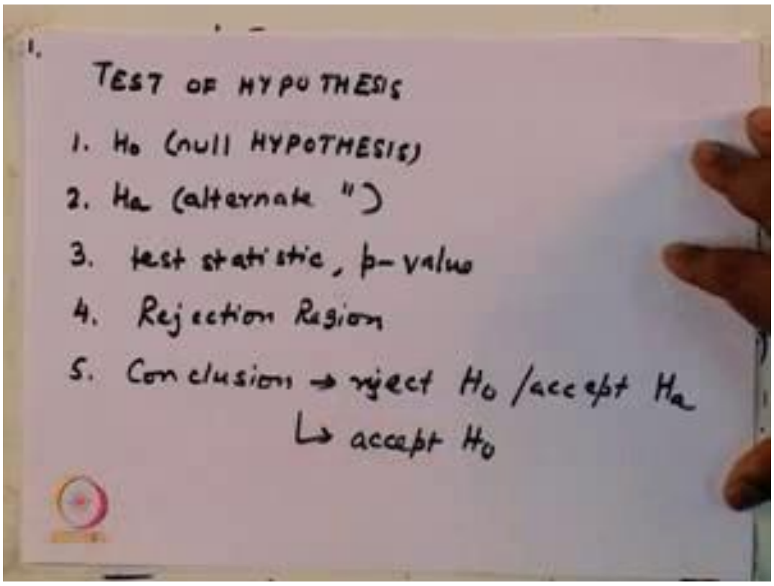

For a test of hypothesis there are 5 components - one is your H naught or null hypothesis. So, null hypothesis H naught is true, you have the alternate hypothesis which contradicts H naught is called the alternate hypothesis. You base your calculation using by calculating the test statistic and something called the p-value which will come to today you define a rejection region and last you have your draw your conclusion which means while you accept you reject H naught or you, so reject H naught accept H a or you accept H naught.

So, let us see how we go about defining H naught and H a. Imagine you want to calculate what is the average daily wage of people in Mumbai.

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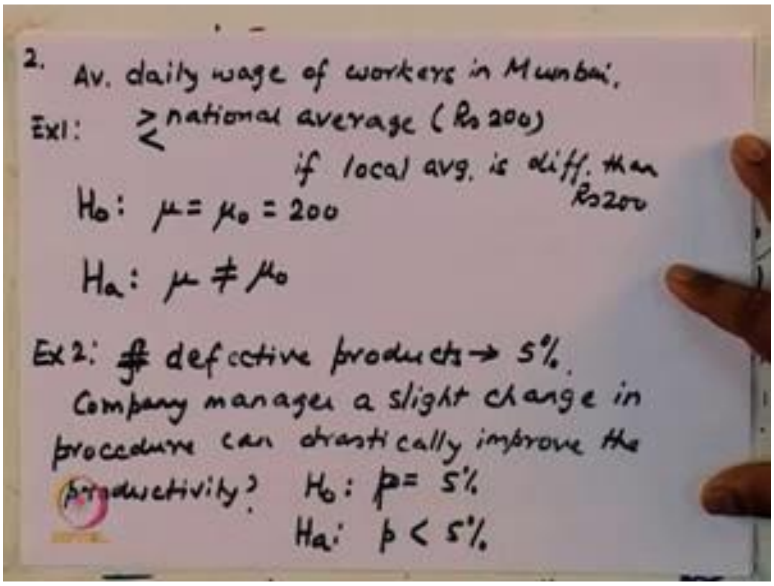

So, you want to calculate the average daily wage of workers in Mumbai and you want to come to the conclusion whether this average daily wage is greater than national or is greater or lesser than national average. And you are given that the national average is rupees 200 per day. So, when you try to proof the alternate hypothesis it is always easier to proof the alternate hypothesis by proving the null hypothesis is false.

So we always begin with a null hypothesis. So, the null hypothesis in this case is that, so we want to basically prove if local average is different. So, different means either it is greater than or lesser than the national average, is differed than rupees 200 which is the national average. So, your null hypothesis assumes that mu is equal to mu naught mu naught is equal to 200; that means, that you assume that the local average is same as the national average. And your H a H a assumes mu is not equal to mu naught. So, this is your null hypothesis and this is your alternative hypothesis. So, what you see is in this particular example we do not discriminate whether the local average is greater than the national average or the local average is lesser than a national.

We want to know whether this average is significantly different than whatever is offered nationally you can so, this is example one. You can have another example let us say, so imagine you have the number of defective products, number or fraction let us say you make the fraction of defective products is roughly 5 percent by a company in a particular line. So, the company; there is a company manager feels a slight change in procedure can drastically improve the productivity. So, by increasing productivity I mean that the number of defective products produced will be significantly less. So, in this case my H naught is mu is equal to 5 percent or I can write p equal to 5 percent and alternate hypothesis is p is less than 5 percent.

So, here I only care whether this has drastically changed whatever is the average. So, there is a difference between this and this. So, in this case I can accept as difference so long as the local average is drastically different than 200. So, answer of 100 may be drastically different or answer of 500 will also qualify as (Refer Time: 06:22). So, long it is not as 180 or 190 we can say with certainty that this is drastically different.

So, let us get back to the earlier example of national average versus local average.

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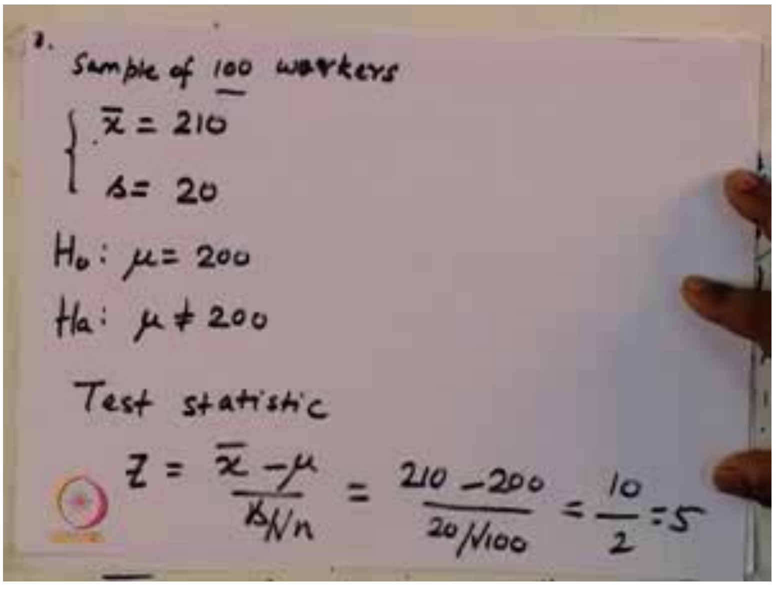

Now let us say there was a sample. So, of 100 wage owners, 100 workers give provided x bar is equal to 210 x bar is equal to 210 and s is equal to 20. So, as before let me rewrite the H naught, H naught is mu is equal to 200, H a is mu not equal to 200. So, how do I draw a conclusion whether this statistics is drastically different and the mean which can be obtained from this statistics is different than this? So, for this we use as test statistic and we know that when your sample size is large. So, n equal to 100 qualifies as reasonably large sample size then I can write that x bar is a normal distribution which standard deviation. So, we can write this test statistic or Z I can define as x bar minus mu by sigma by s by root of n. So, this Z I can compute the value x bar is 210 minus mu is

200 by s is 20 and root of 100, so you get 10, is equal to 10 divided by 2 equal to 5. So the test statistic Z gives me a value of 5.

In other words what this tells me is the value that I obtained the value that I obtained is 5 standard deviations away from the mean.

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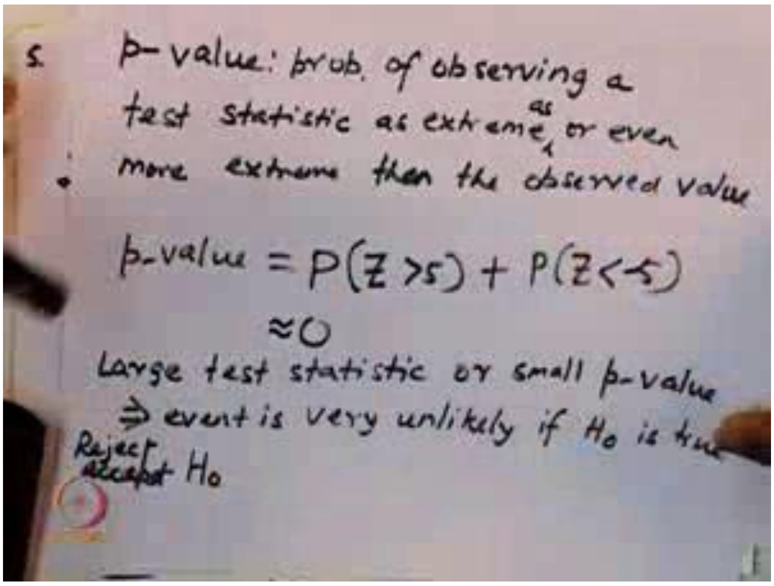

So, the p-value is taken as the probability of observing a test statistic as extreme or even more extreme then the observed value. So, this, our p-value is nothing but a probability right, so I can calculate the p-value as per this definition. So, in the above case I had obtained Z equal to 5. So, my p-value then becomes p-value is the probability of observing a test and as extreme as or even more extreme than the observed value. So, p-value is probability of Z greater than 5 or probability of Z less than minus 5 ok.

So, you know for 5 standard deviations away this value will be negligible. So, for all practical values p-value is very close to 0. So, if you have a large Z value, large test statistic or small p-value would indicate that event is very unlikely if H naught is true. So, this tells me that in this case this is a very unlikely event if the average was to be 200. So, in this particular case I can say H I would accept H naught, I will accept H naught because this probability is very low. So, between the two cases accept or reject is based by dividing the entire possibility of test statistics into two regions.

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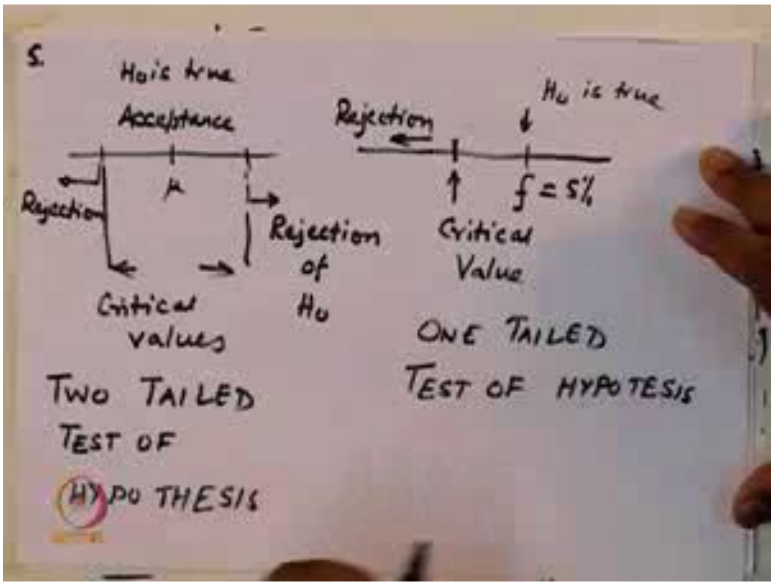

So, in the case of salary for example, so this is your mean salary you can have either less or higher. So, you can say if it is beyond this value I would reject or rejection region, rejection of H naught is either if this value is very far off and this is the acceptance region. So, this this sorry sorry, so this in the last case this has to be I have to reject H naught since this is 5 standard deviations of you forming. So, this is the acceptance region where H naught is true this assertion is accepted versus for these two points you have a rejection. So, this or these values are referred to as critical values and this test because you have two zones possible this is called a two tailed test of hypothesis.

In the other example we decided, we had discussed briefly that you have a fraction f of 5 percent which is so this corresponds to H naught is true. So, you will, you want to lower the defective pieces. So, your critical value is somewhere to the much to the left of it and this is your rejection region this is your rejection region and this is a critical value. So, this is an example of a one tailed test of hypothesis. So, these are the two possibilities.

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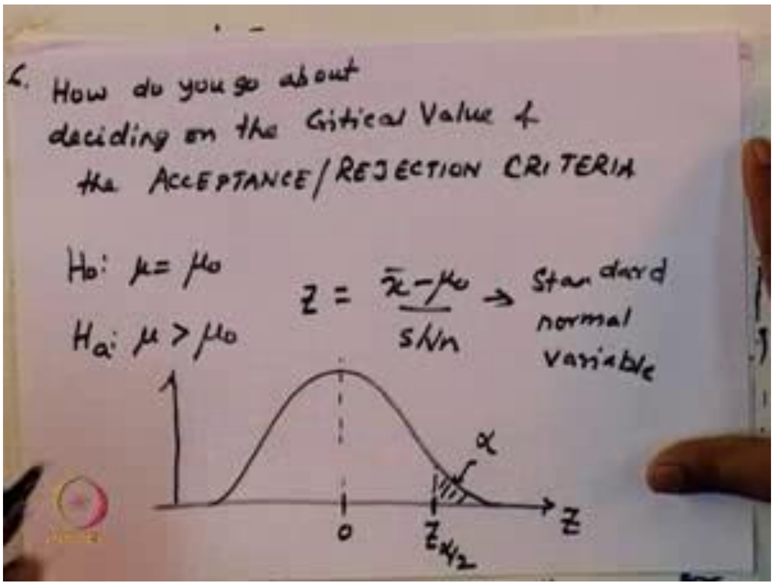

Now we want to see how do you go about deciding on the critical value and the acceptance or rejection. So, as one would expect, as one would expect these depends on how much certainty we want to have in our assertion. So, let us take a sample case. So, let us take again H naught is mu equal to mu naught and we take H a is mu greater than mu naught. So, I would go about by calculating the test statistics Z given bar x bar minus mu naught by s by root of n. So, if I would to draw the whole curve, so this is my 0 since I am plotting the Z is a standard normal variable, so I can say that if my Z is more than certain times I can this is a probability, this is the, let us say we call this as alpha or the significance level of rejection.

So, till this zone, in this particular case since mu is greater than mu naught is considered. So, this part of the curve does not even come into existence, but for this part of the curve, this alpha will stipulate a critical value of Z alpha by 2. So, depending on the confidence level that we want we can chase this value of Z of alpha by 2 as we have done for our confidence intervals.

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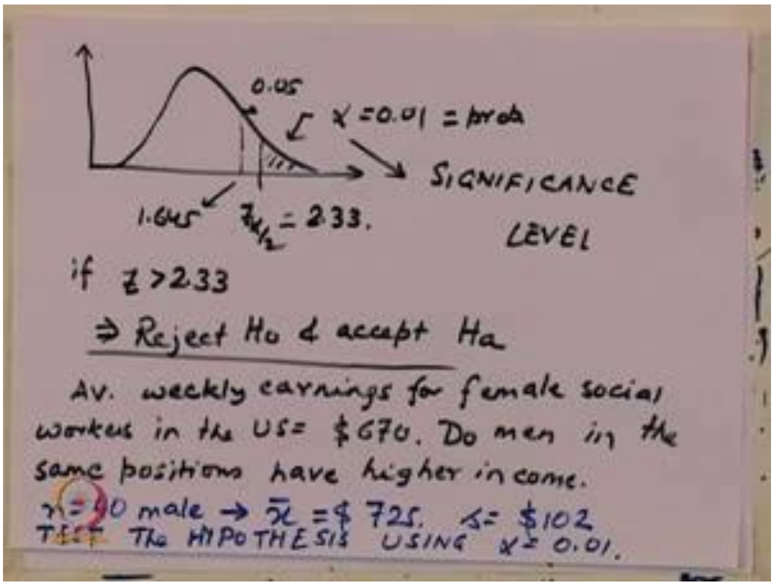

So, if I draw it again if for example, I want alpha equal to 0.01 alpha is nothing but this area under this curve. So, alpha is the probability this probability and this is the area under this curve and this is my Z of alpha by 2. So, if I choose alpha equal to 0.01 then Z of alpha by 2 becomes 2.33. So, if for the sample my Z value is greater than 2.33 then I would reject H naught I would reject H naught and accept H of a. So, alpha is nothing but the area under the curve alpha is called the significance level, so alpha is also called the significance level. So, Z of alpha by 2 is 2.33 corresponding to alpha equal to 0.01. So, similarly I can write. So, if alpha, if I want alpha is equal to 0.05 alpha equal to 0.05 I will get a value of Z alpha by 2 of, so if this is 0.05 then Z of alpha by 2 is 1.645.

So let us take an example. So, the average weekly earnings for female social workers in the US equal to dollar 670. So, the question is to find out do men in the same positions have higher income. So, random sample of n equal to 40 individuals 40 male individuals has given the statistics of x bar equal to dollar of 725 and s equal to dollar of 102. So, you want to calculate. So, test, test the appropriate hypothesis, the hypothesis using alpha equal to 0.01.

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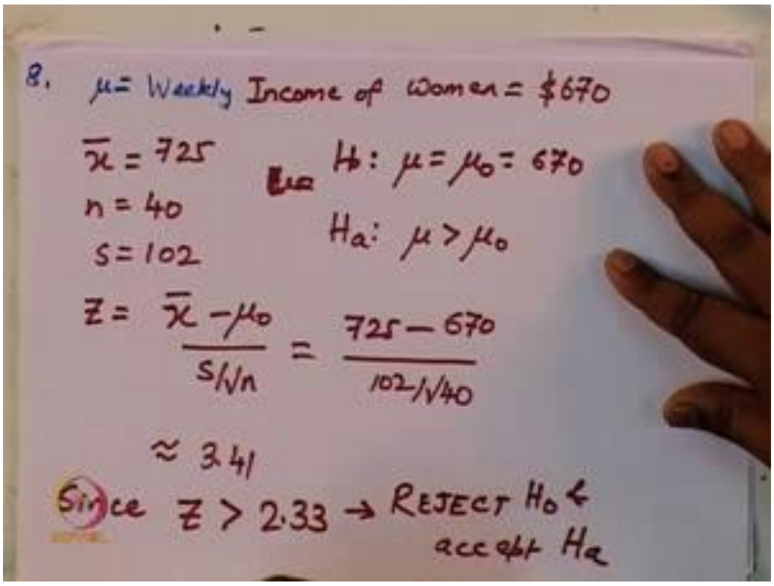

So, what you have been given is mu is equal to average weekly salaries, weekly income of women equal to dollar 670. The x bar is the salary of average weekly when of 40 males, so you have n equal to 40, s equal to 102 and x bar is 725. So, as before, as before we want to calculate the test statistics which is given by Z, before doing that let us write what is the null hypothesis. Null hypothesis is mu equal to mu naught equal to 670 and H a is mu is greater than mu naught. So, everything is based on the test statistics. So, I can find out Z equal to x bar minus mu naught by s by root of n is 725 minus of 670 by 100 and 2 by root of 40. So, the value of Z comes out to be roughly 3.41. Now for significance level of alpha of 0.01 for significance level of alpha 0.01, I have Z alpha by 2 equal to 2.33.

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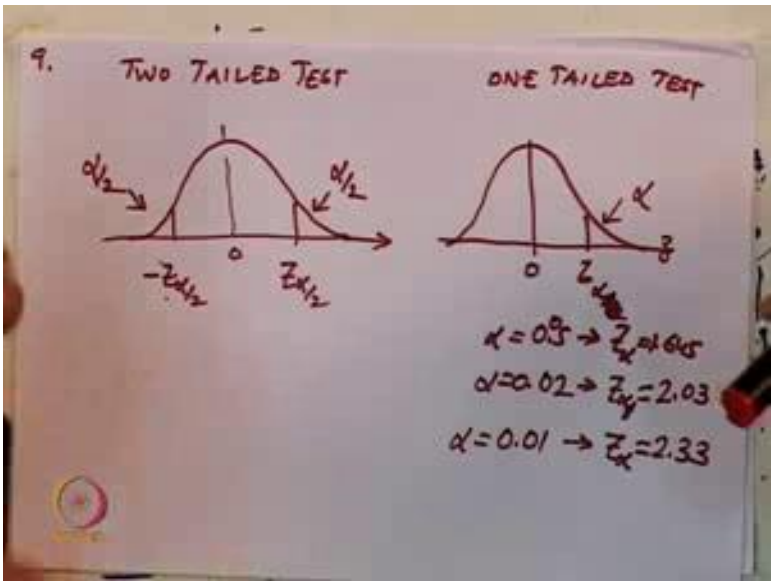

So, since Z is greater than 2.33 I can reject H naught and accept H a. So, there is a slight difference as again for a two tailed test, so for a two tailed test, for a two tailed test - 0 and for a one tailed test, you can write this. So, this area is alpha, for a one tailed test and these two areas are alpha by 2 for the two tailed test.

So this gives you, this gives you. So, for the one tailed test if I put down the values, for alpha equal to 0.05 I have Z of alpha equal to 1.645, for alpha equal to 0.05. 0.02 I get Z of alpha equal to 2.03 and for alpha equal to 0.1 I get Z of alpha equal to 2.33. So, these values for a two tailed test I refer a Z of alpha by 2 for a single test single one tail test you only represent by Z of alpha.

So, with that you can I conclude our lecture for today. We have introduced the concept of testing of hypothesis and we showed that you have 5 parts of a hypothesis testing. You start with the null hypothesis, you always write the null hypothesis as I have done here with an equality. So, null hypothesis will never end in equal to sign in this signal. So, this is an example of a null hypothesis where I write mu equal to mu naught. You have an alternate hypothesis which seems, which intends to negate or contradict a null hypothesis you can write as mu naught equal to mu naught or mu greater than mu naught or mu less than mu naught as the case may be.

And you if based on the statistics or the sample size you will probably be given the sample mean the number of you know observations in the sample, in the sample standard deviation you will go ahead and calculate the test statistics. And once you have observed the test statistics you will calculate either if your Z value is greater than either Z of alpha by 2 probability of Z of alpha by 2 and probability Z and less than minus Z of alpha by 2 that is one that is going to be alpha or in for the one tailed test probability of Z of Z greater than Z of alpha. Then you can calculate you can calculate, you will see the value of Z if it is Z is greater than Z alpha Z is less than Z alpha you can accordingly accept your null hypothesis or reject your null hypothesis when Z is greater than Z alpha.

With that I conclude today's lecture and I will continue in the next day's lecture.

Thank you.